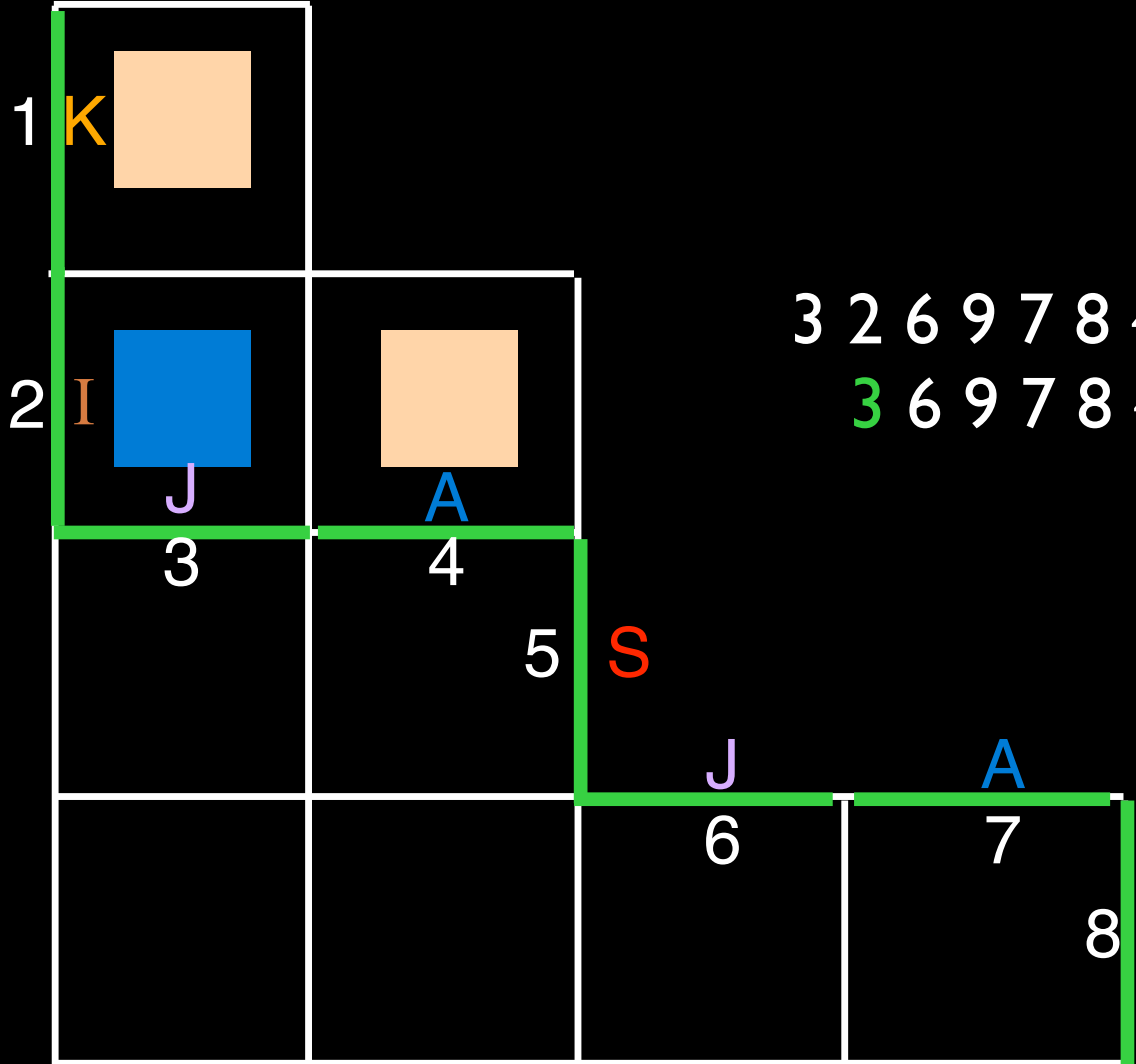
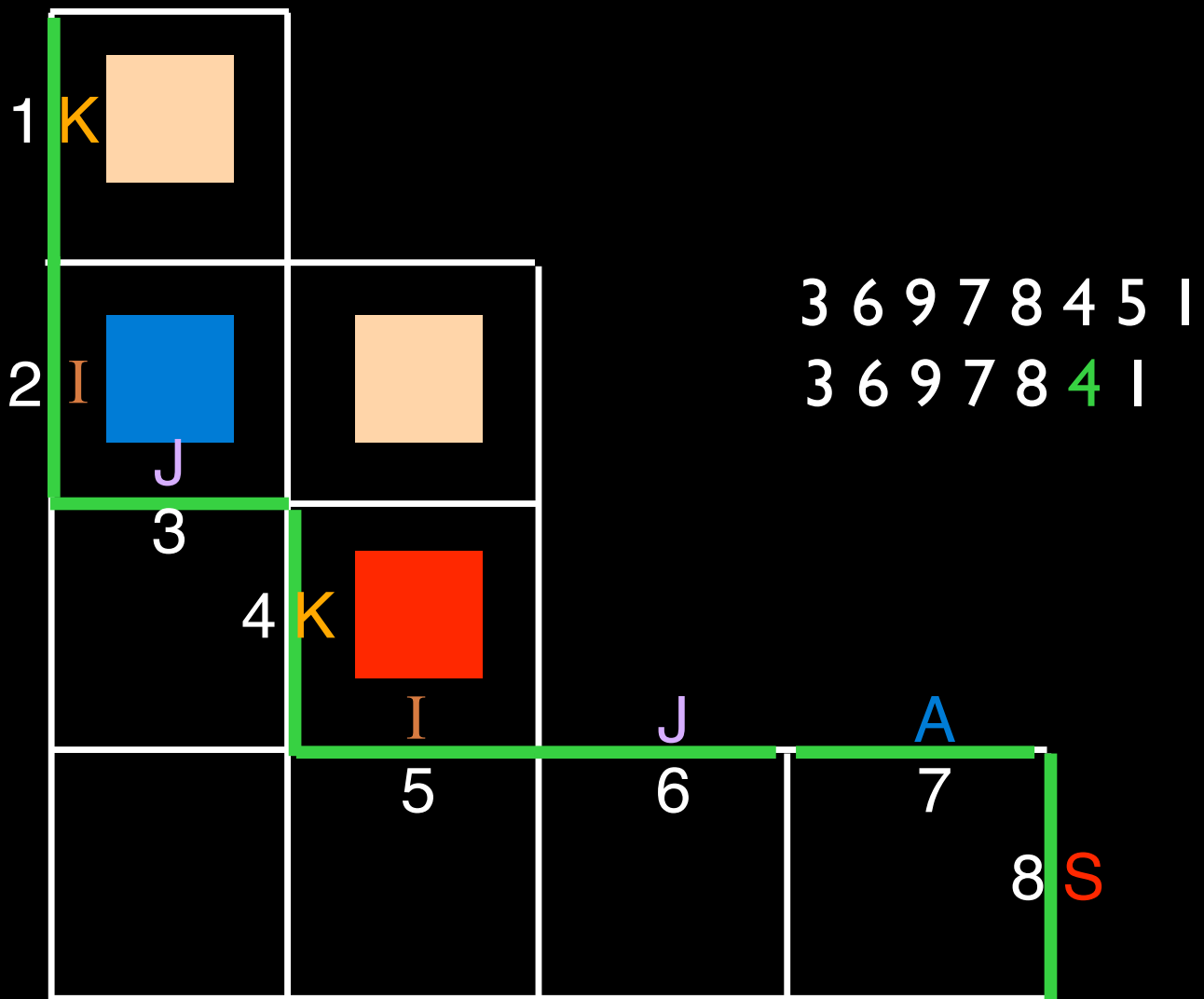
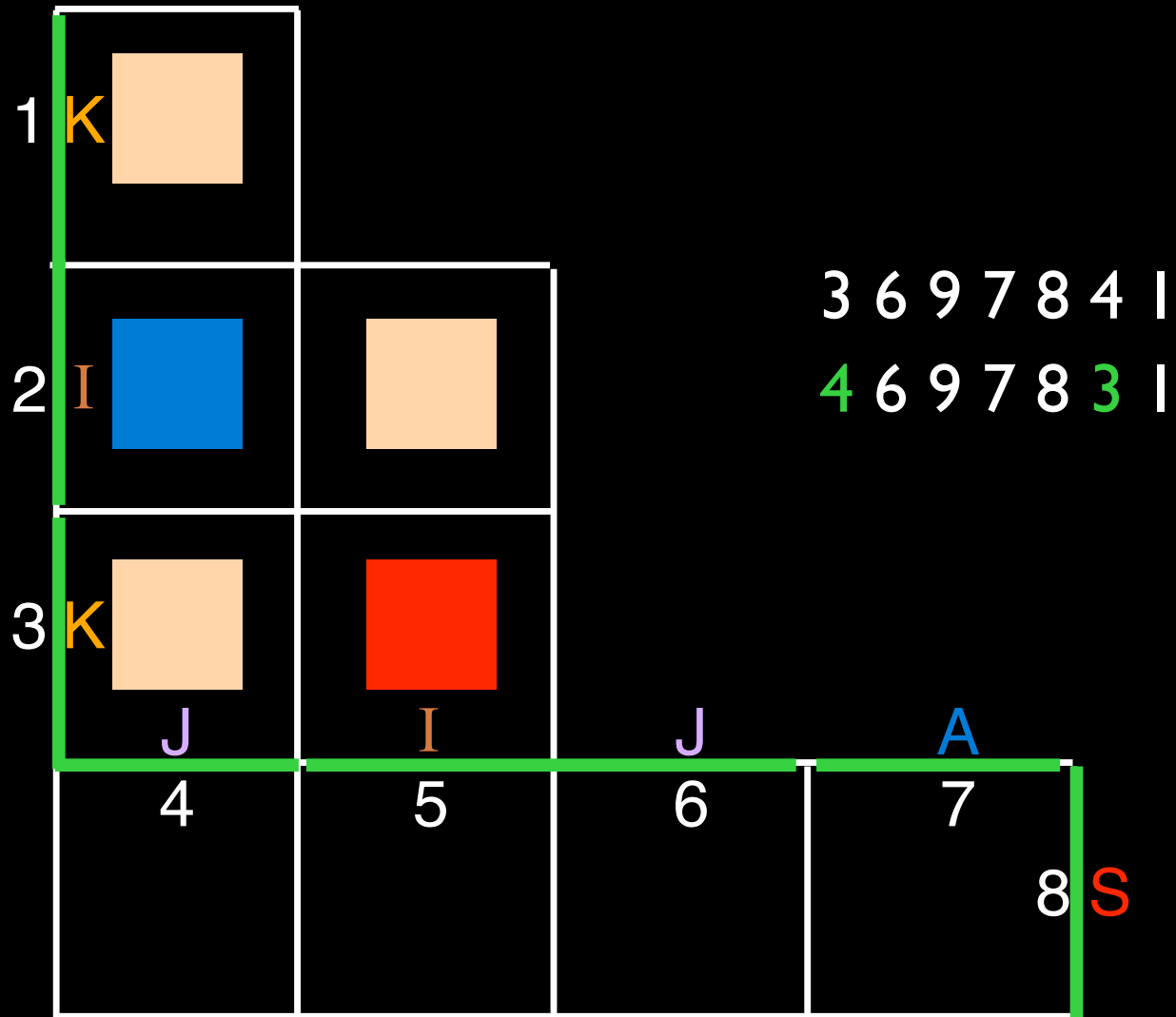


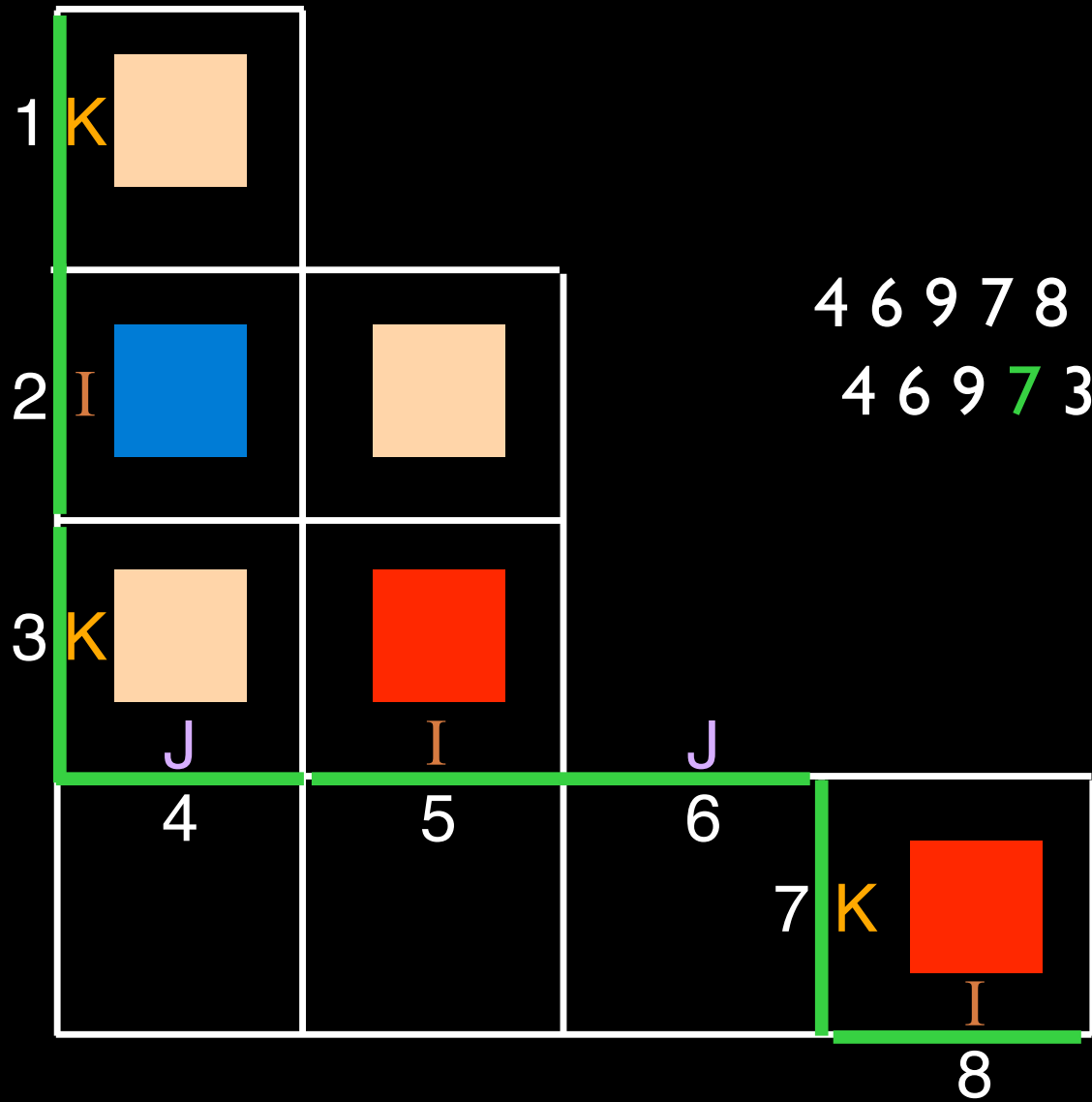
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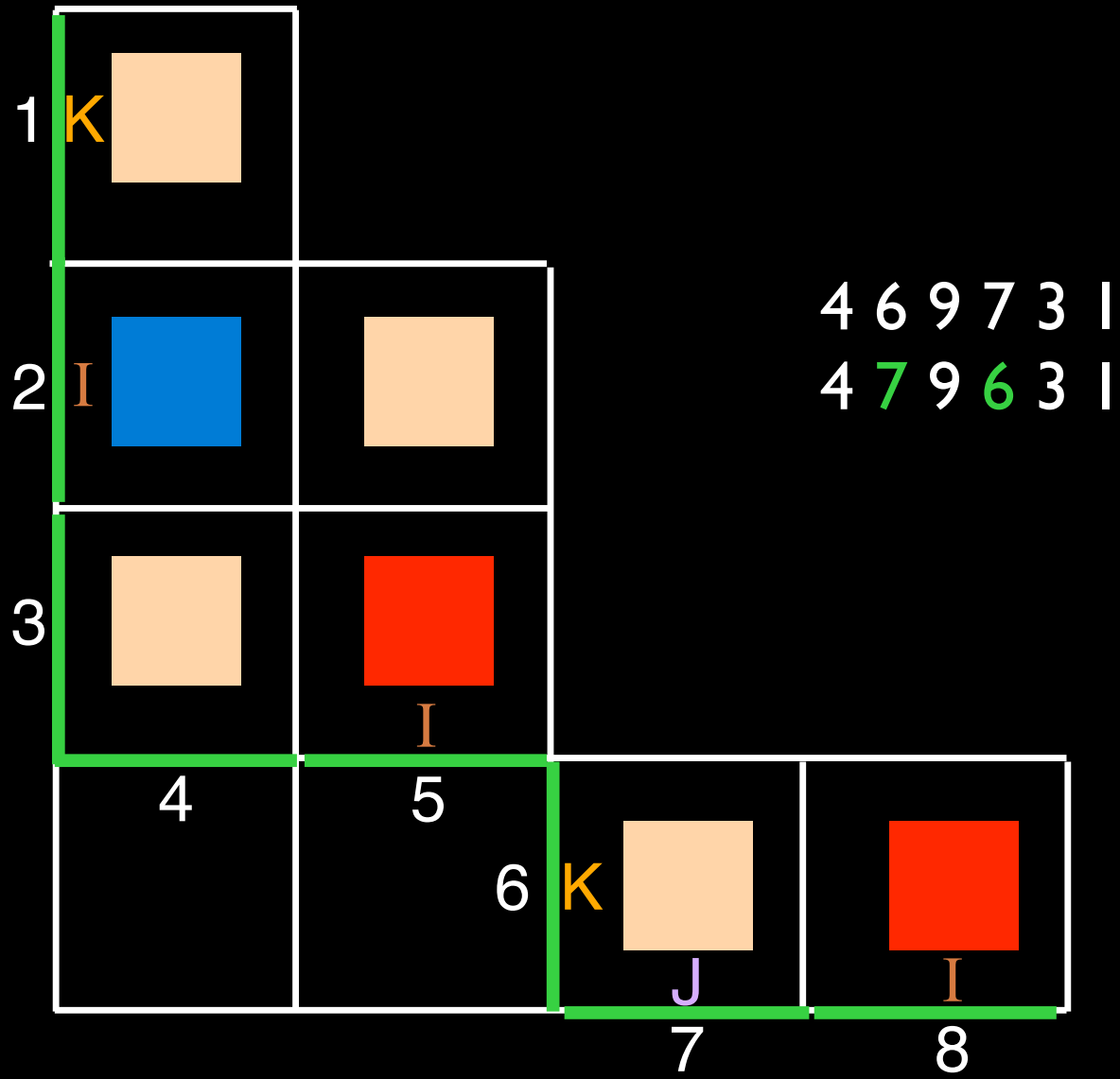
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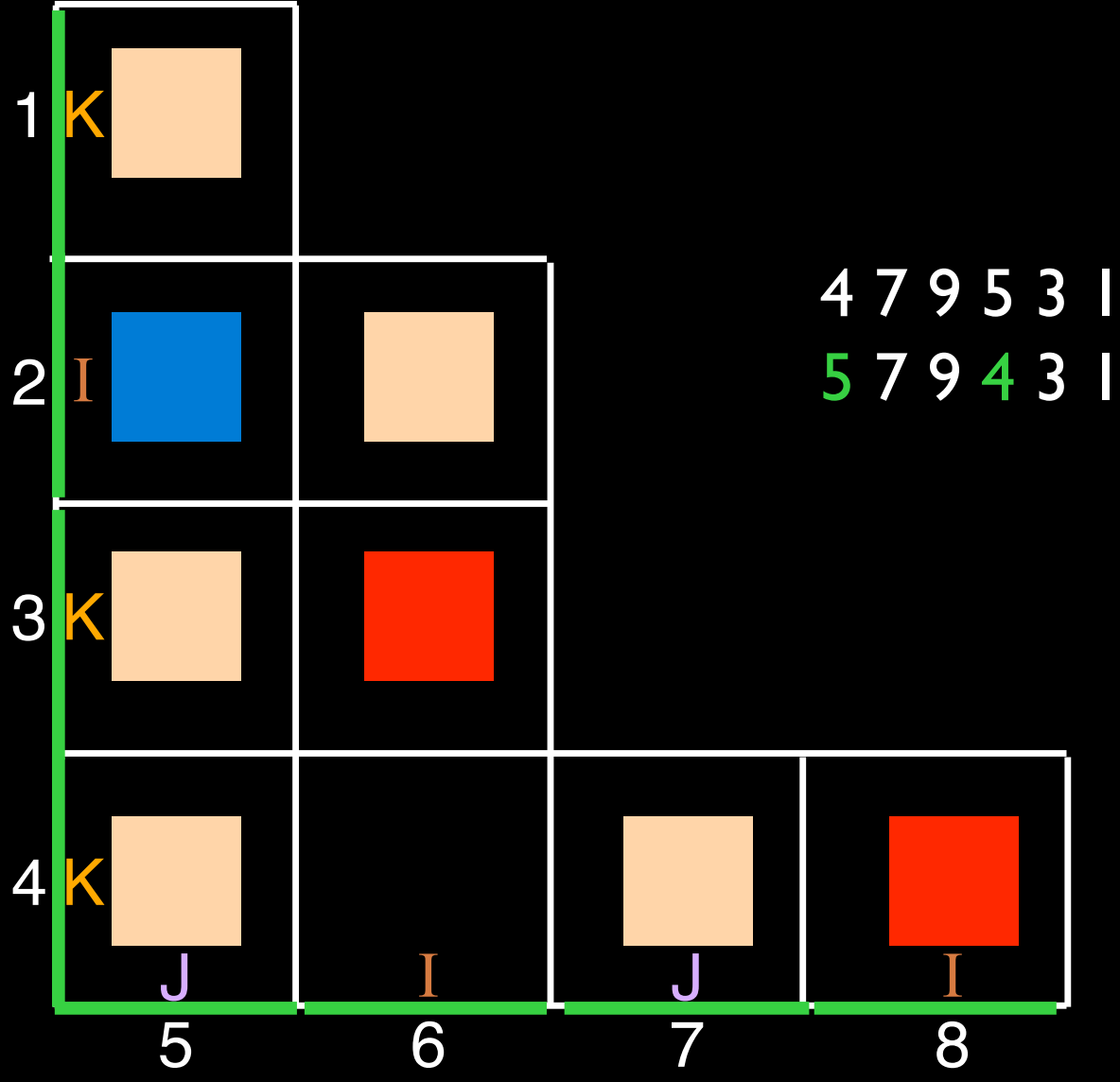


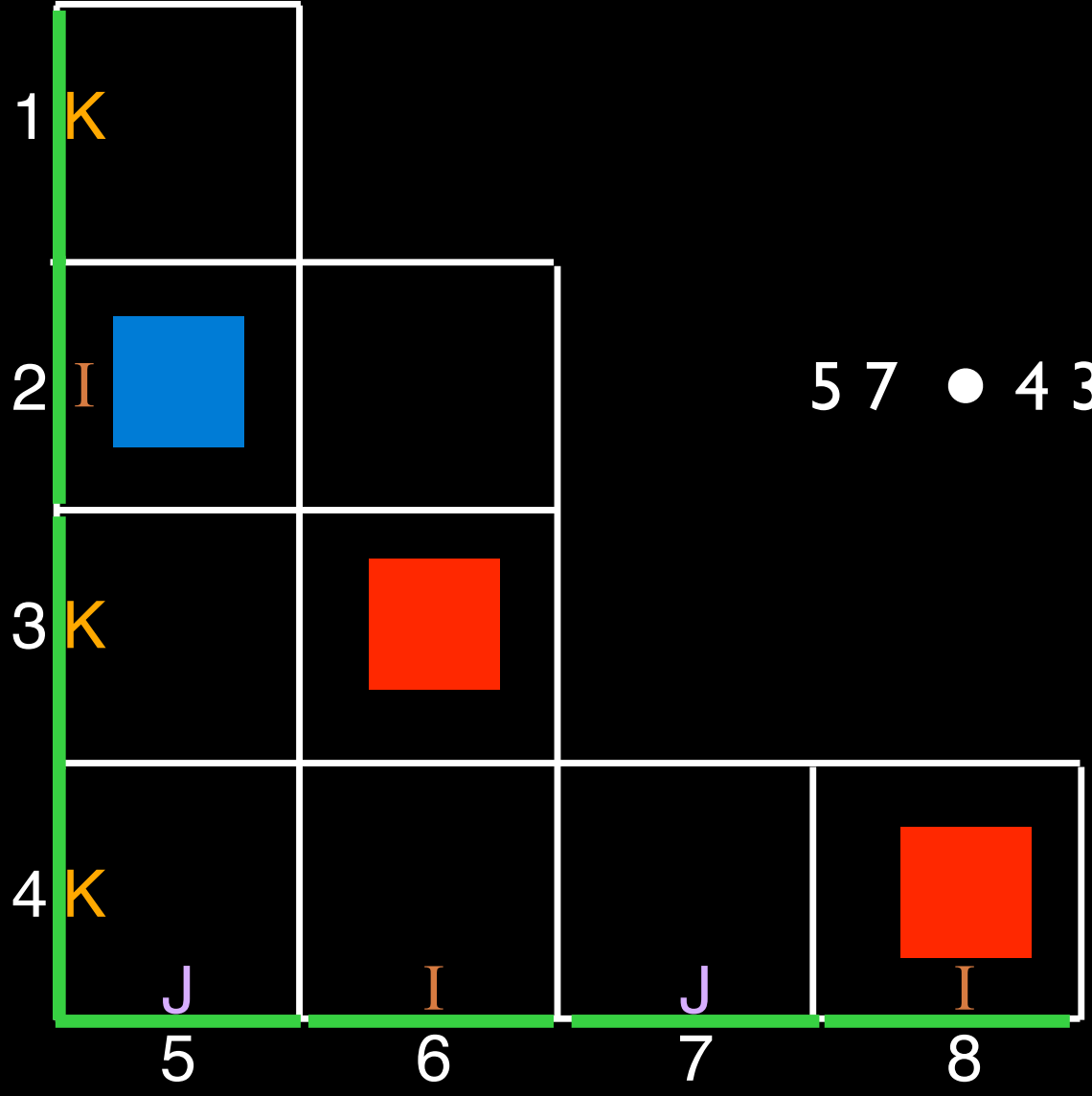
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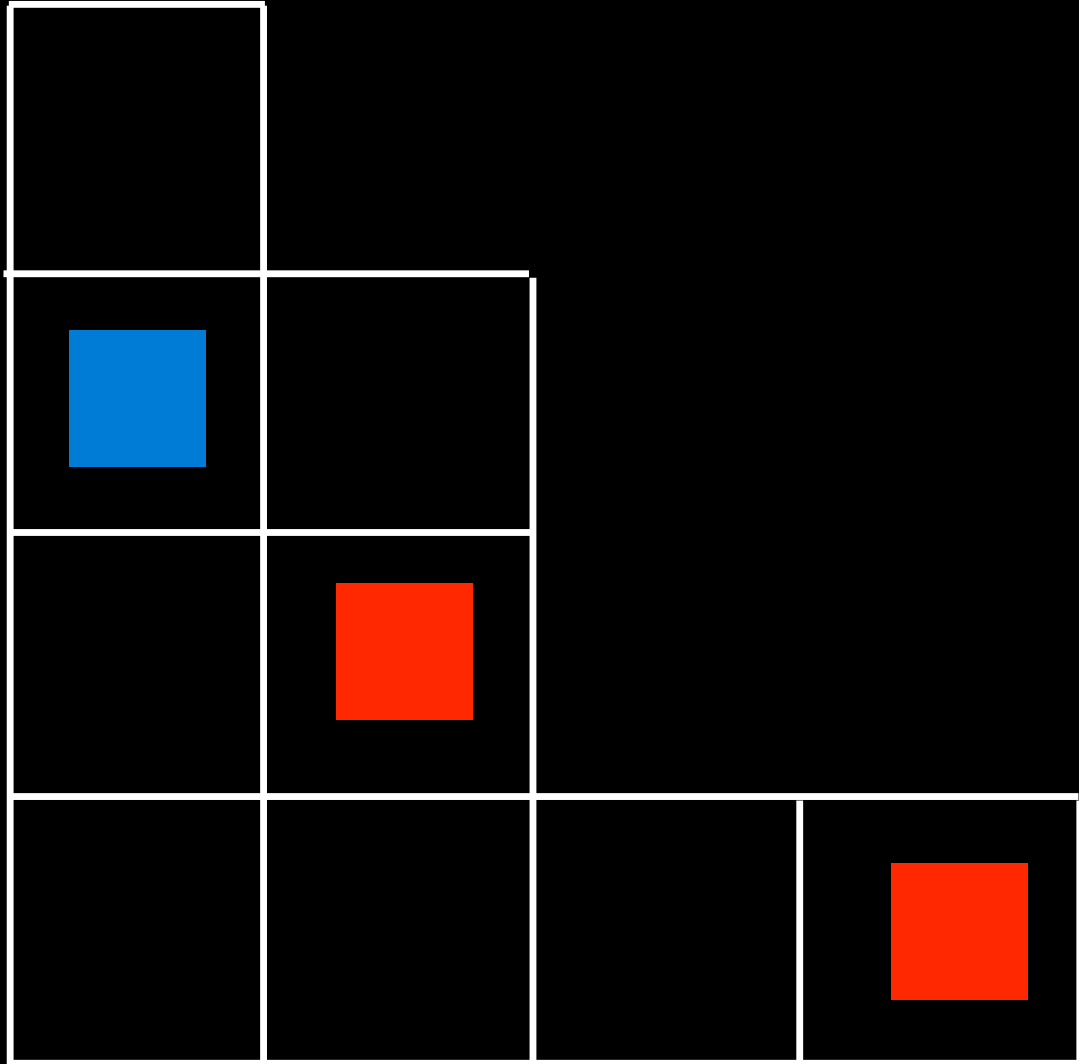


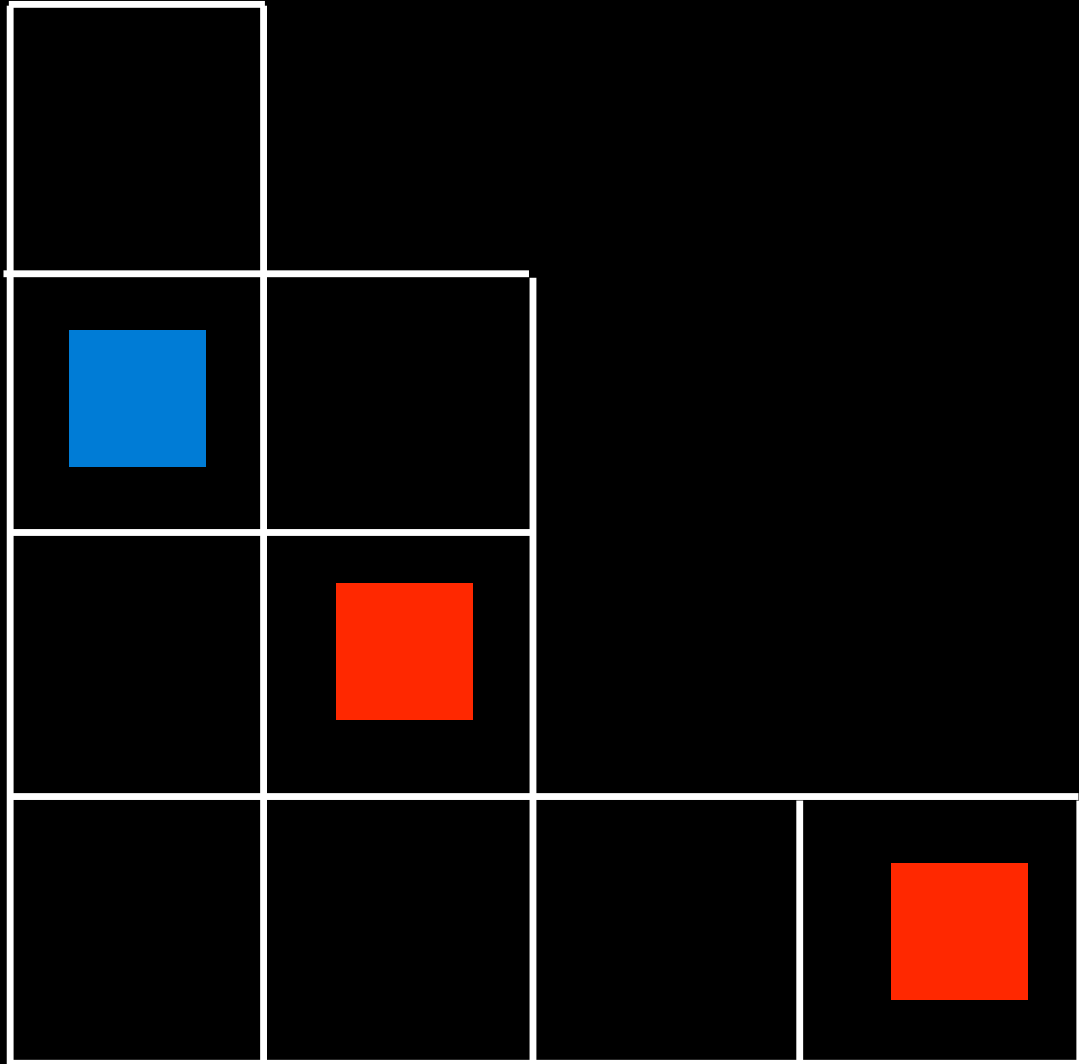


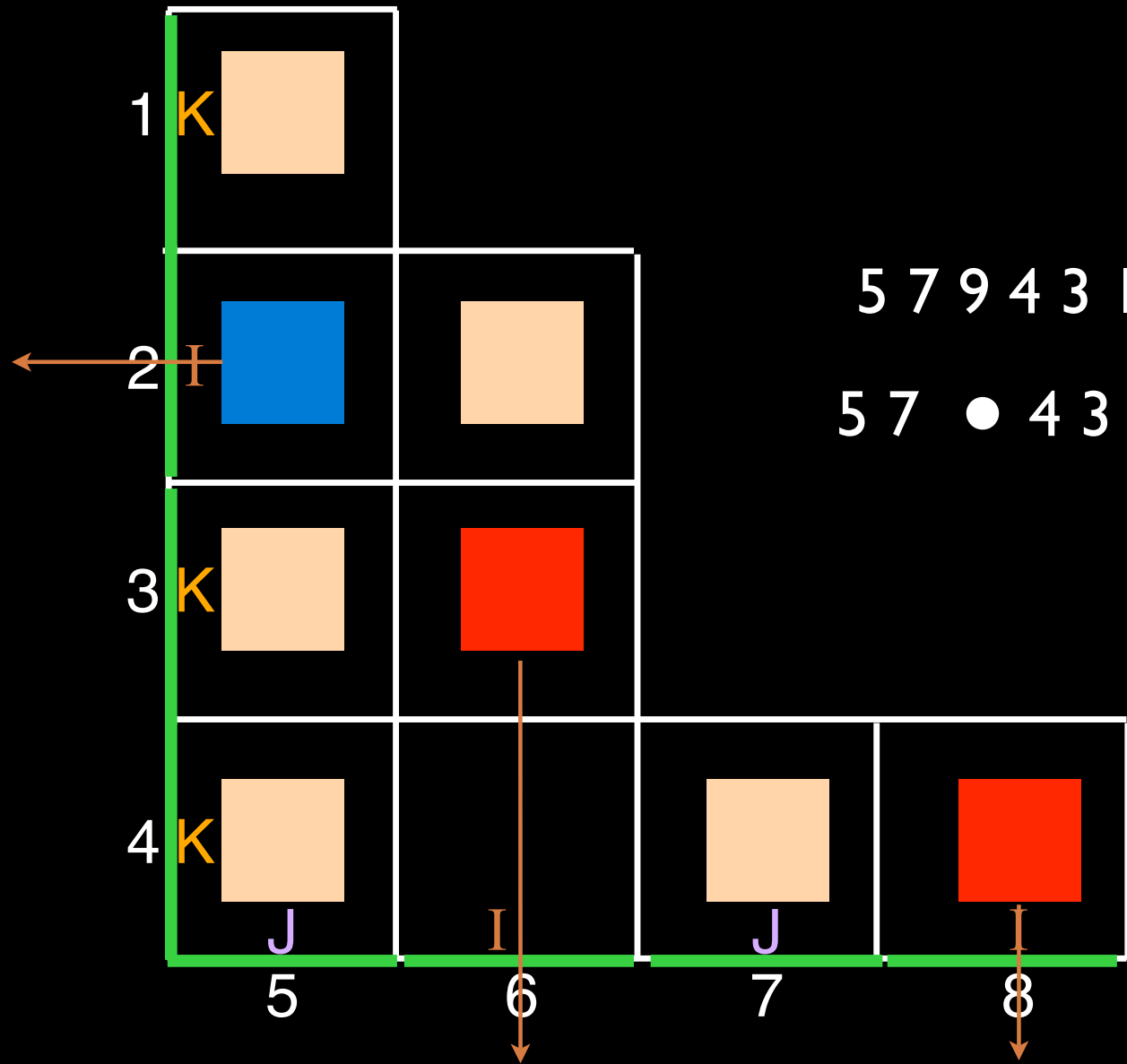




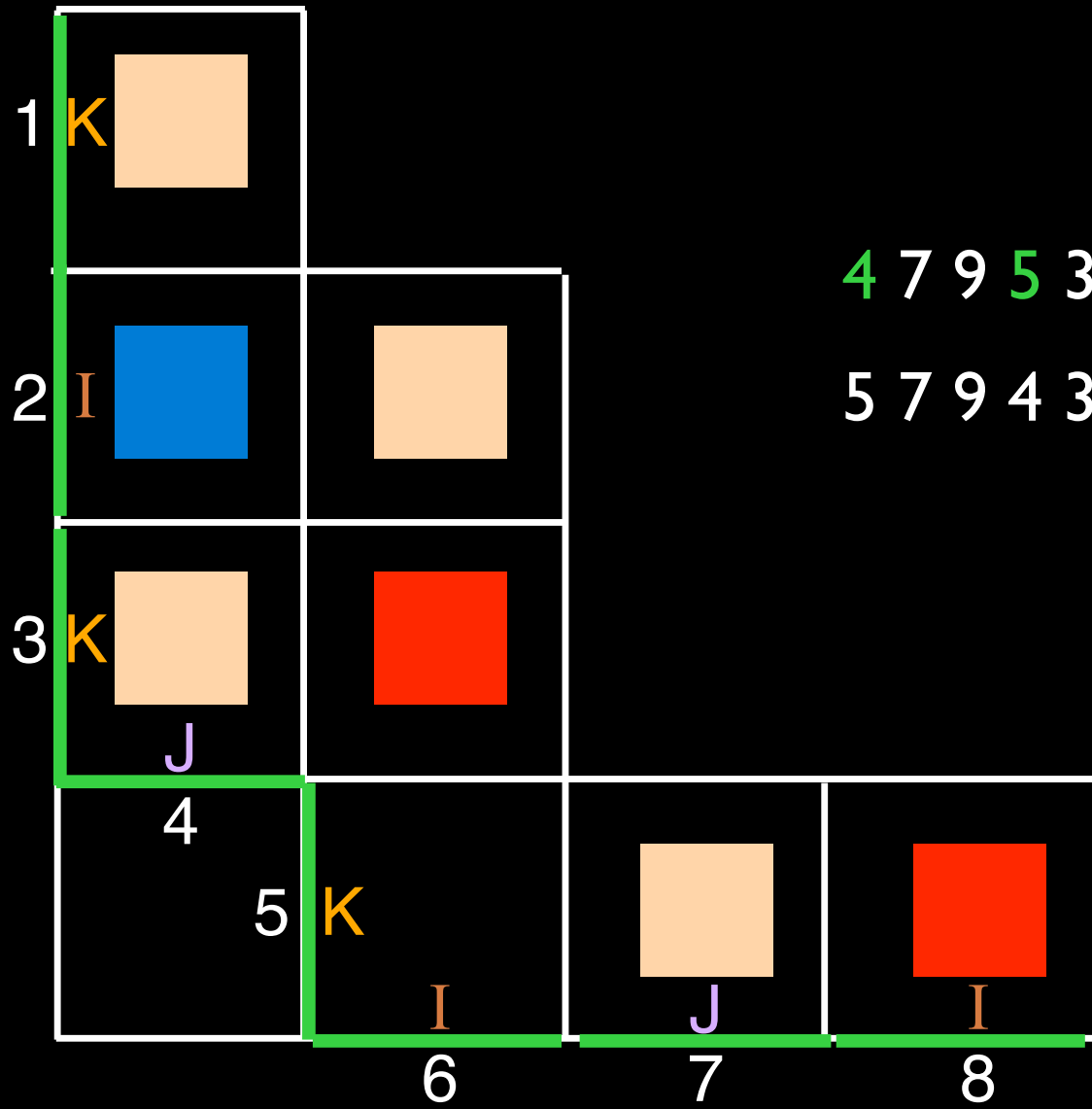






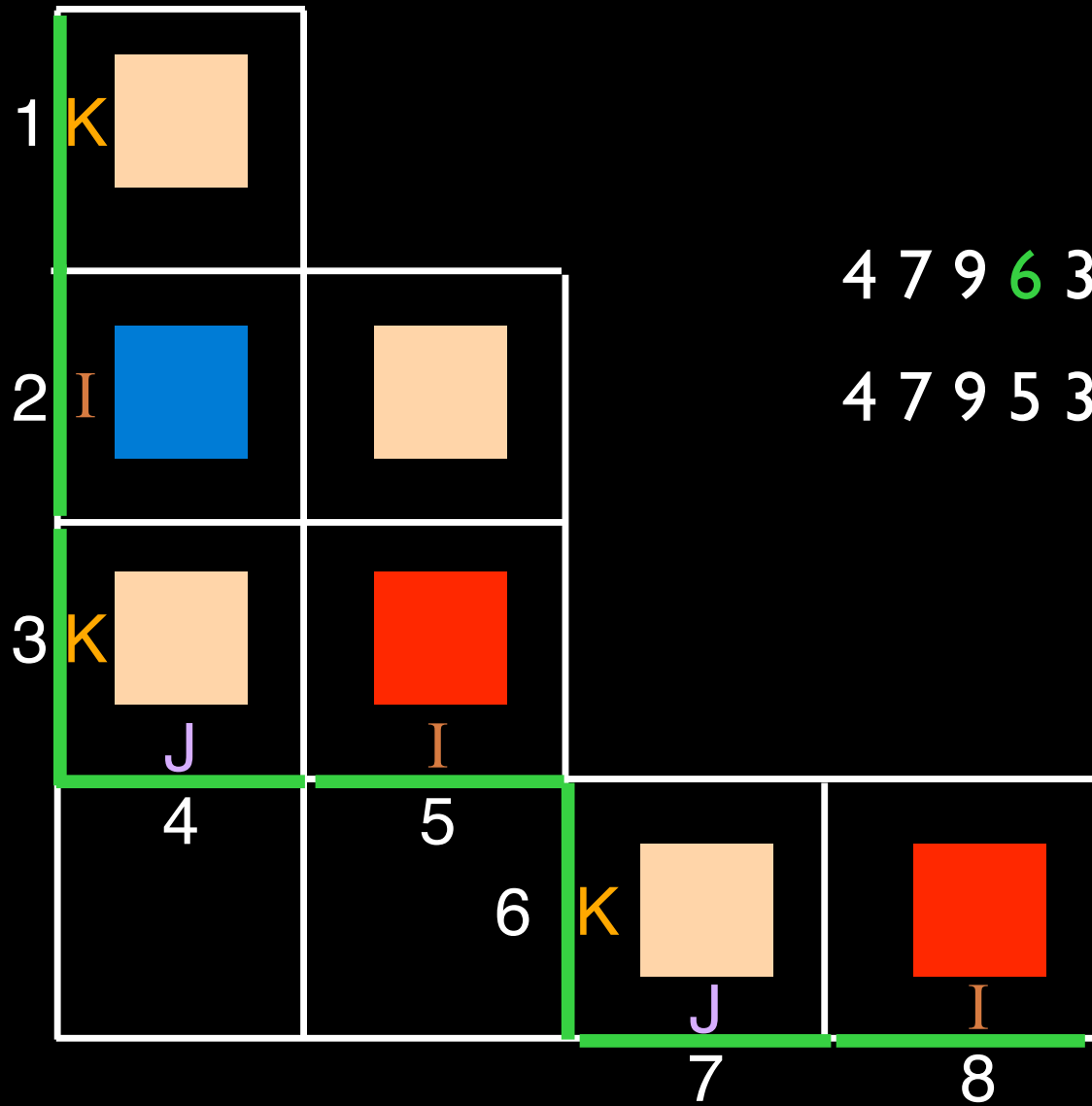


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4 7 9 5 3 1

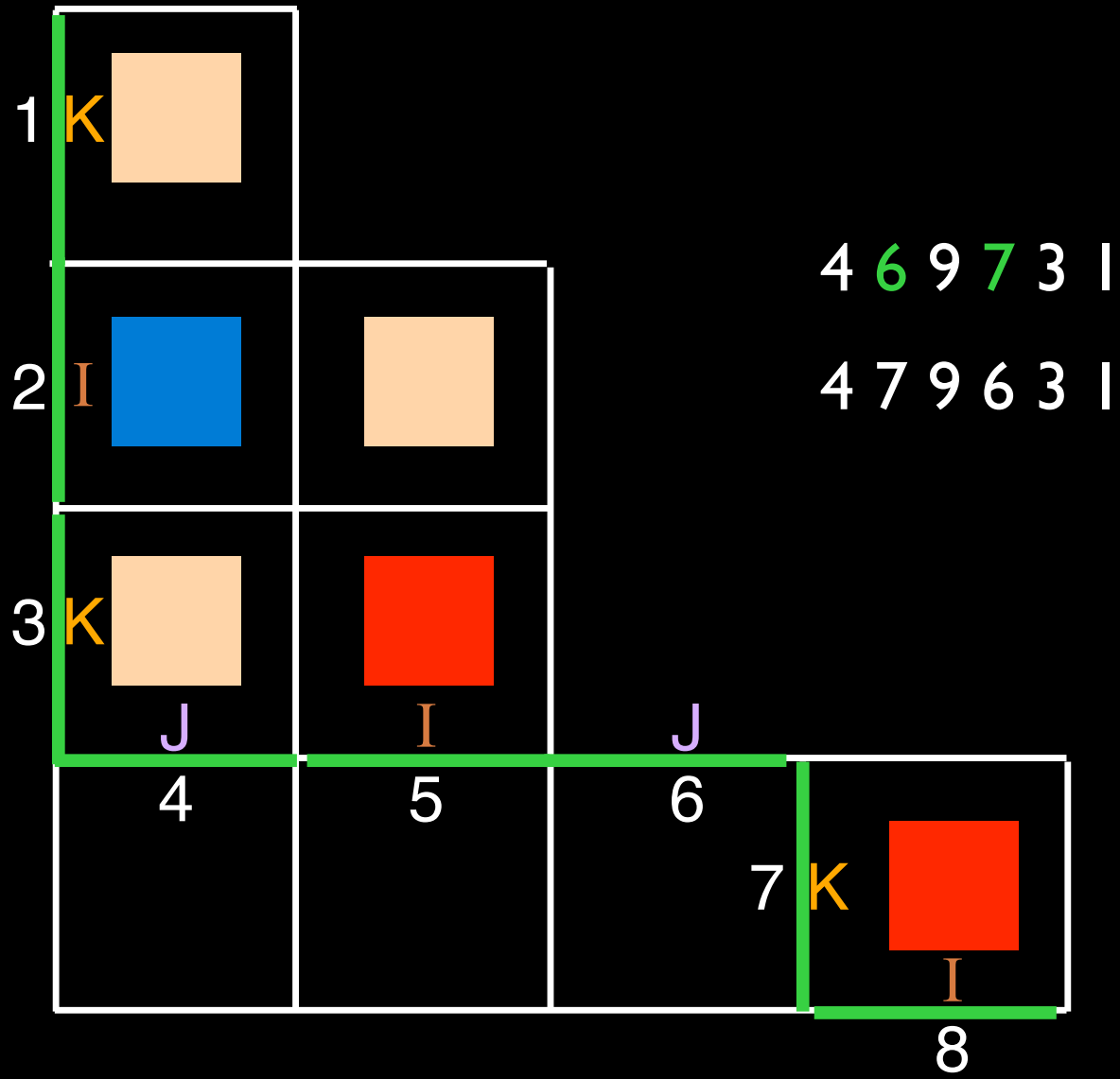
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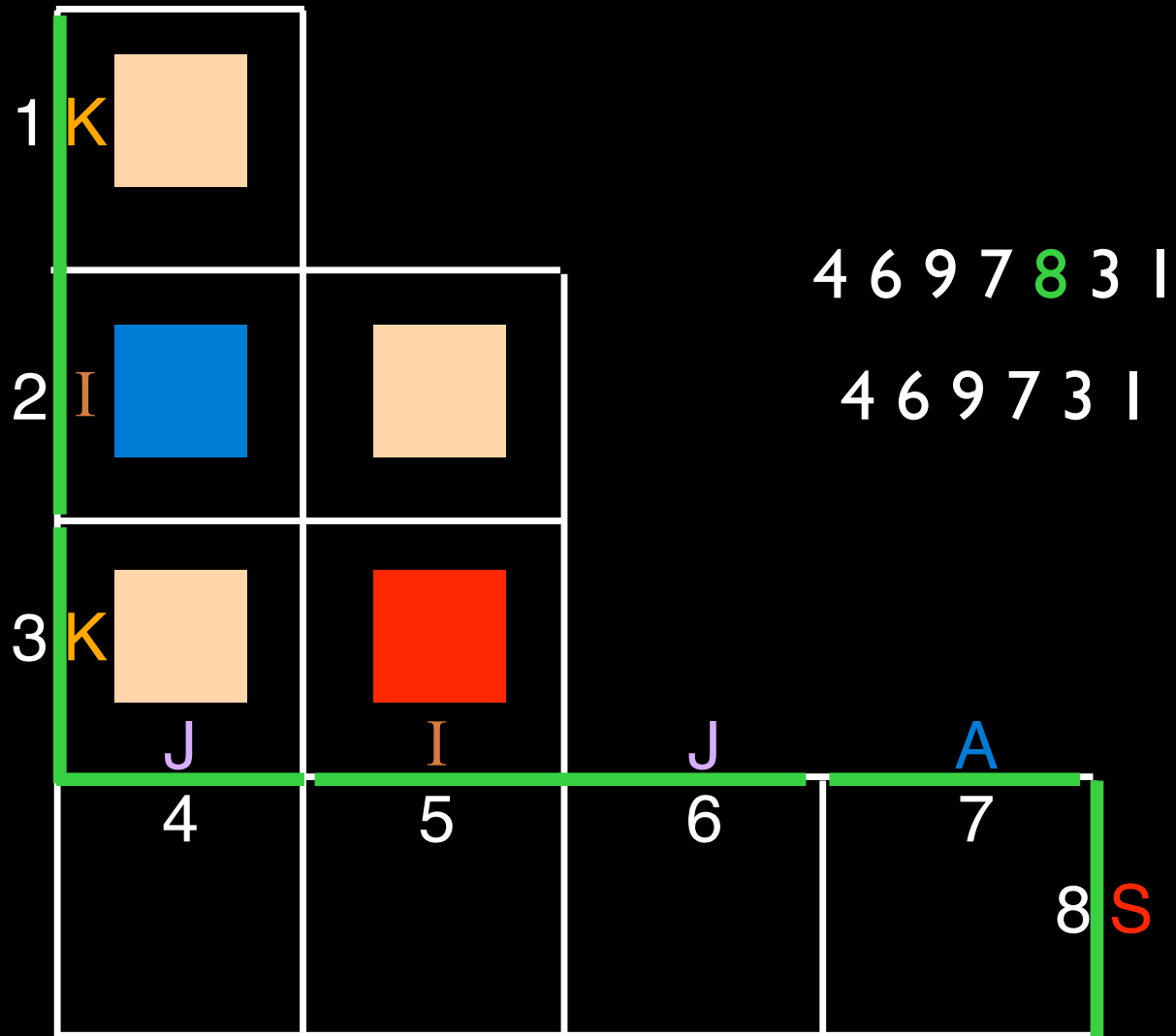
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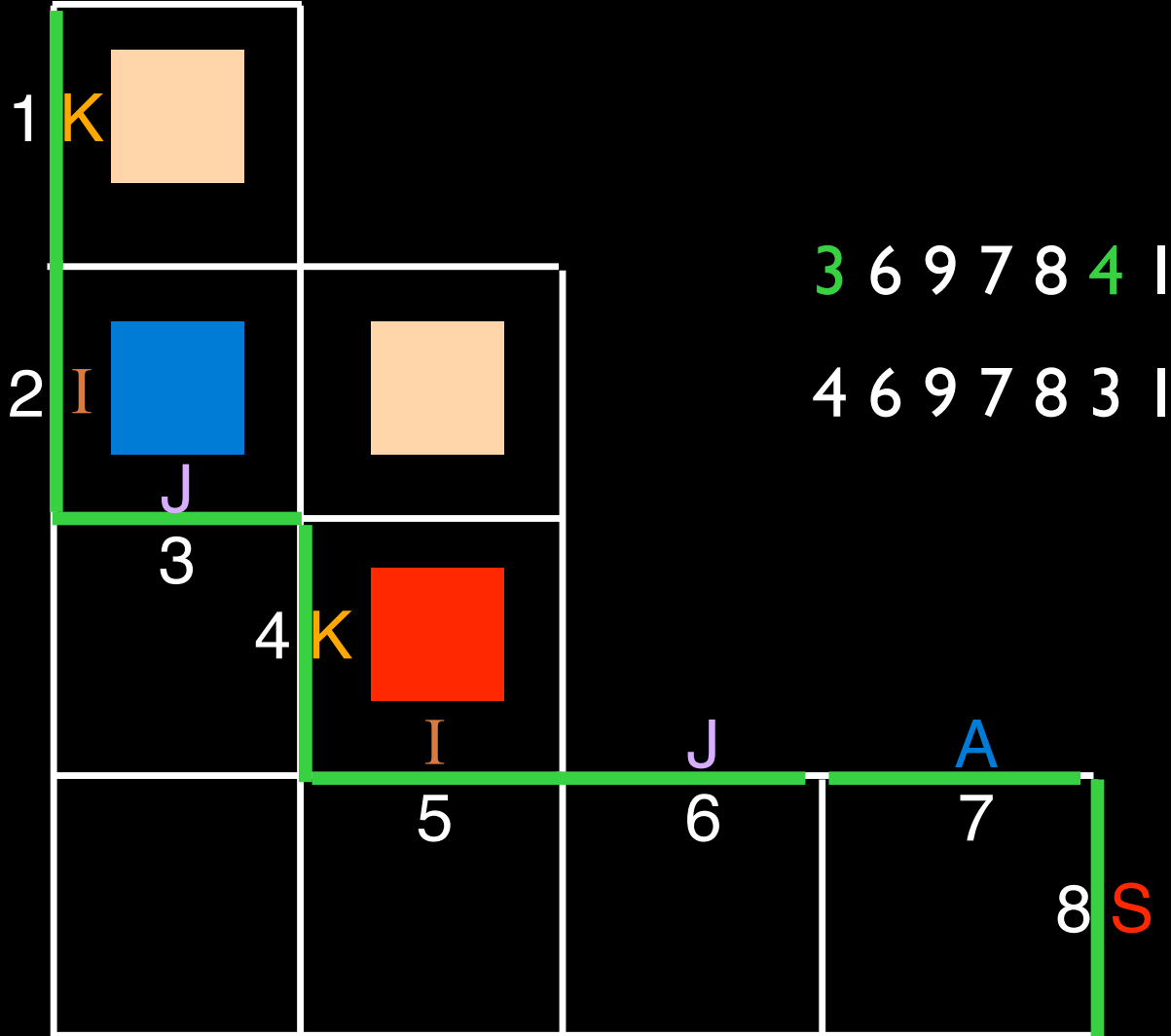


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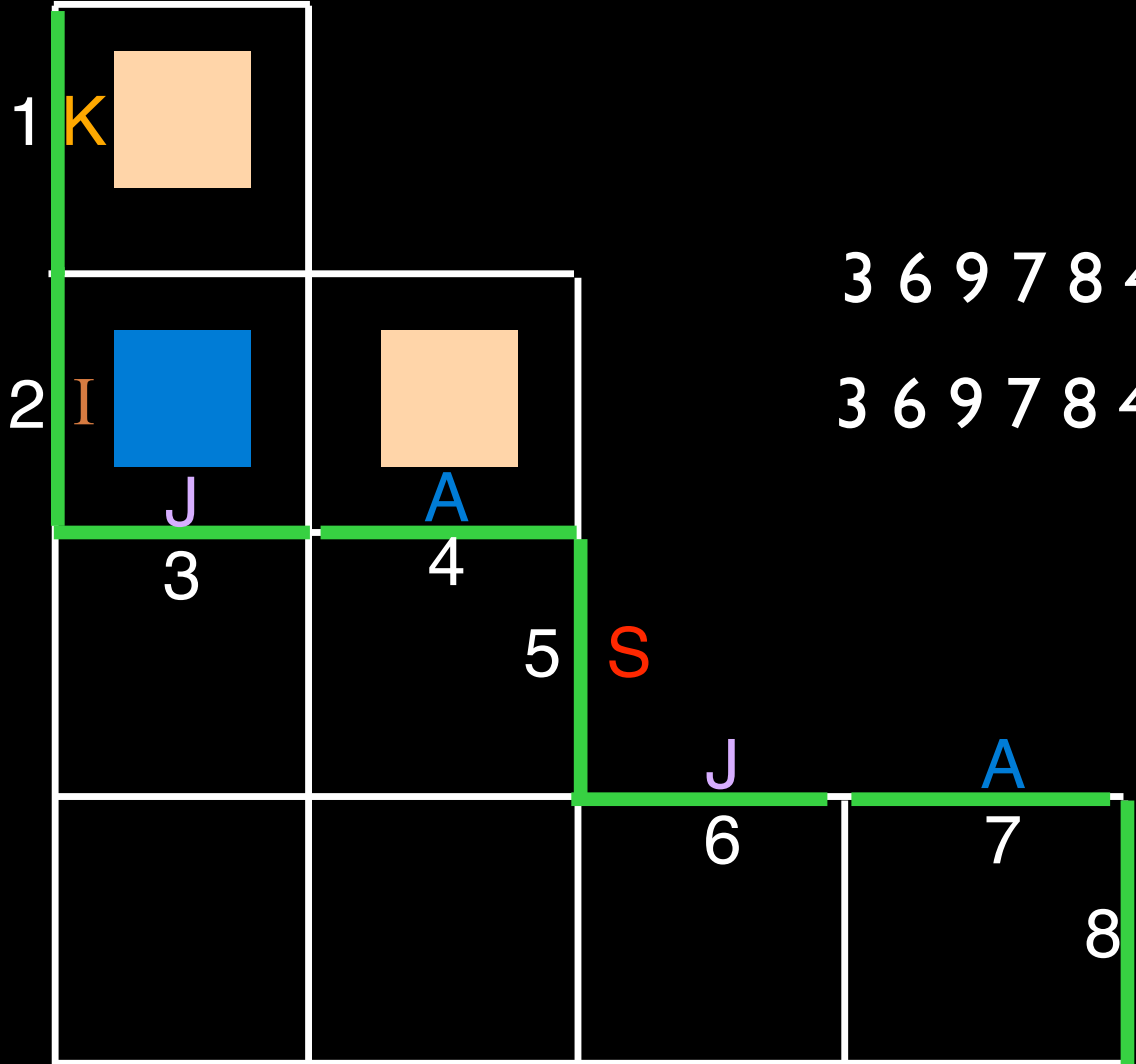


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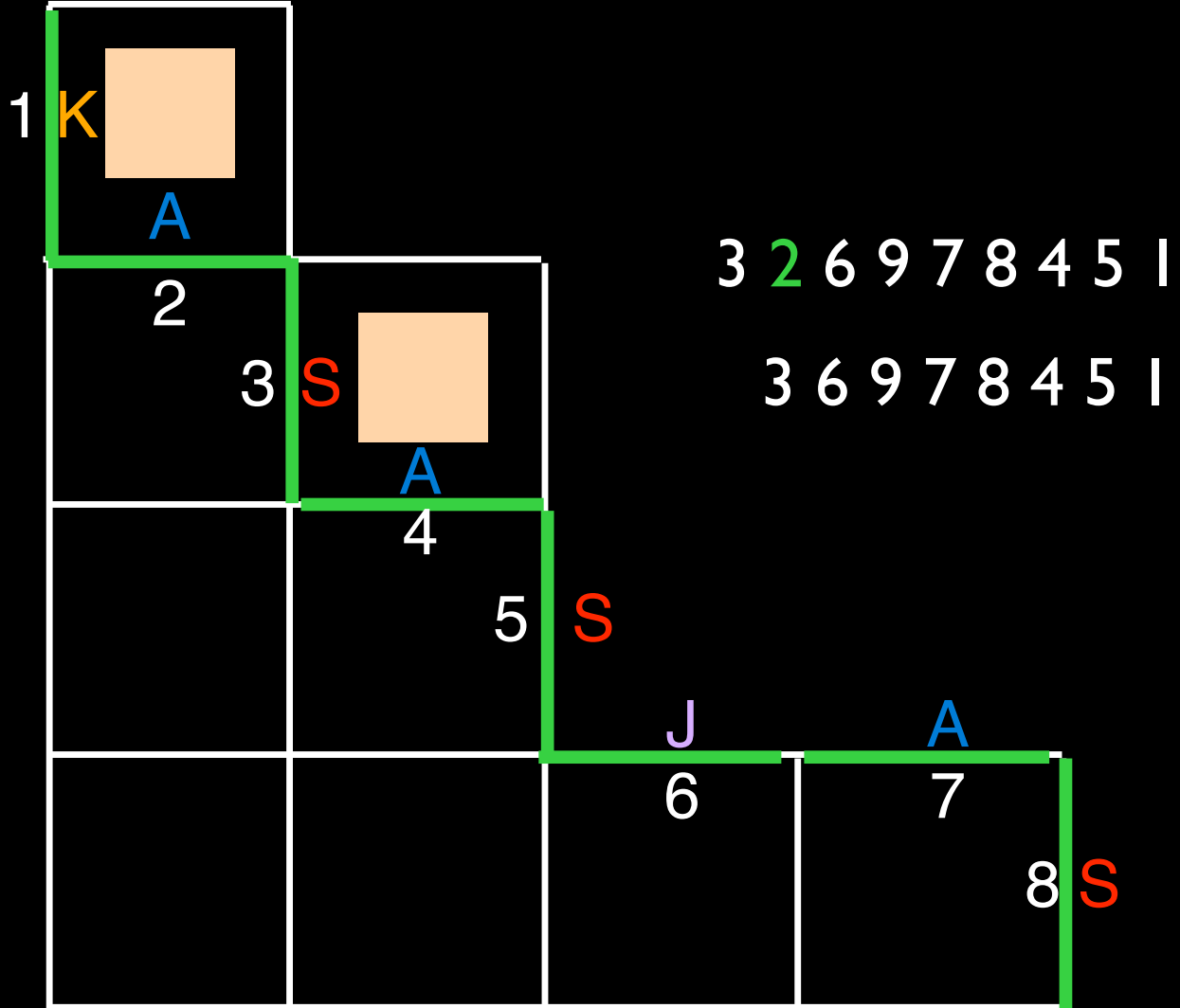
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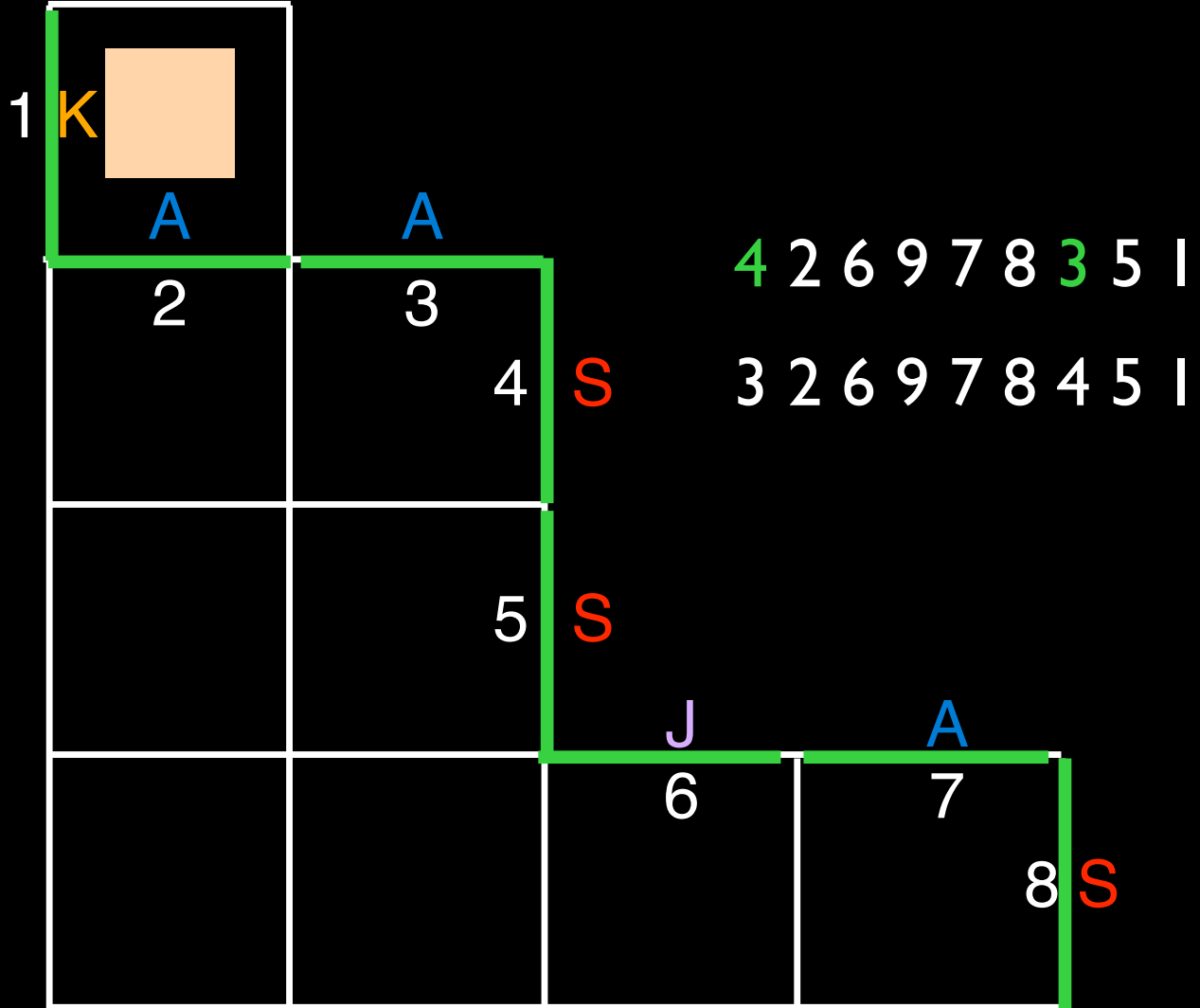


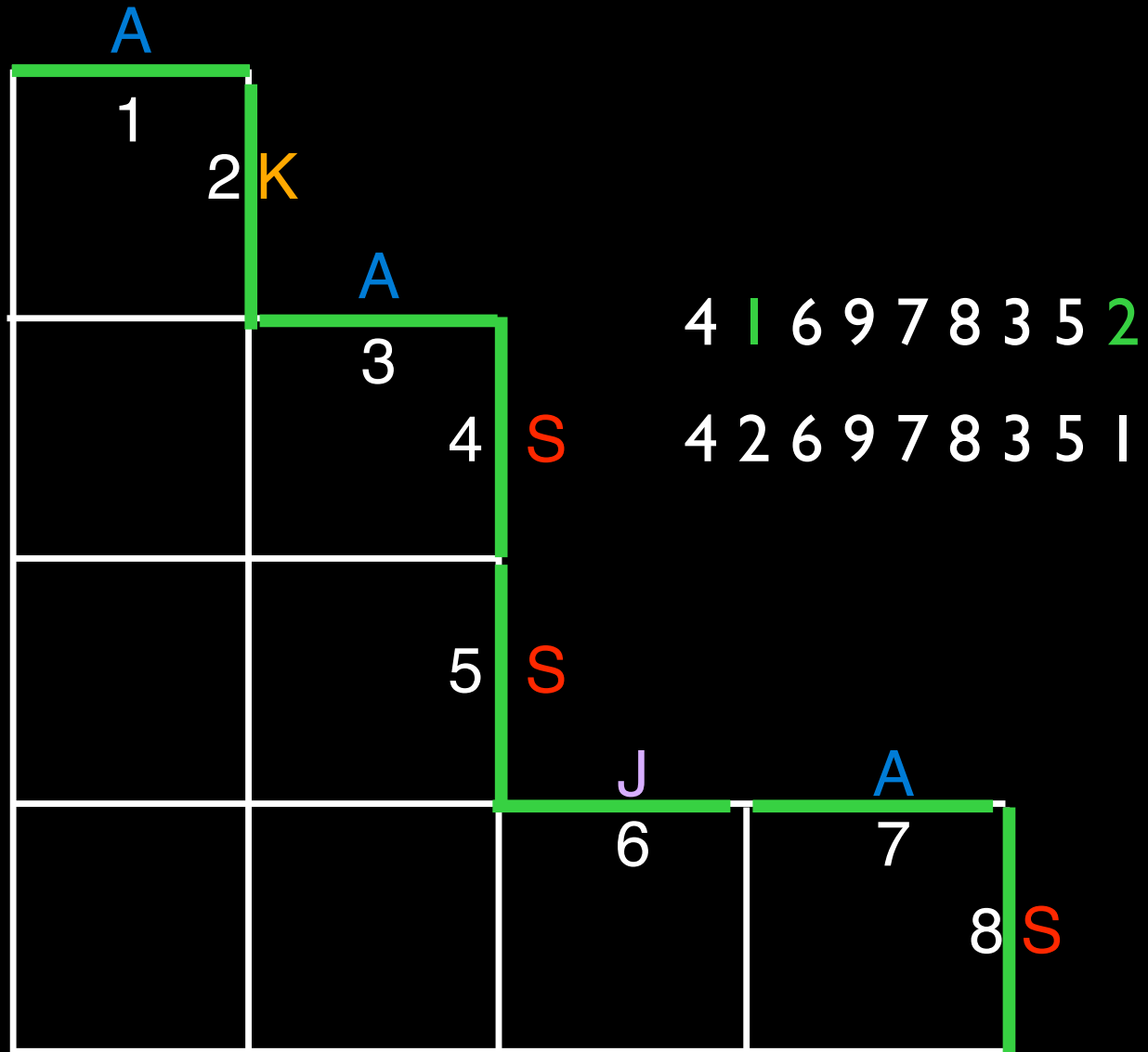
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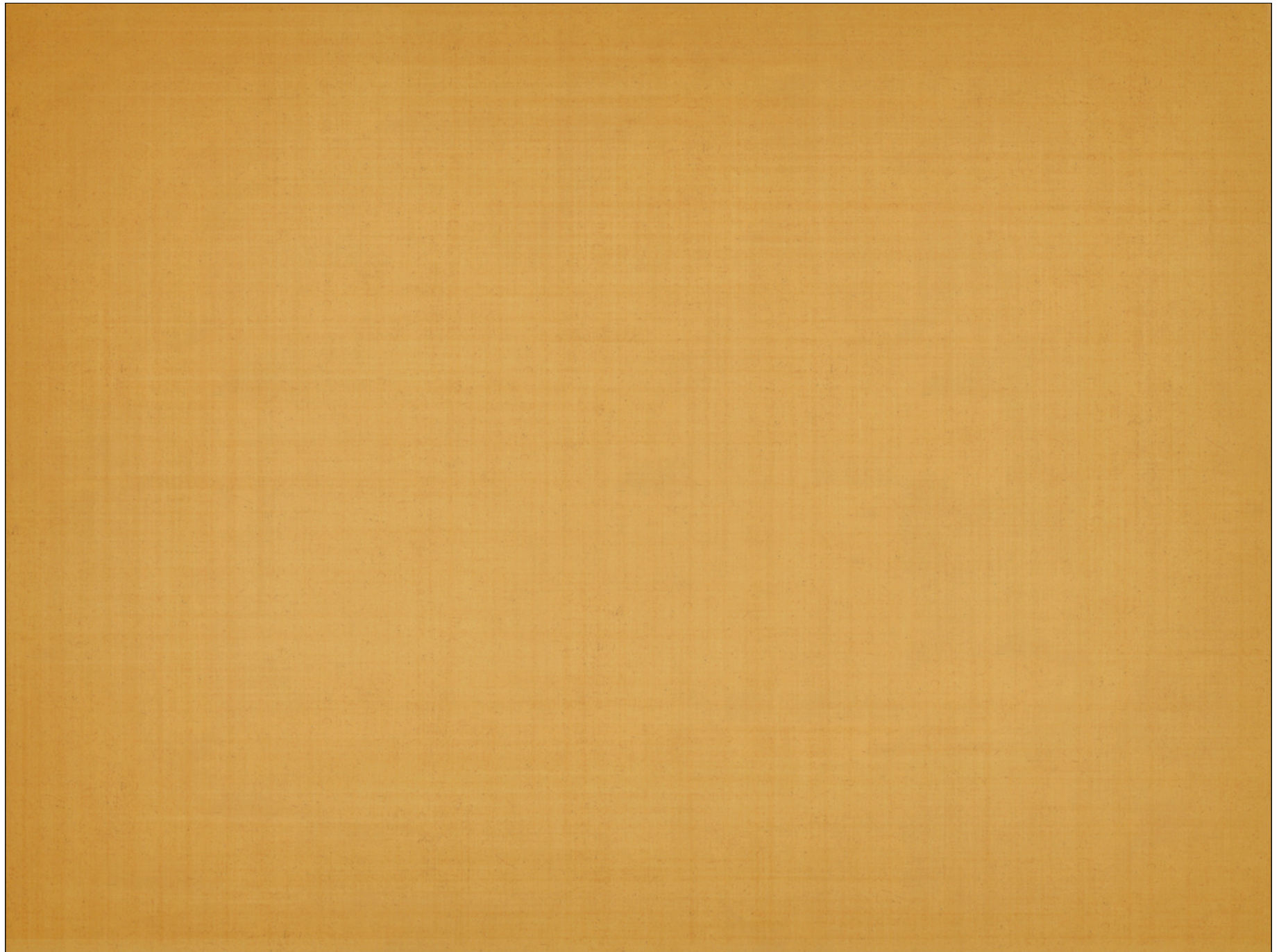


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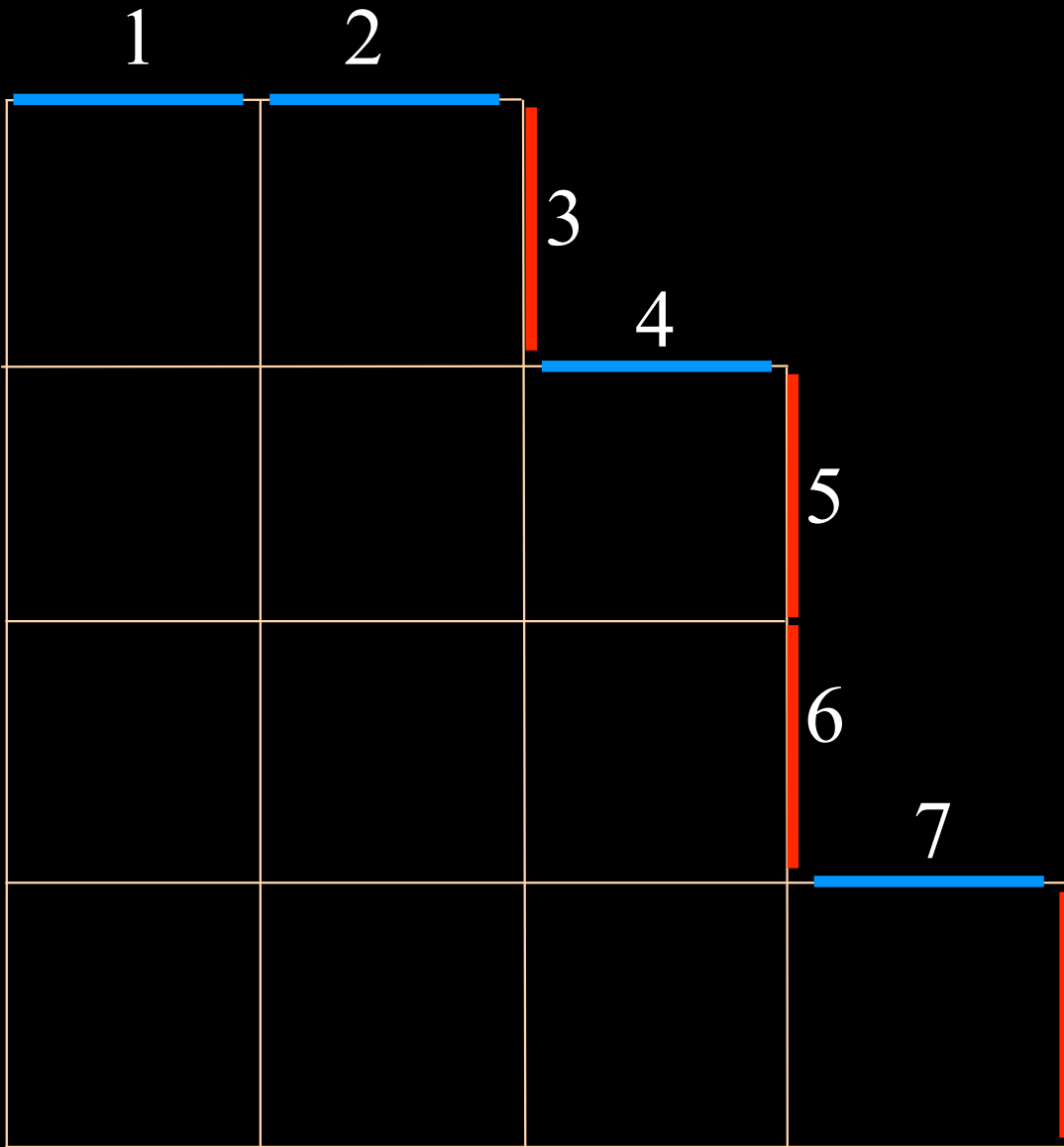








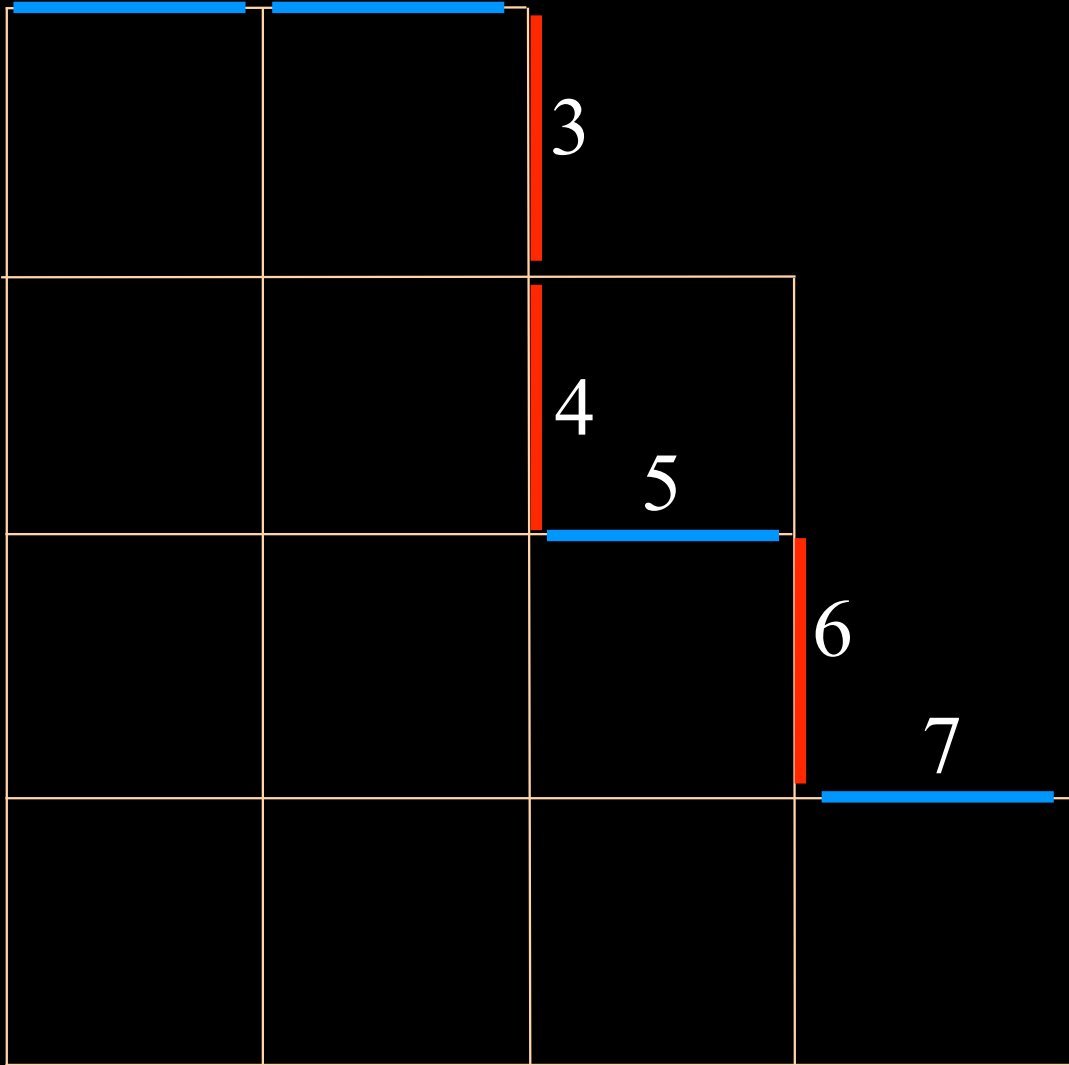




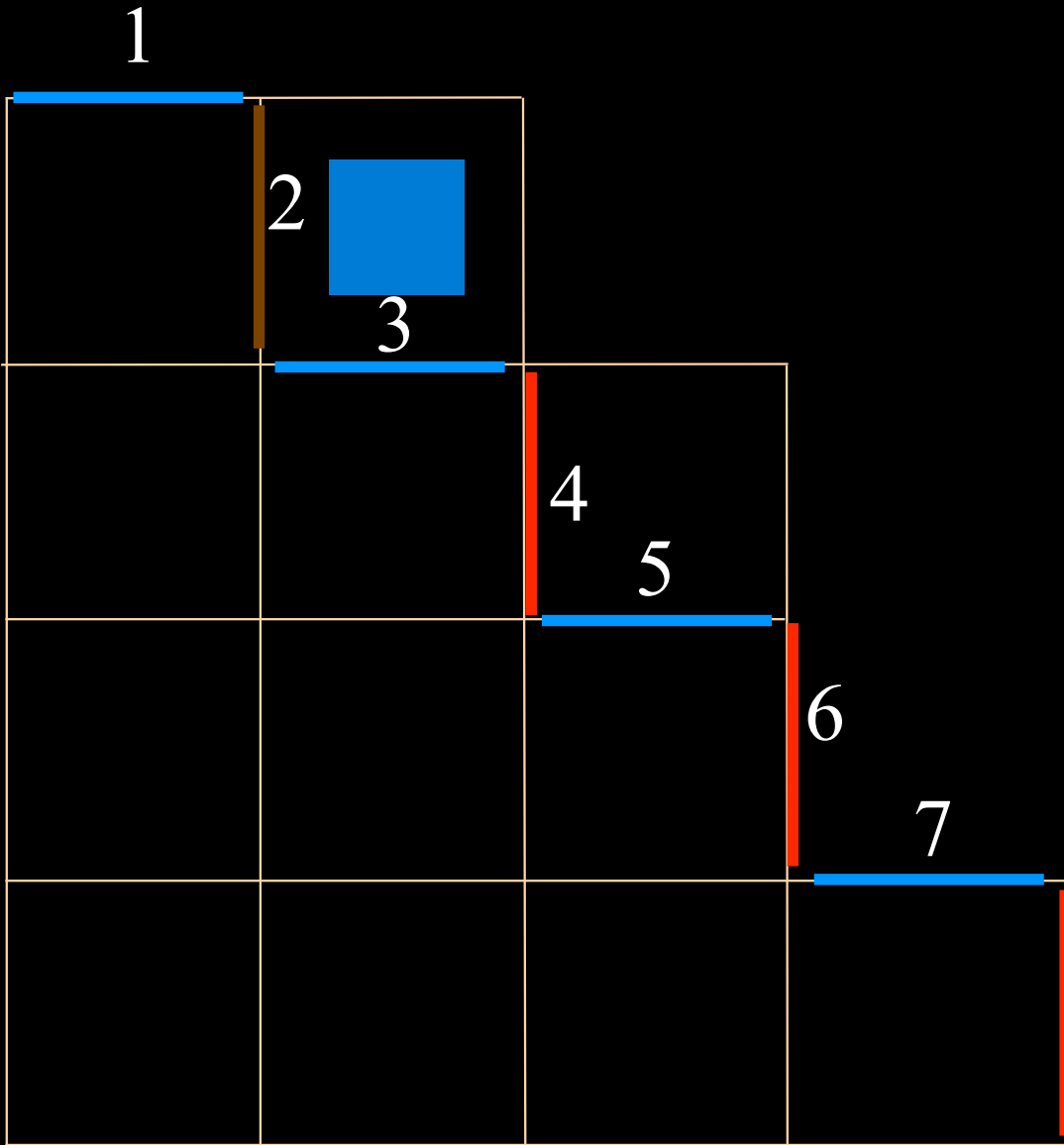
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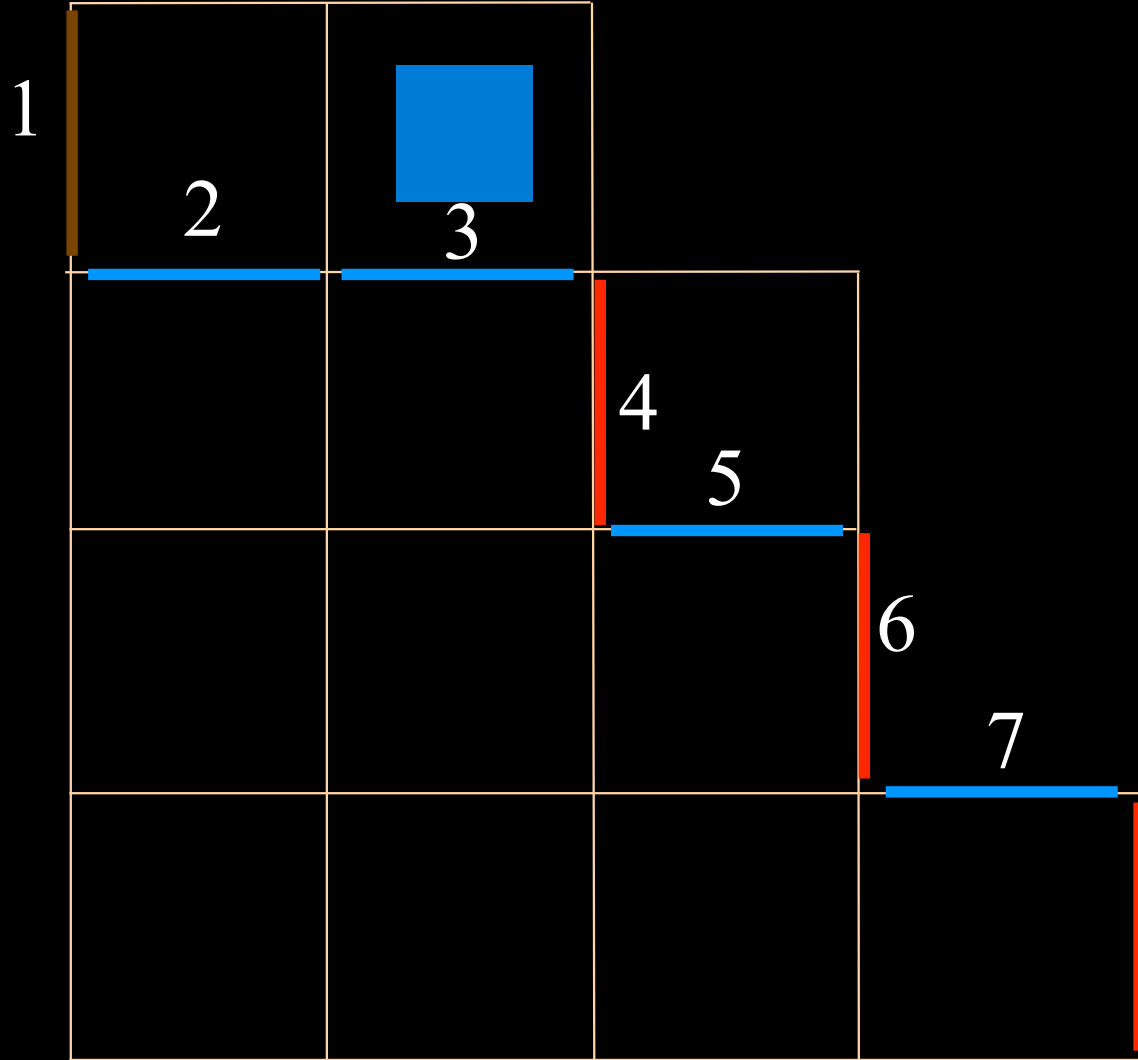
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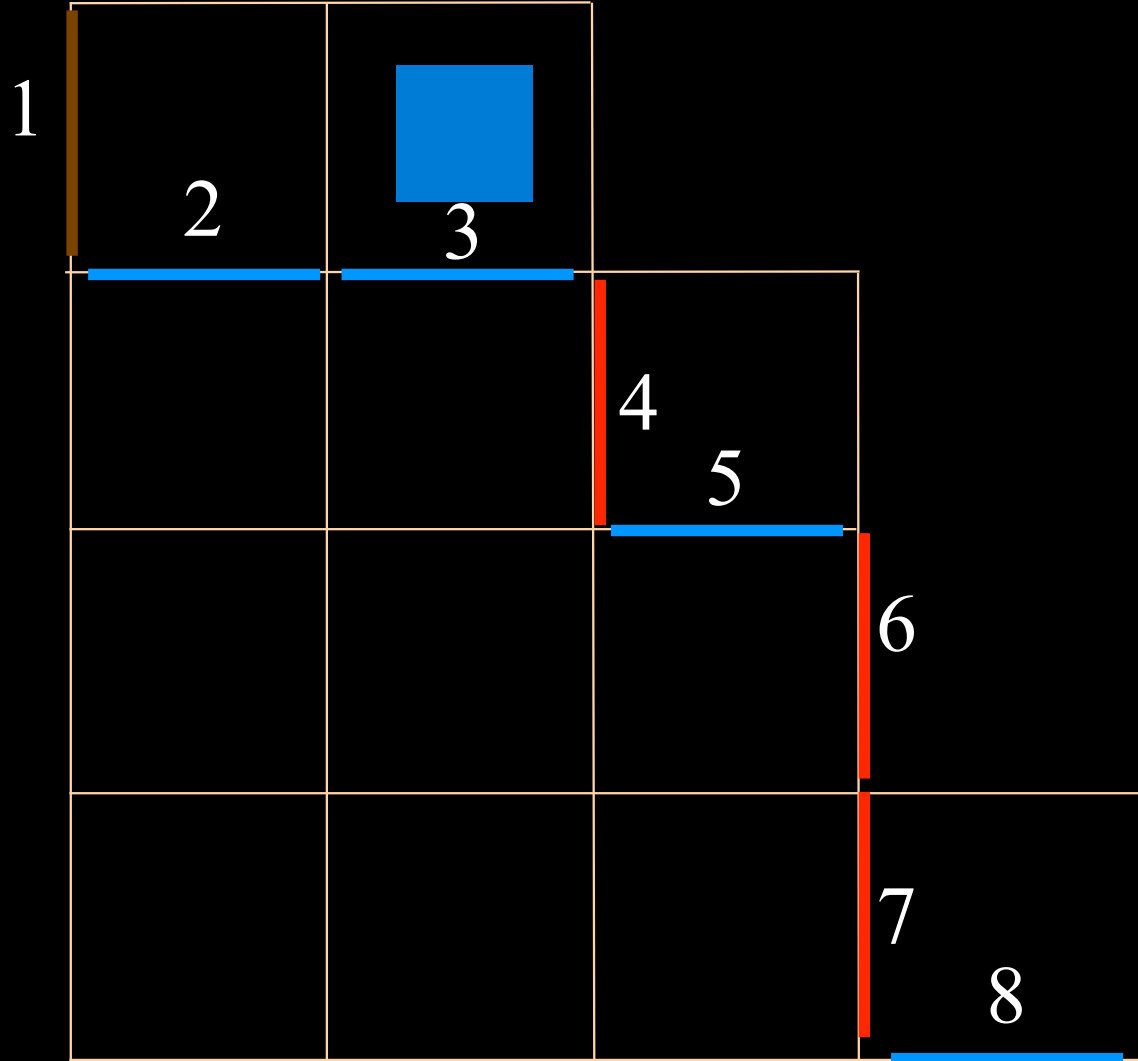
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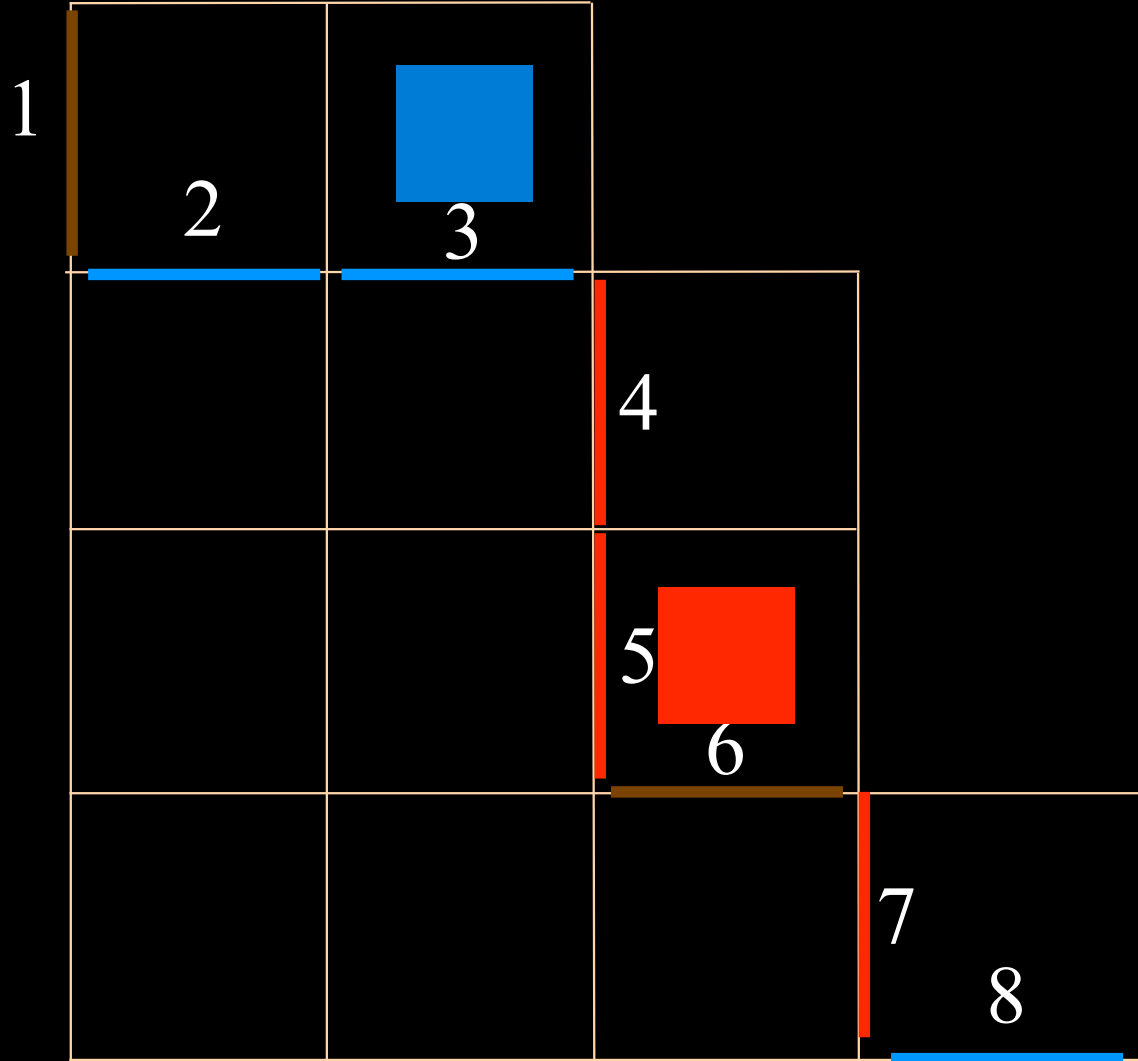
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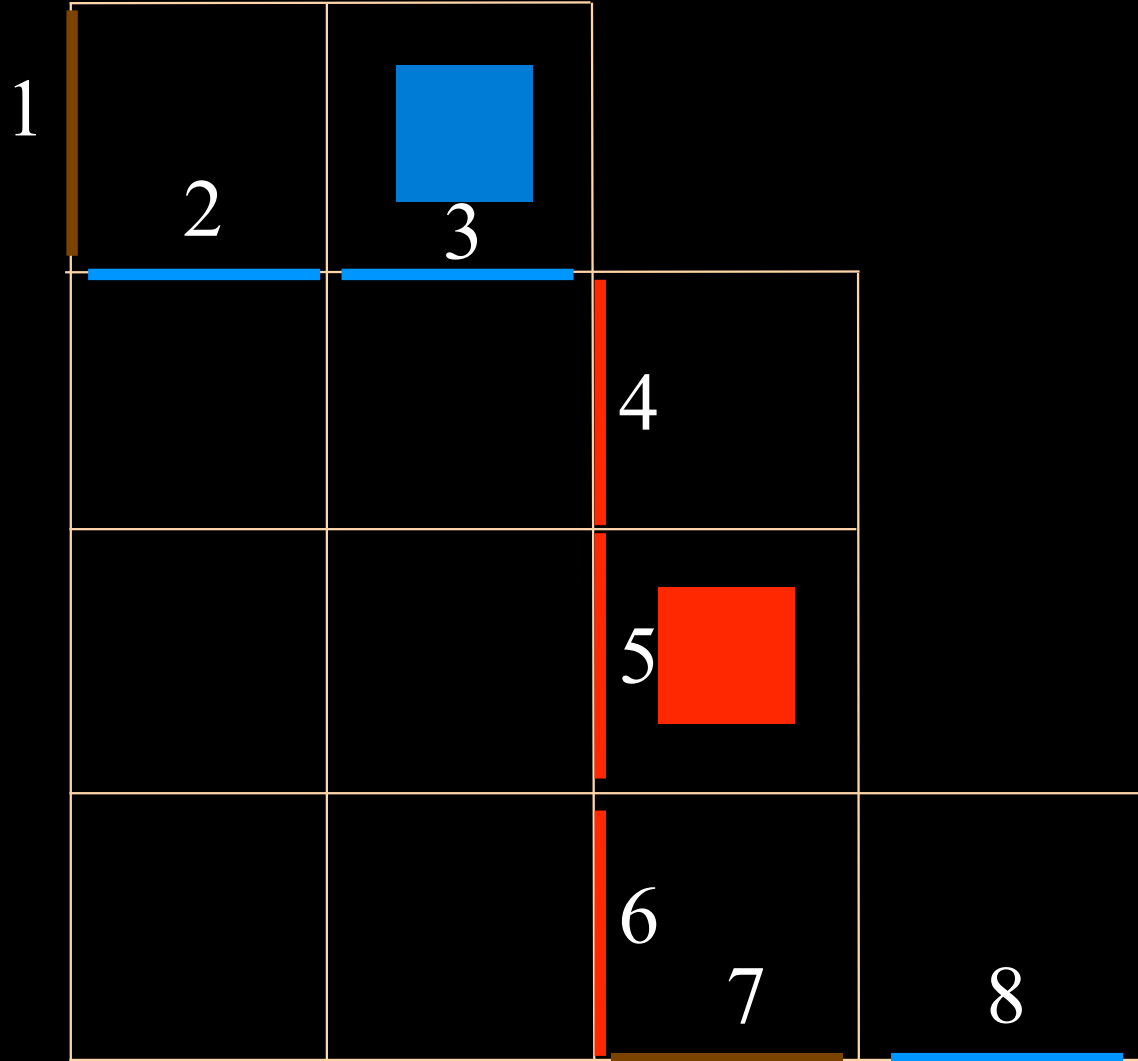
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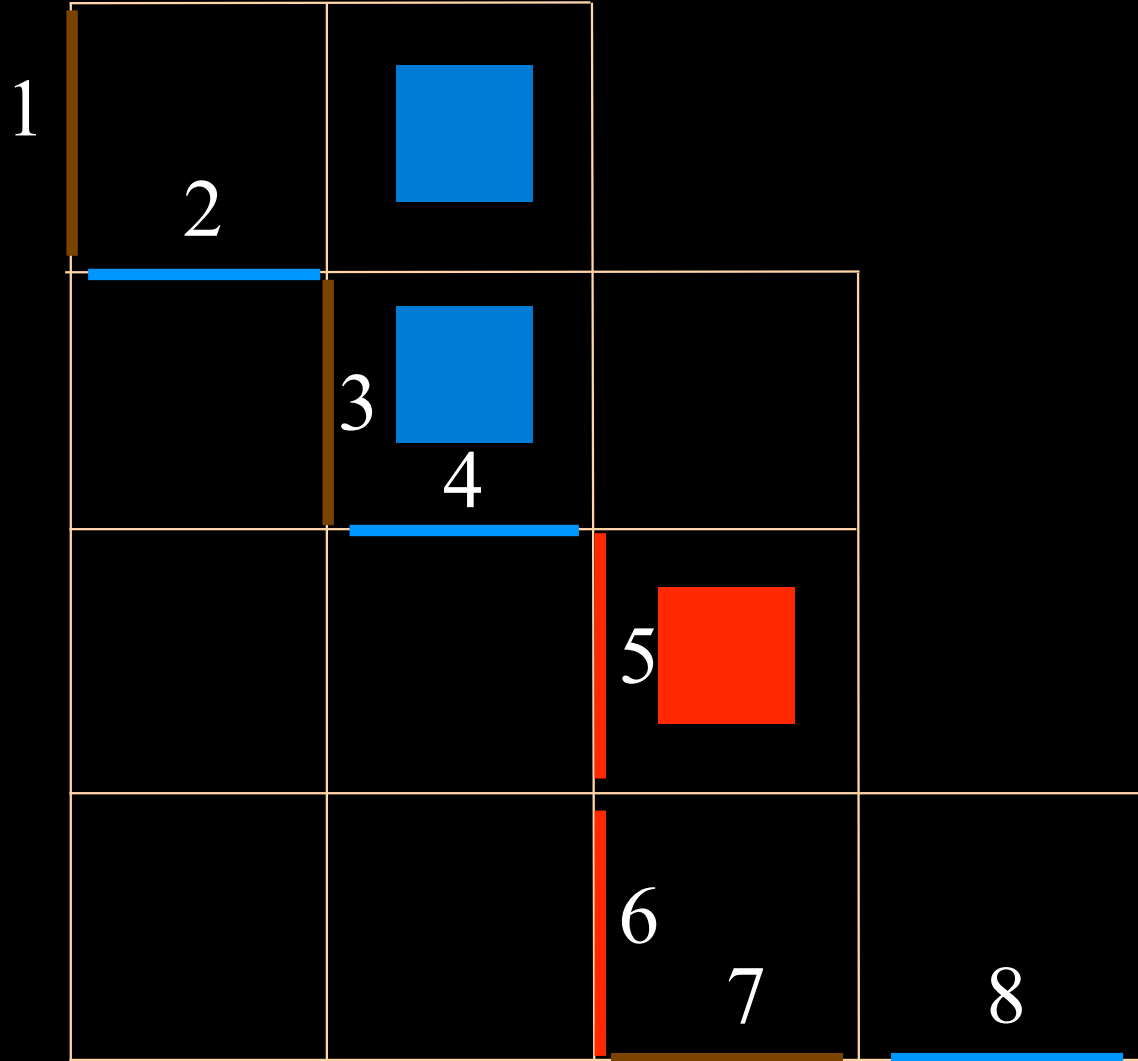
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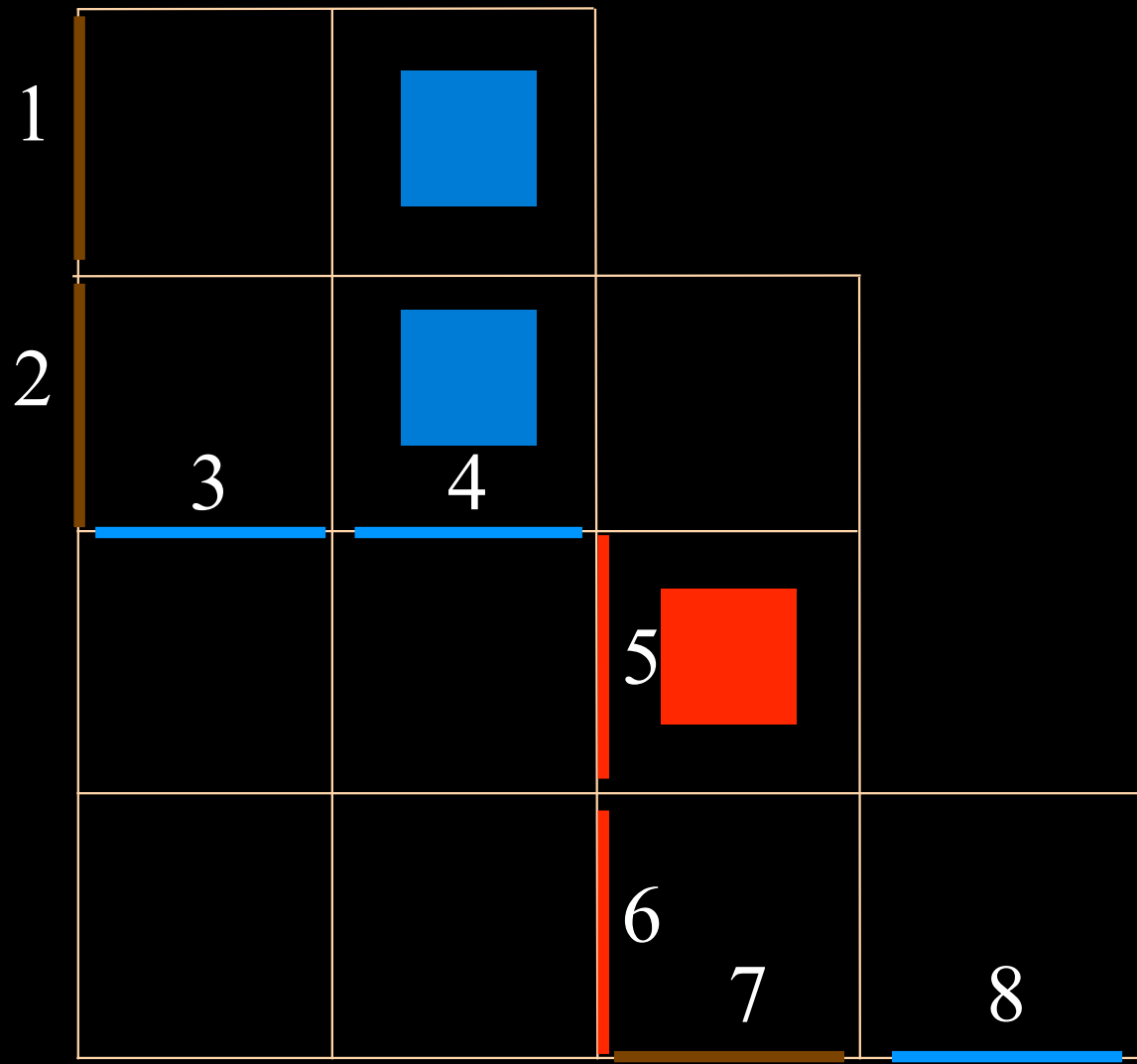


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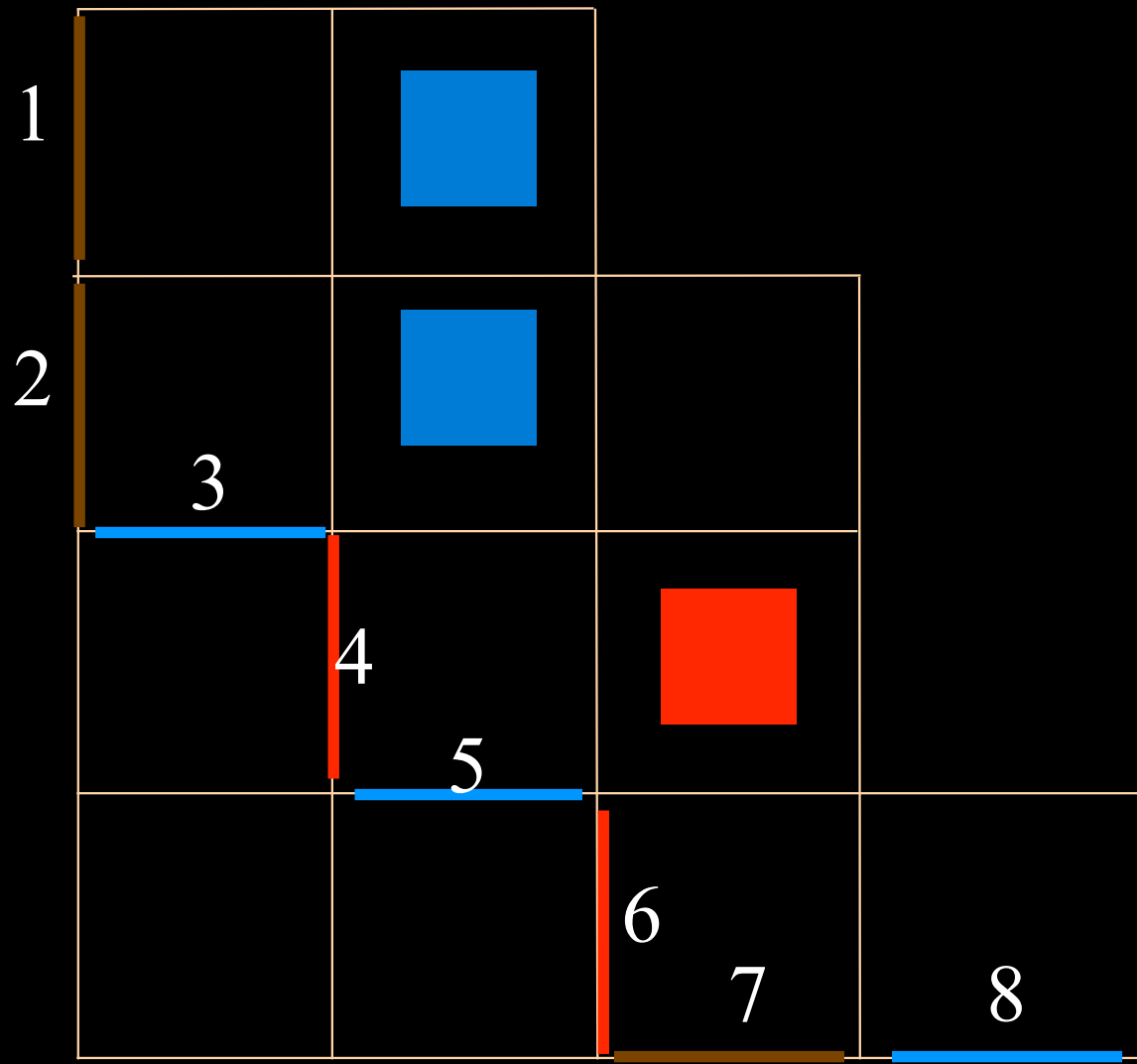


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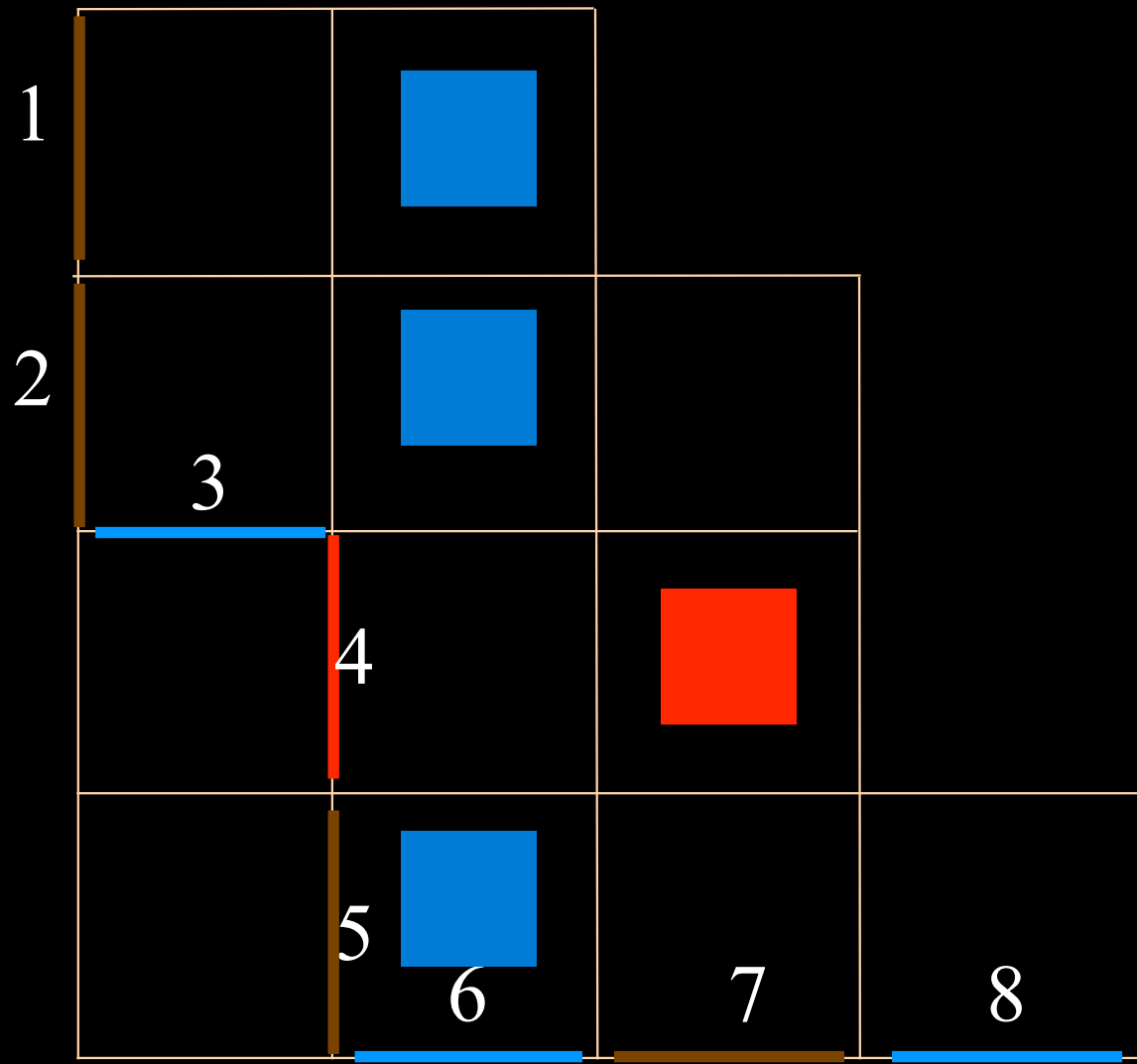




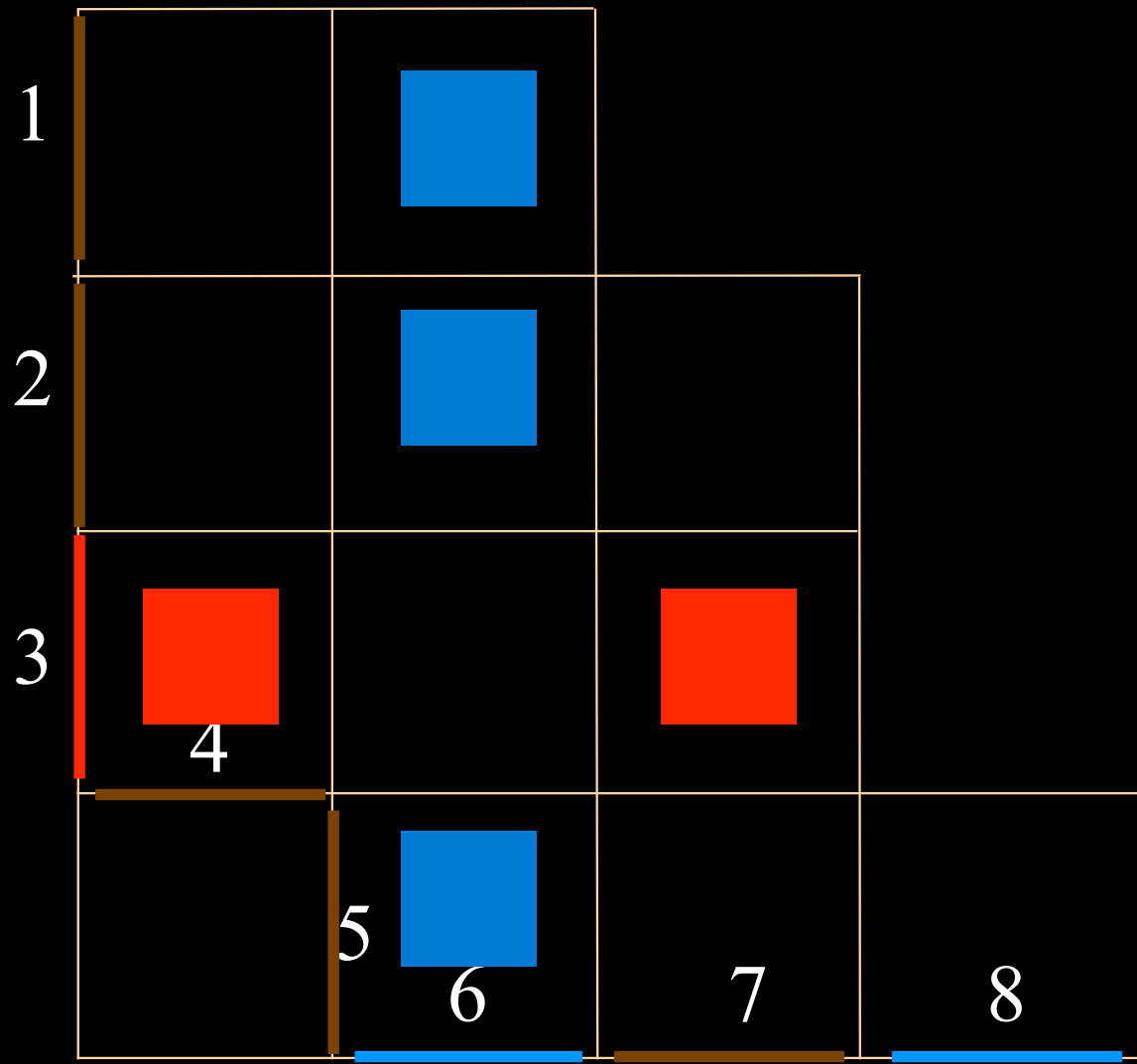
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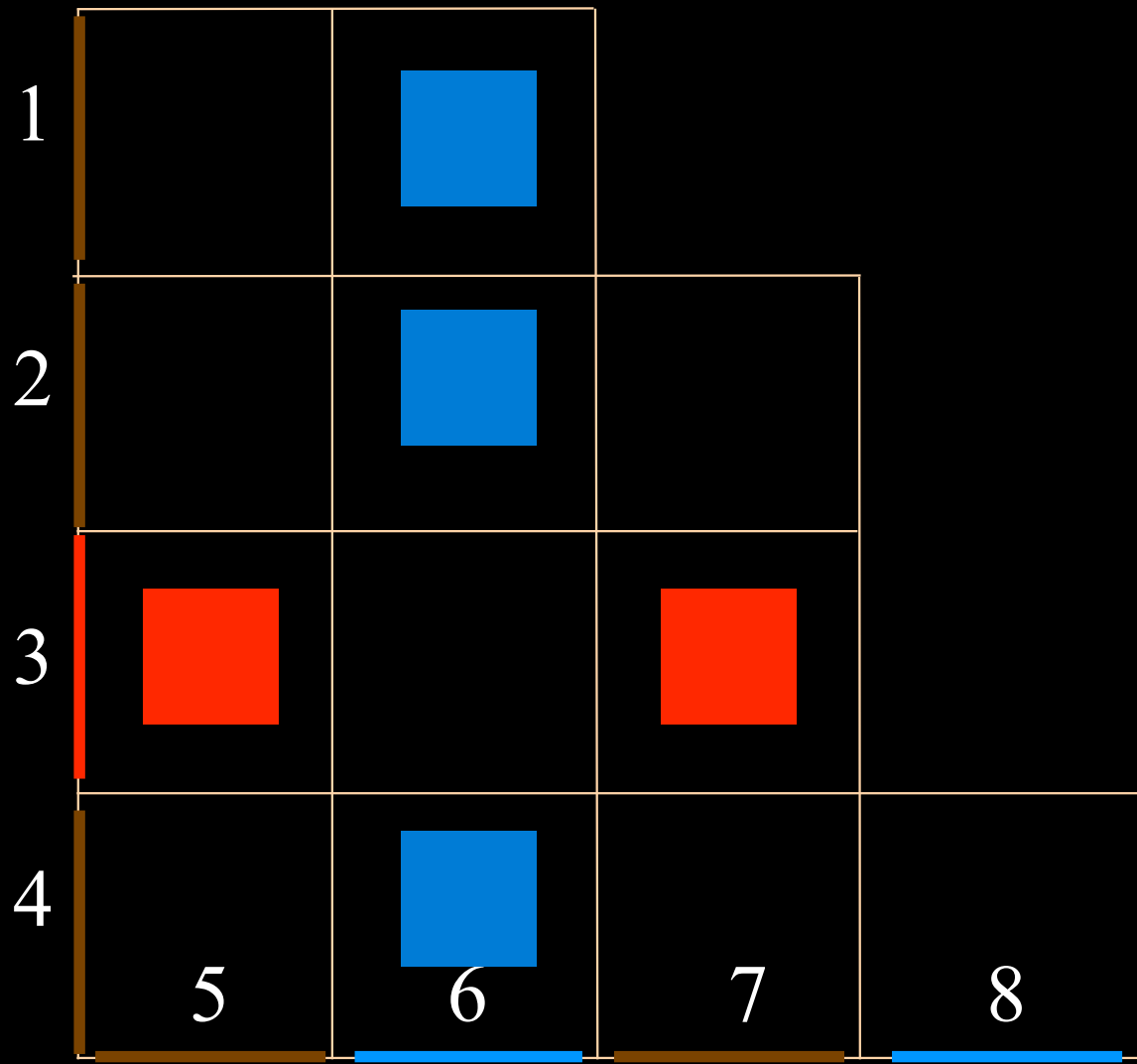
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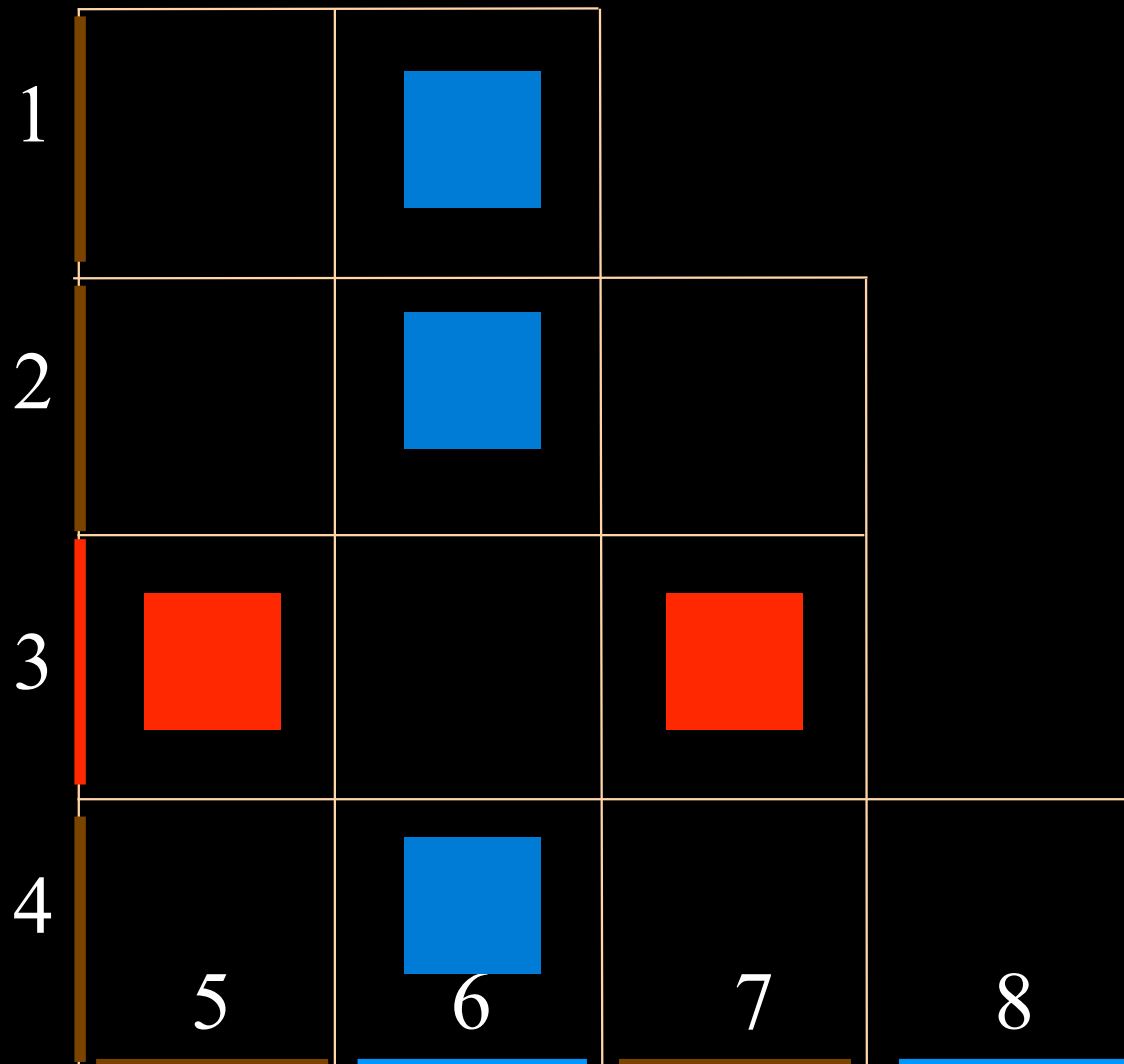
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8	4	2	3	7	9	1	5	6
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8 5 3 2 7 9 1 4 6  
 8 4 3 2 7 9 1 5 6  
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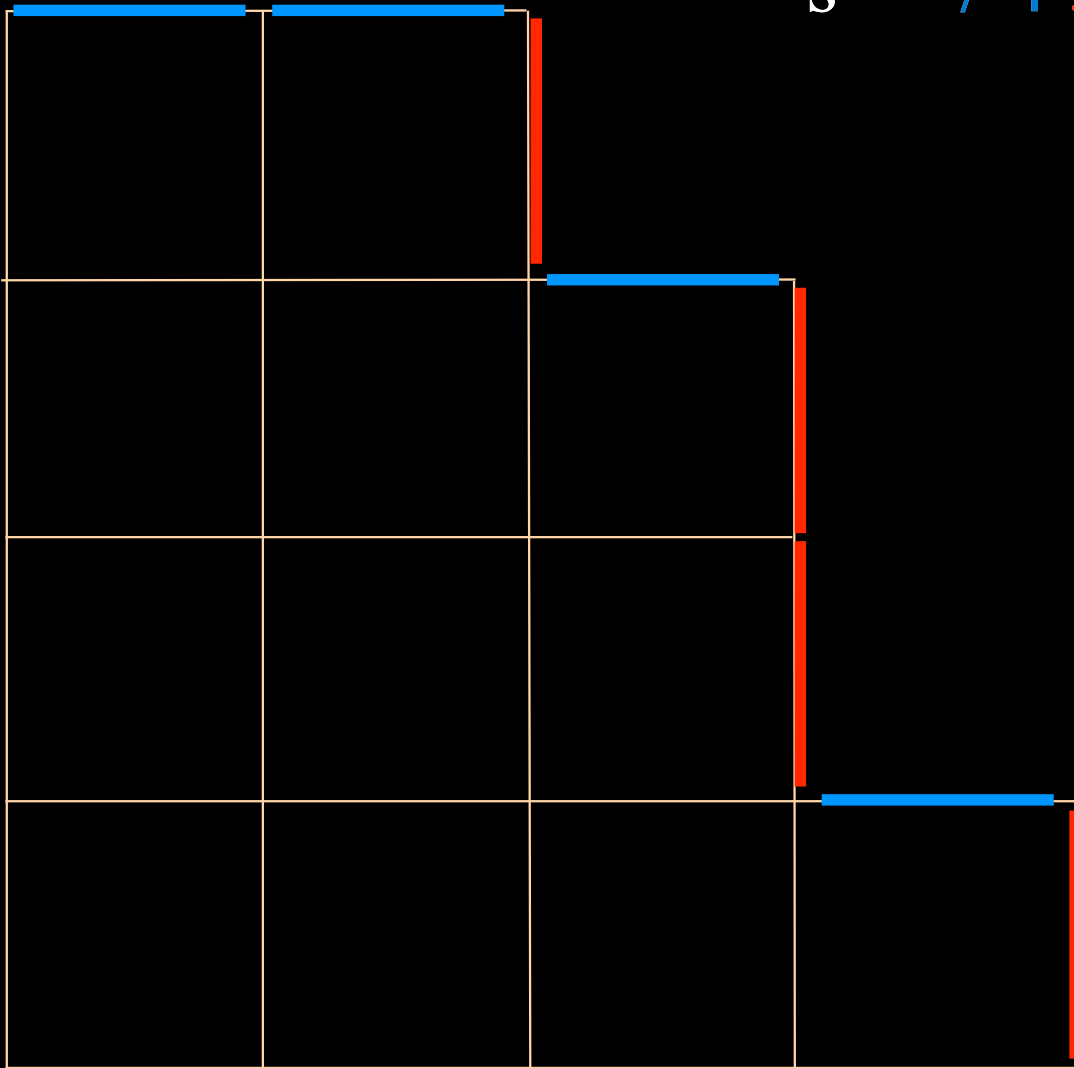


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8	4	3	2	7	9	1	5	6
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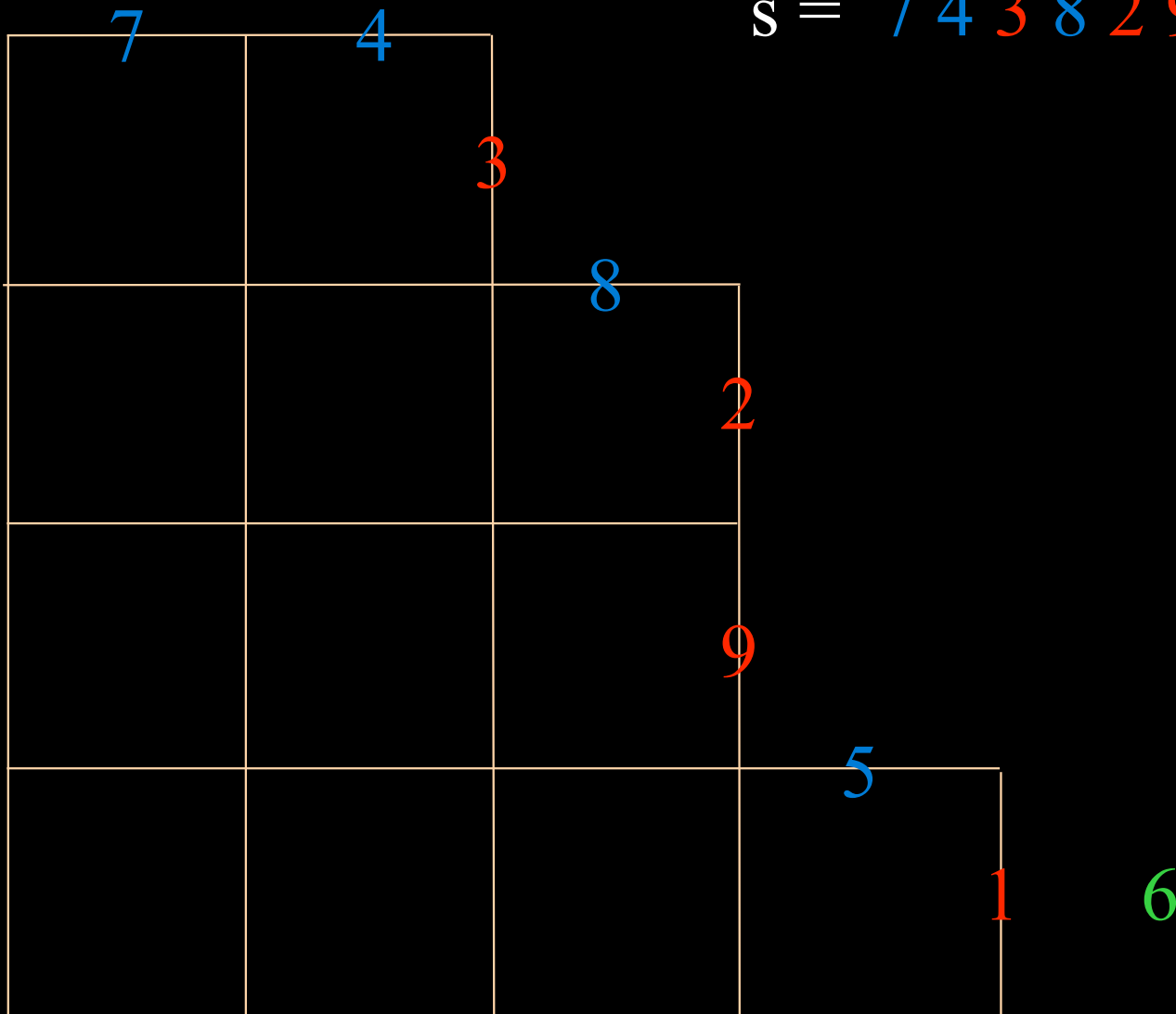
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s = 7 4 3 8 2 9 5 1 6



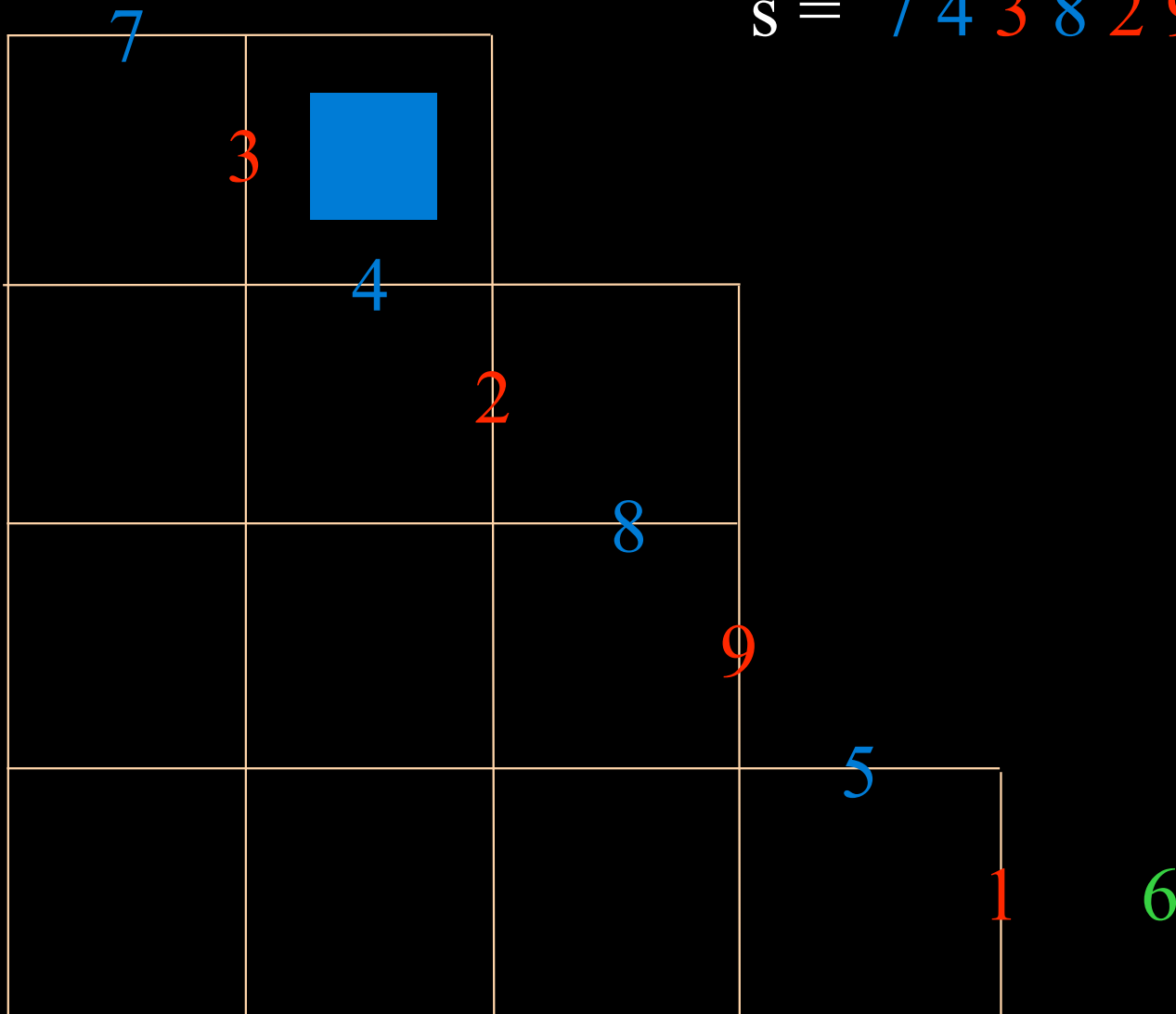


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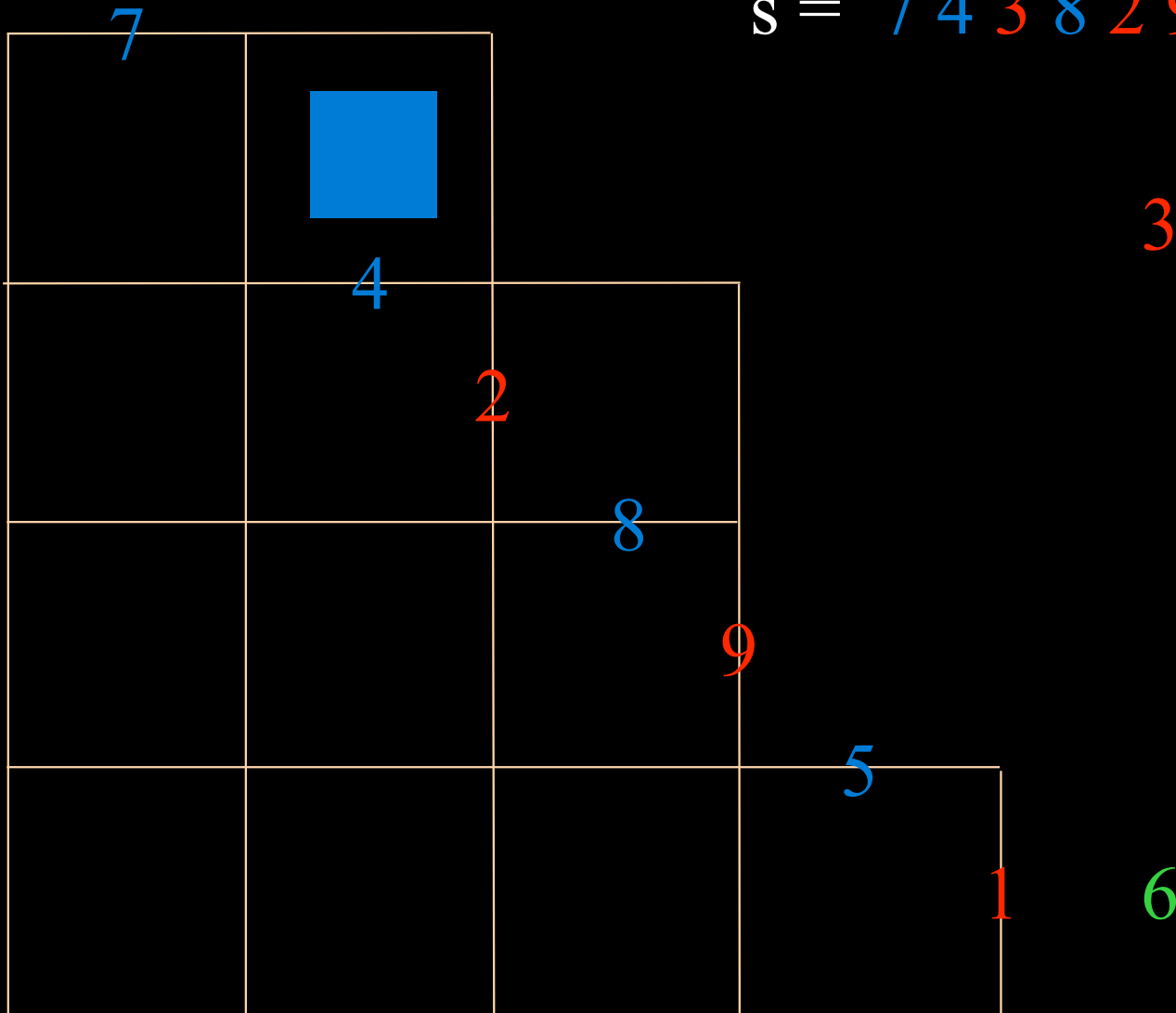




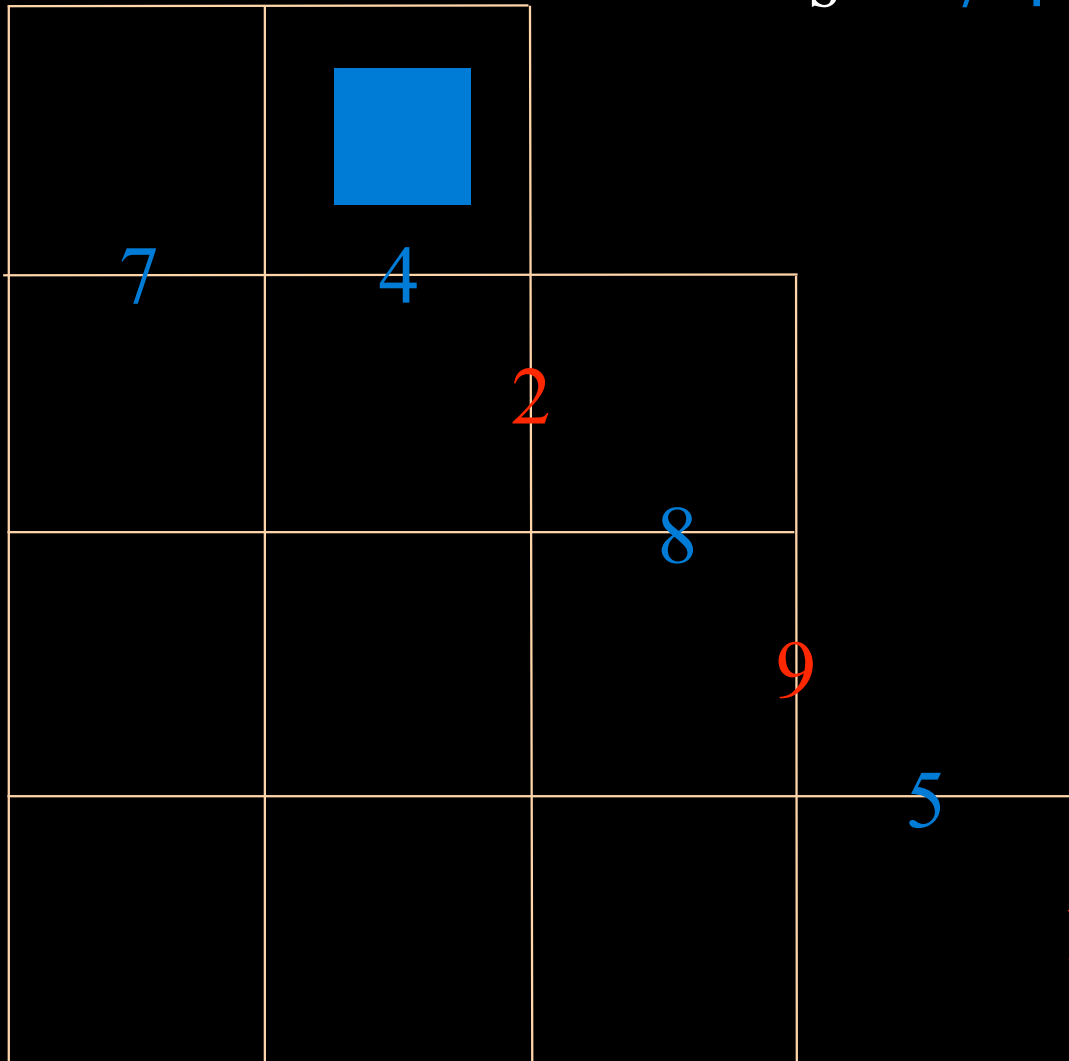
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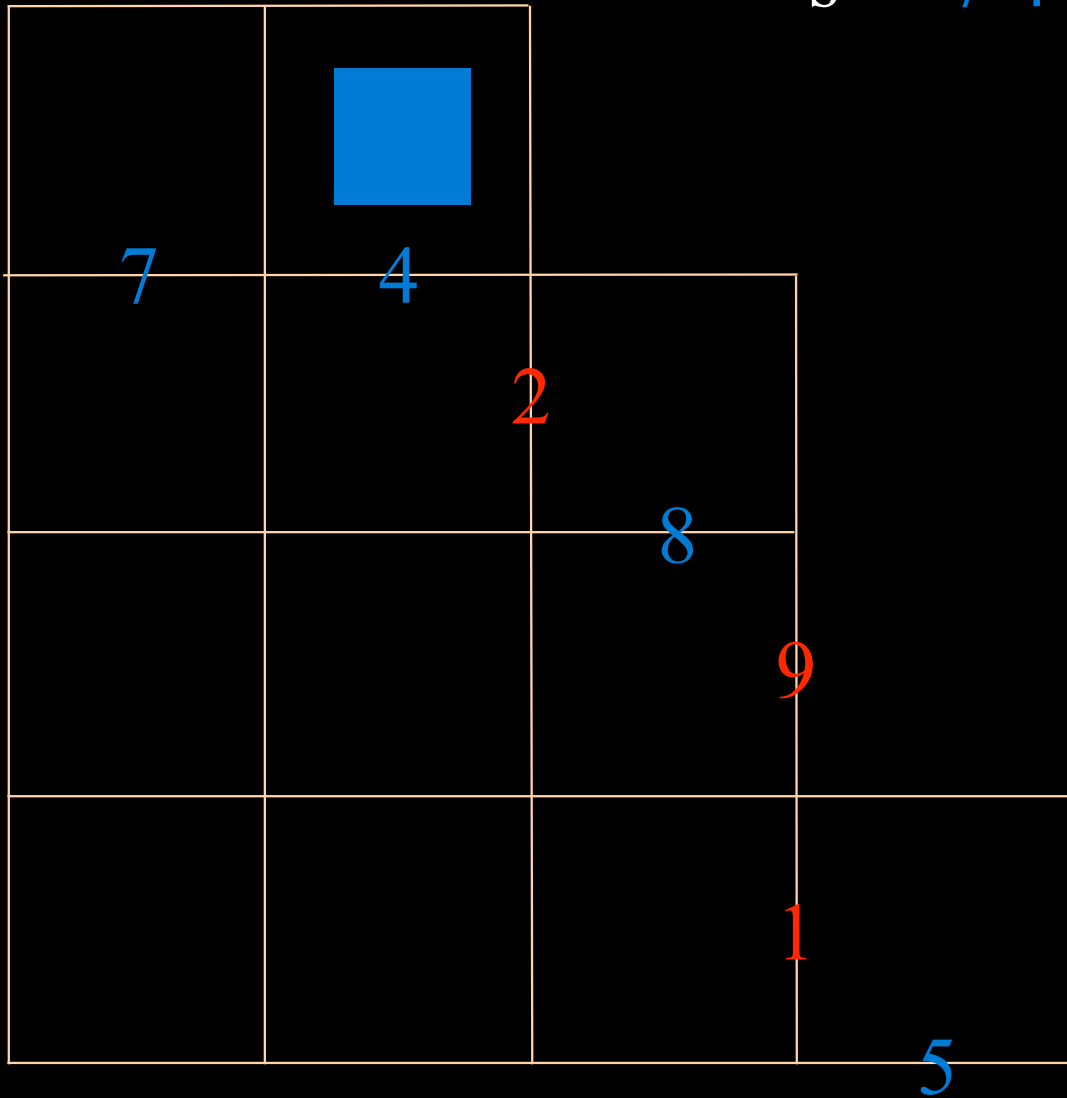
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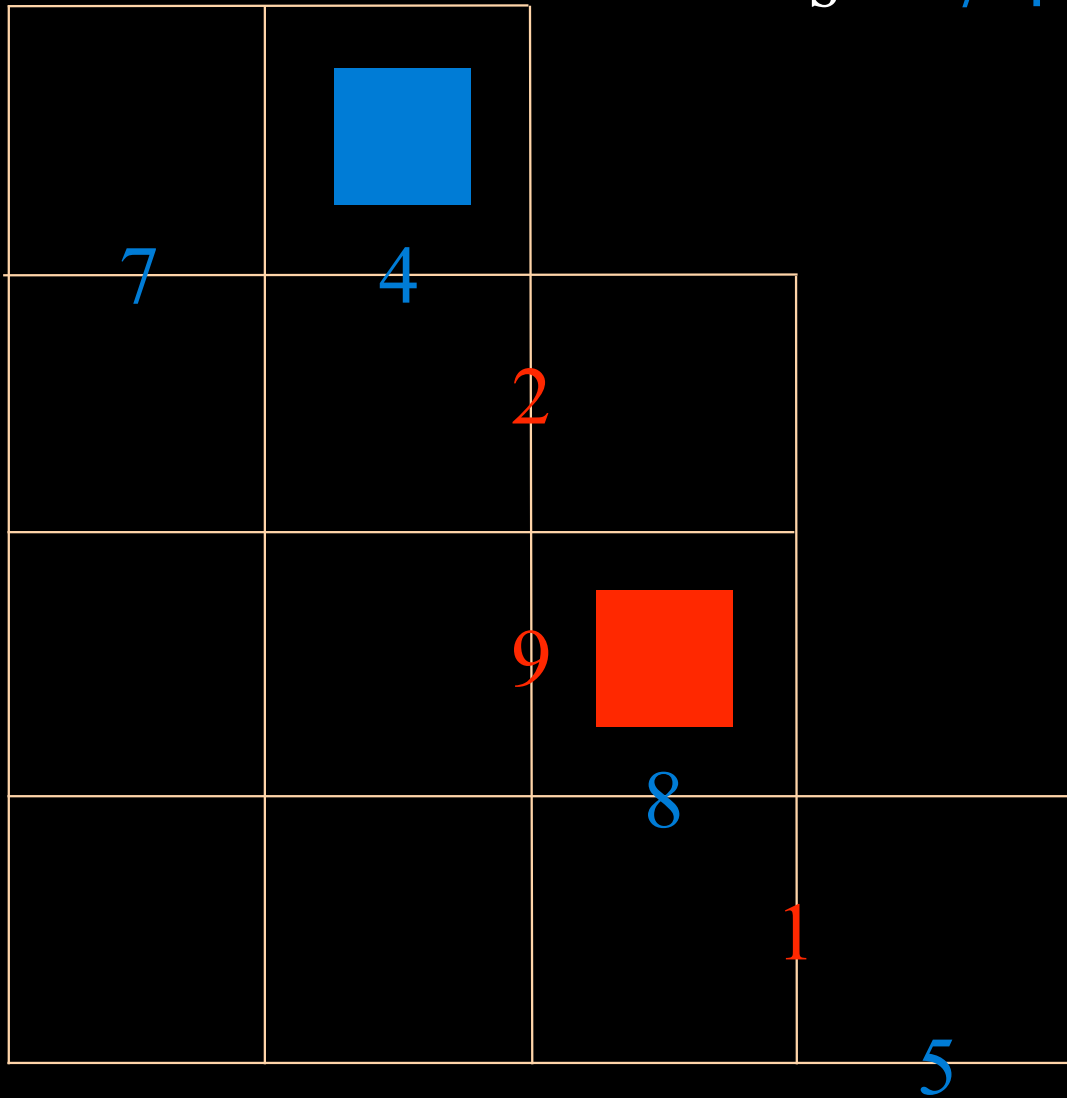
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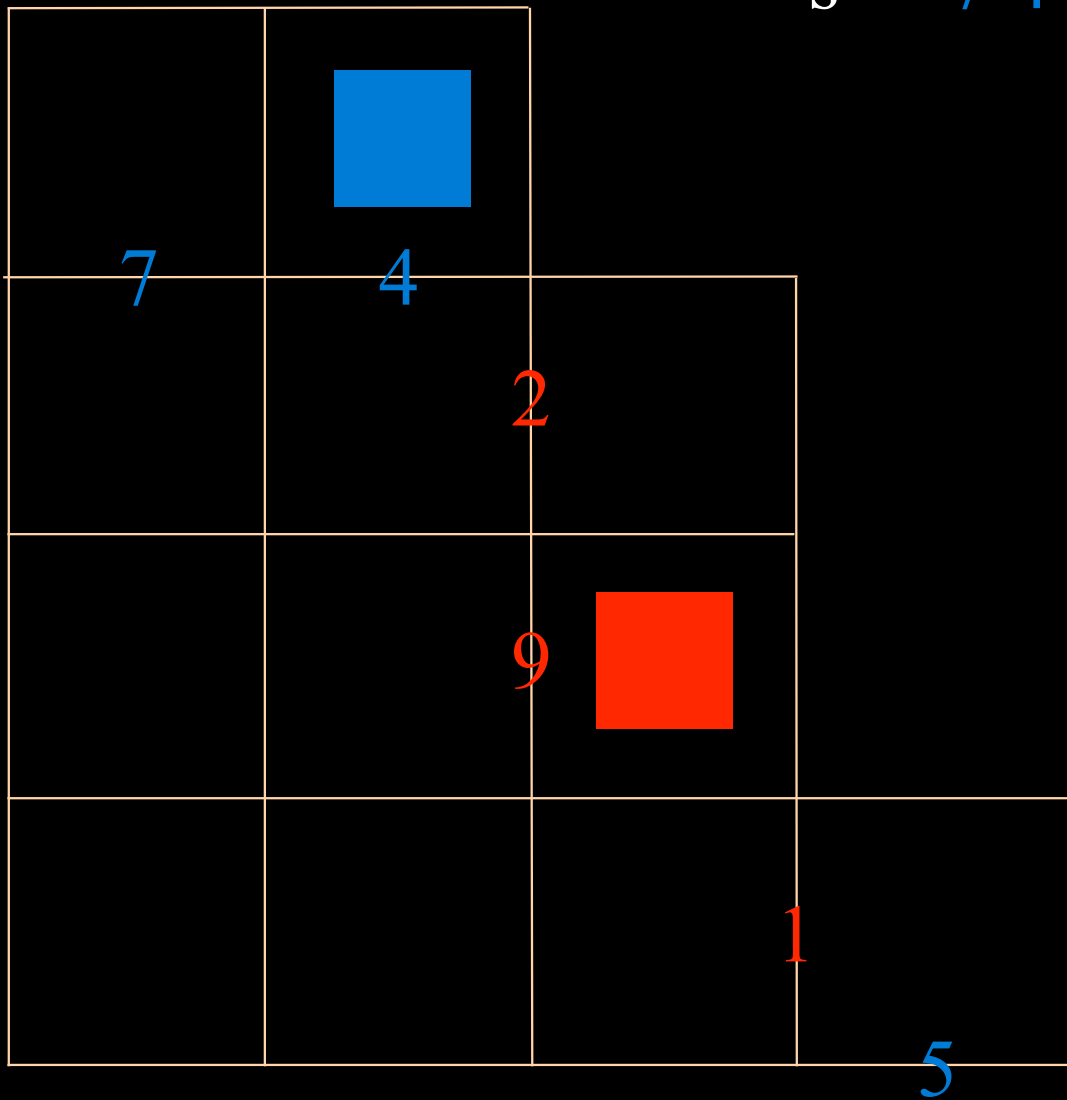
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s = 7 4 3 8 2 9 5 1 6



s = 7 4 3 8 2 9 5 1 6



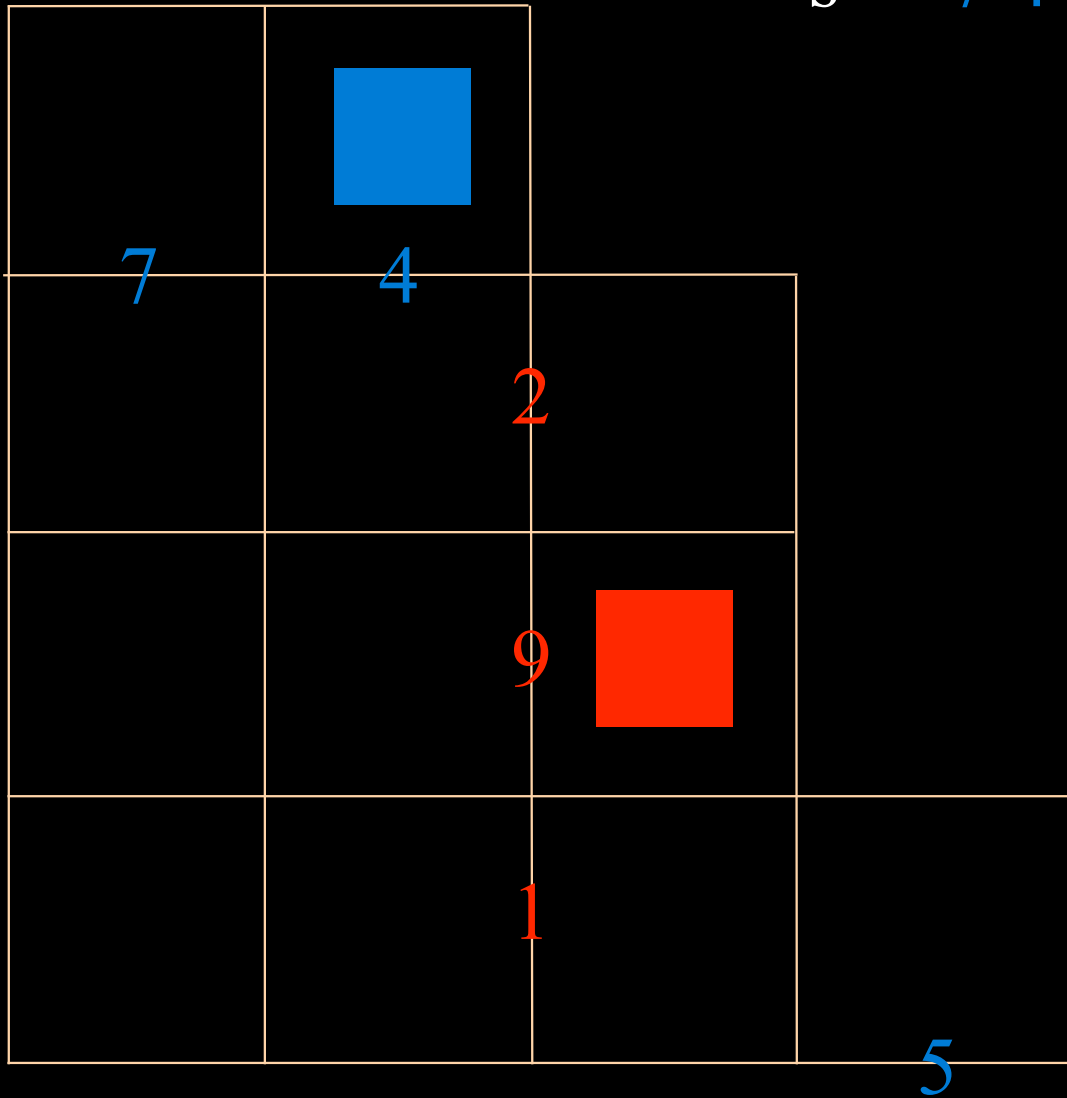
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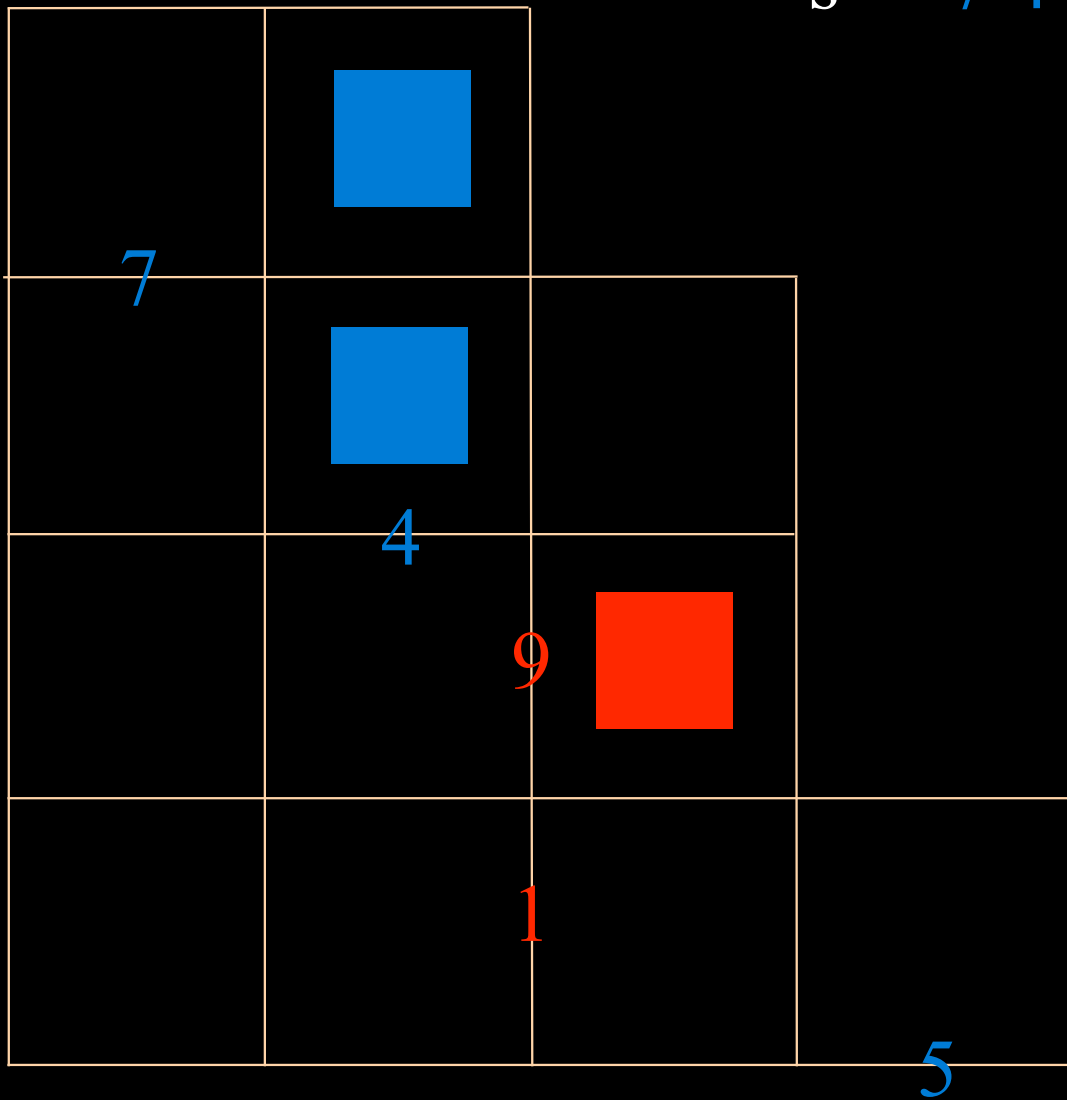


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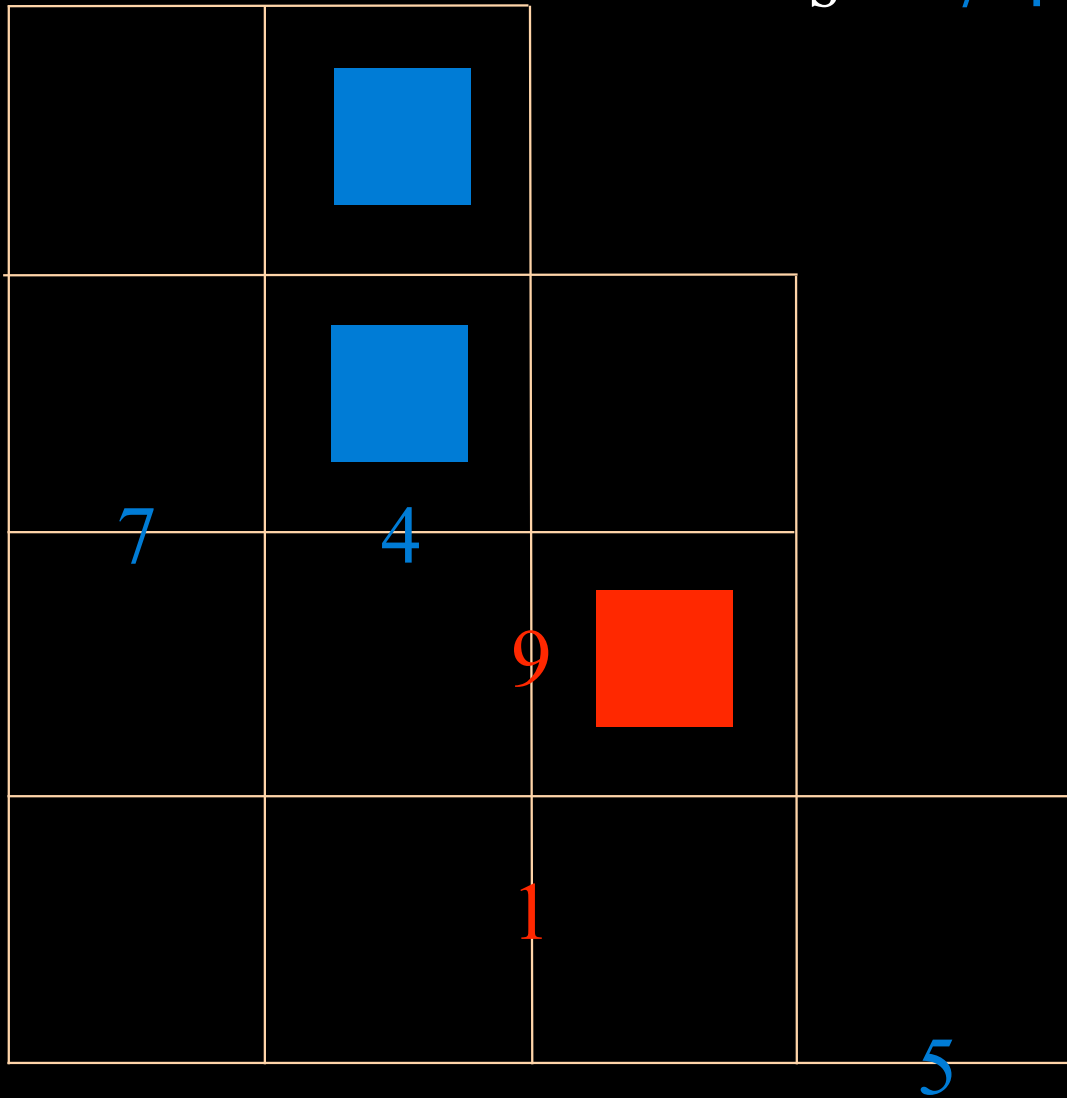
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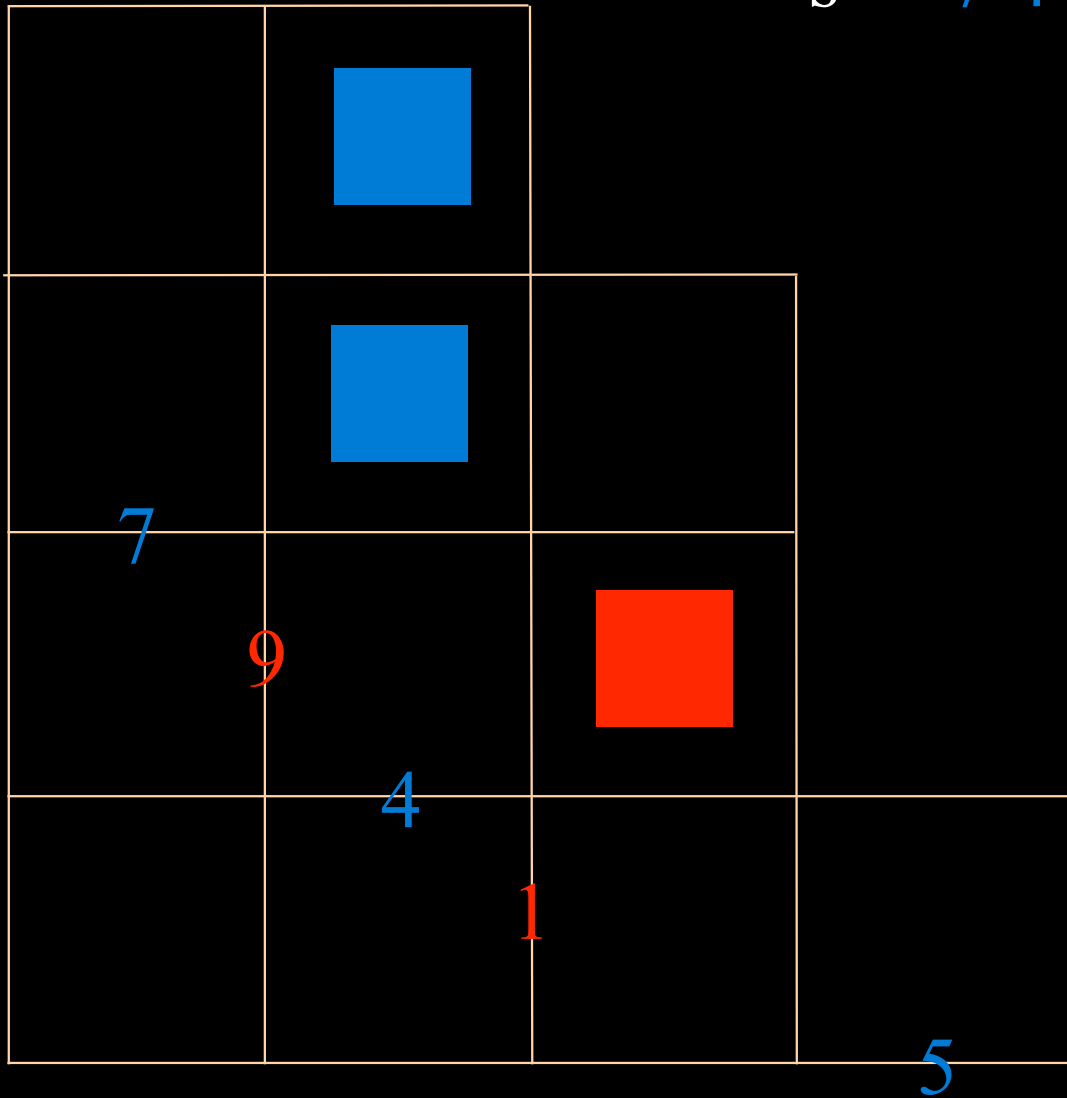
s = 7 4 3 8 2 9 5 1 6



s = 7 4 3 8 2 9 5 1 6



s = 7 4 3 8 2 9 5 1 6



3

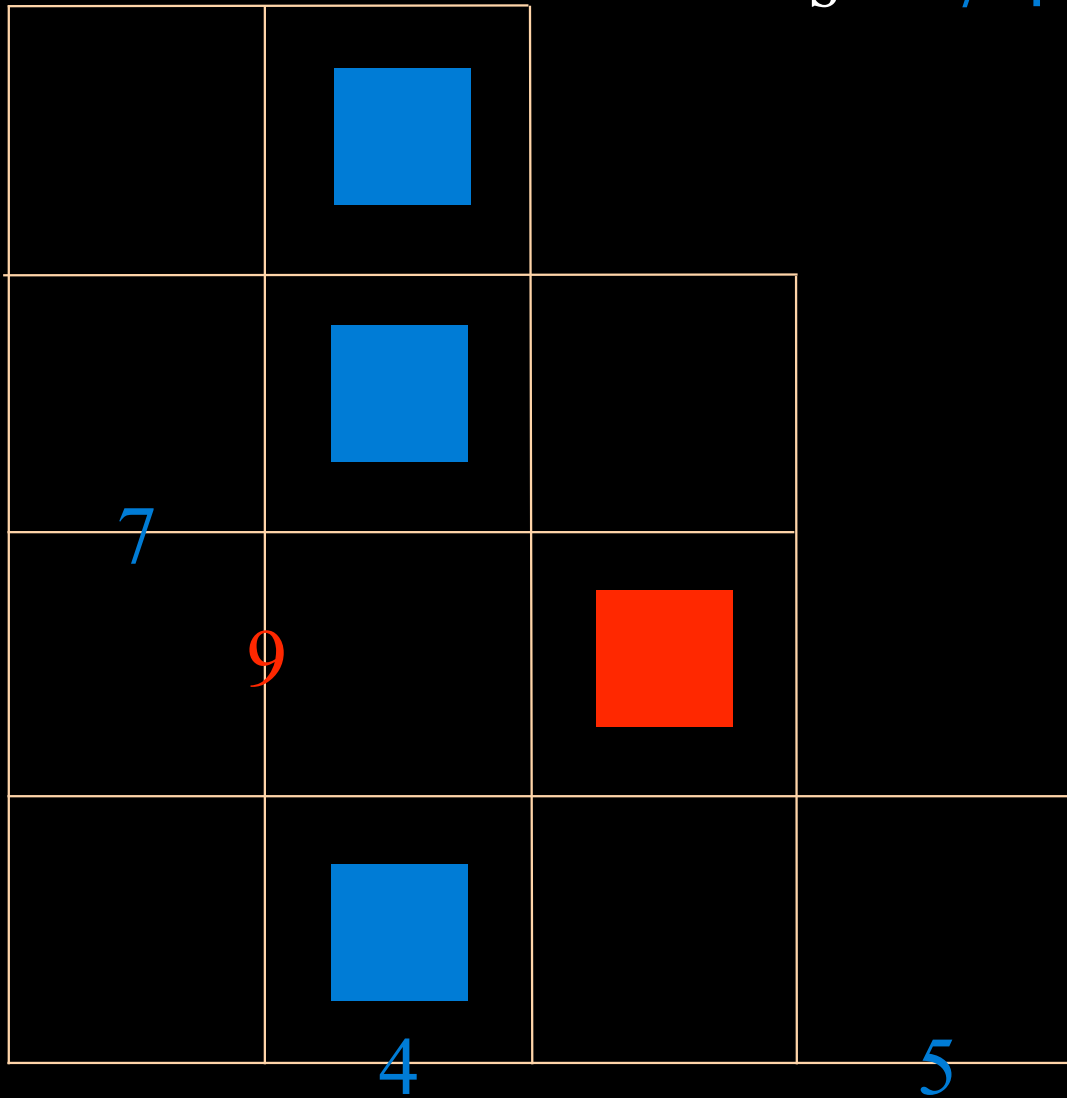
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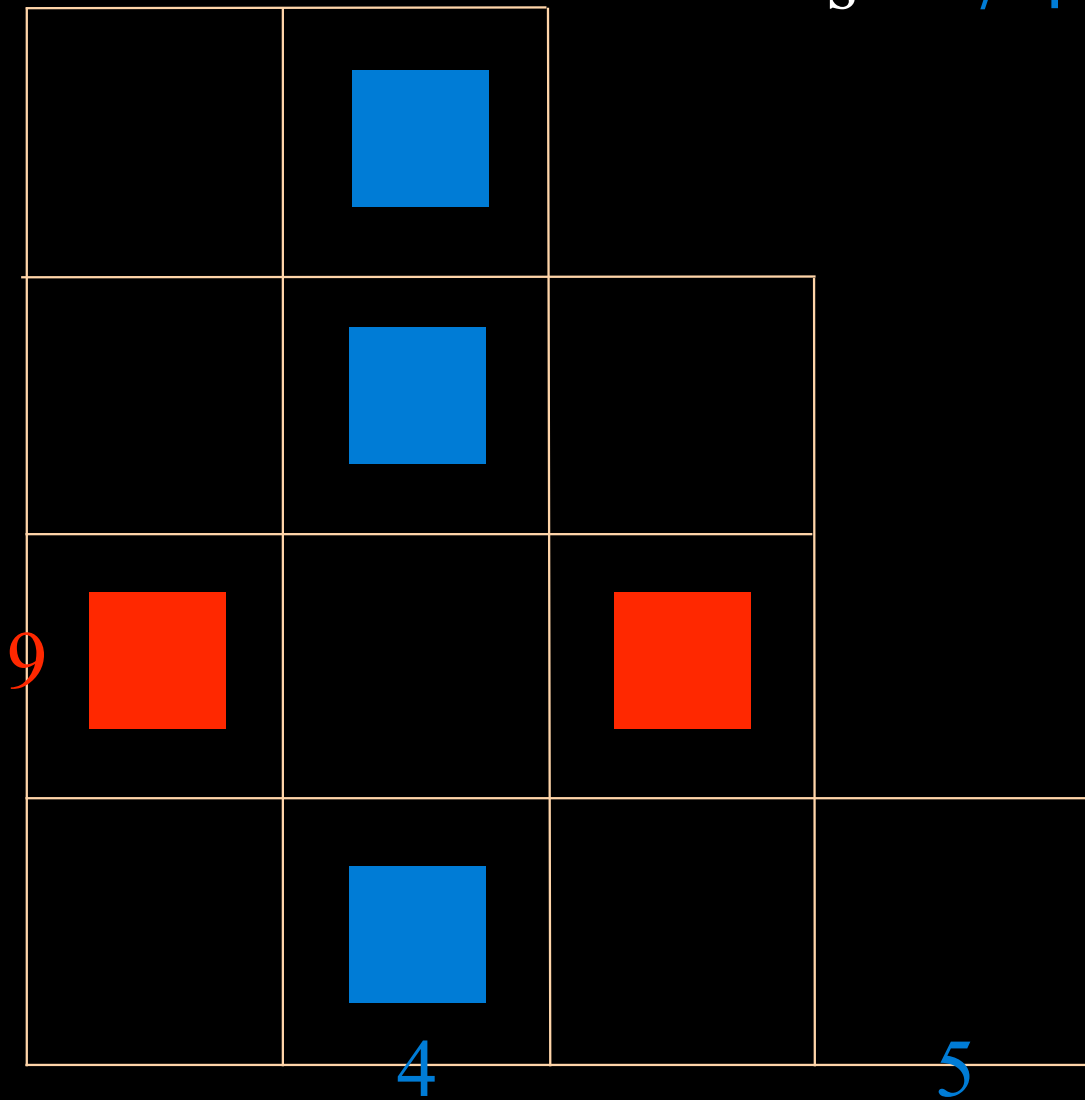
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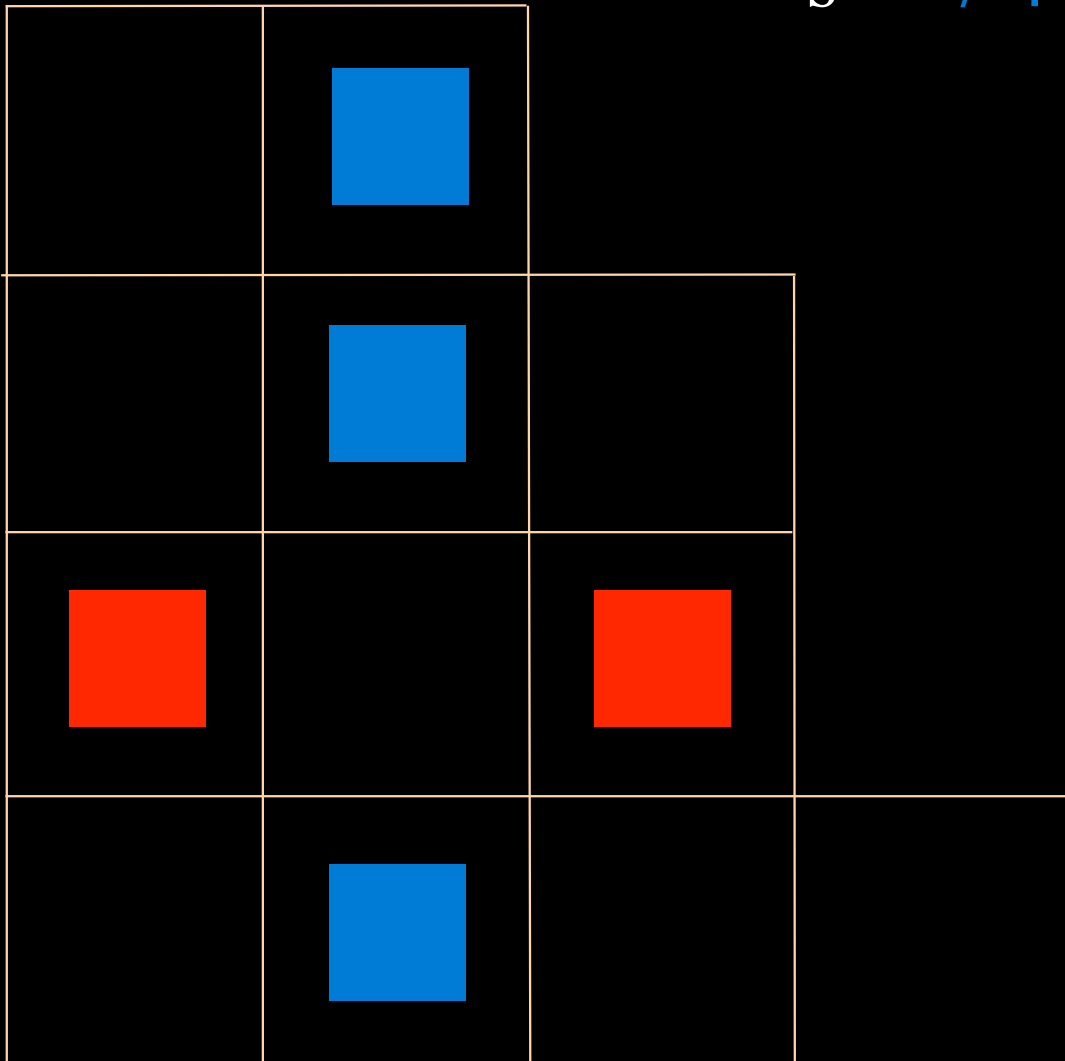
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6

s = 7 4 3 8 2 9 5 1 6



s = 7 4 3 8 2 9 5 1 6



9

3

8

2

1

7

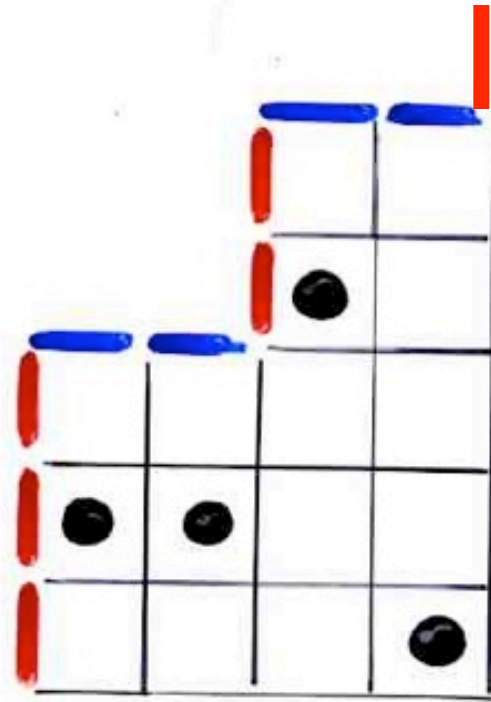
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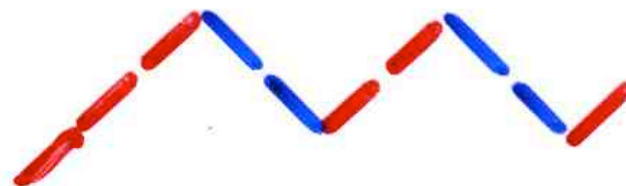
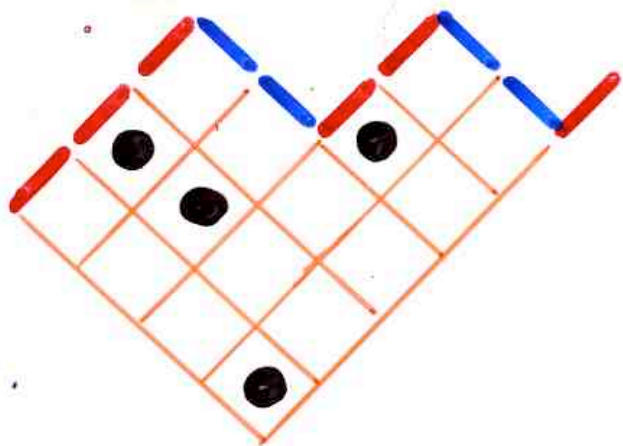
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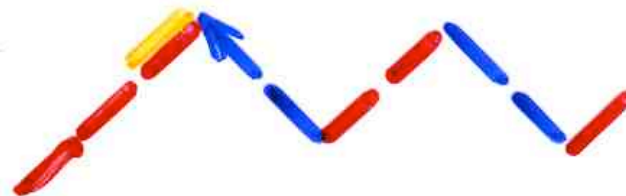
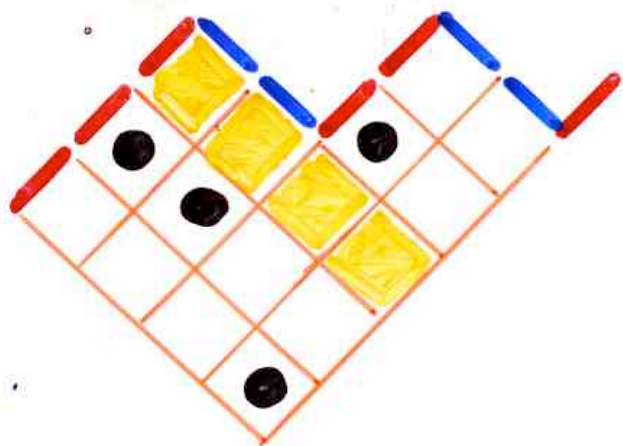
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bijection  
Catalan tableaux  
binary trees

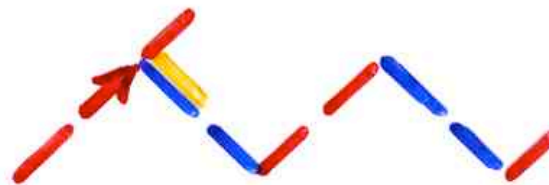
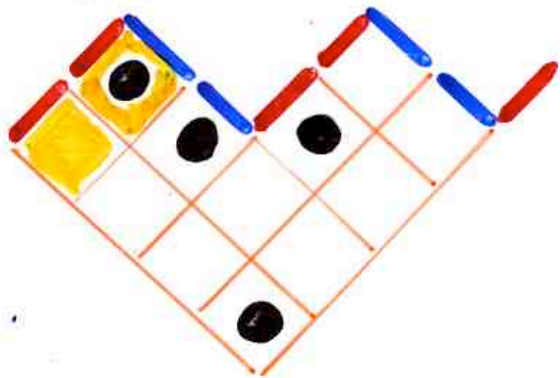


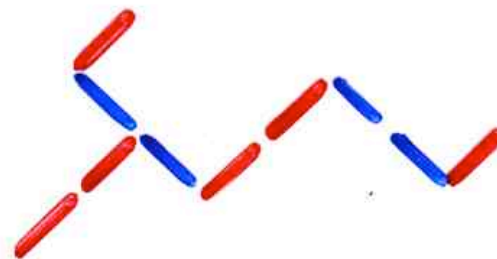
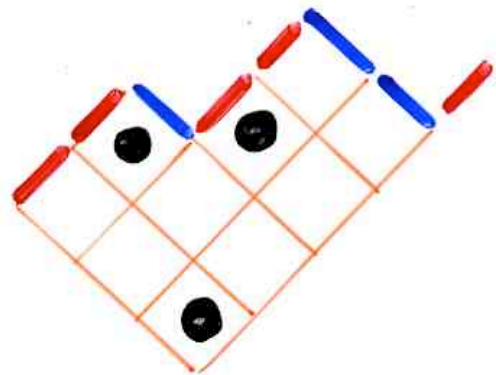


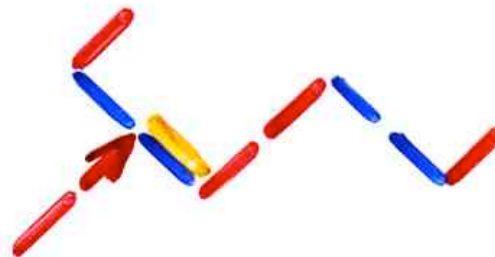
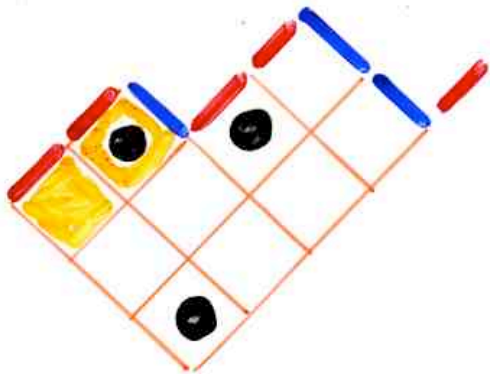


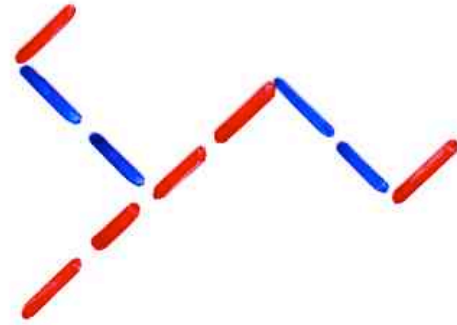
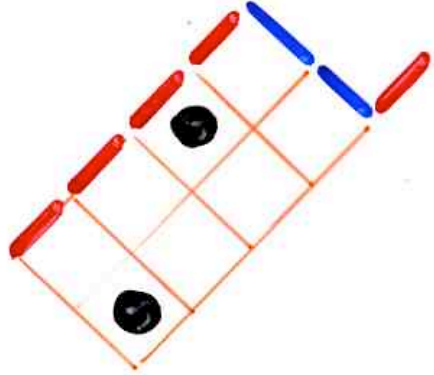




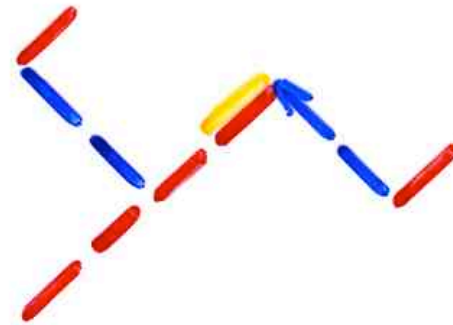
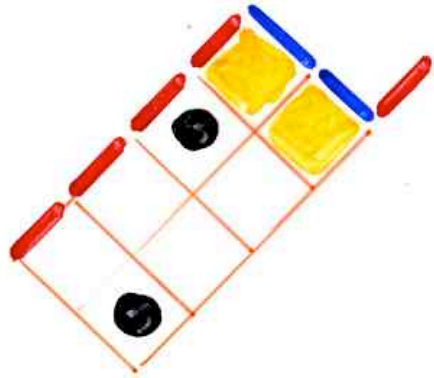


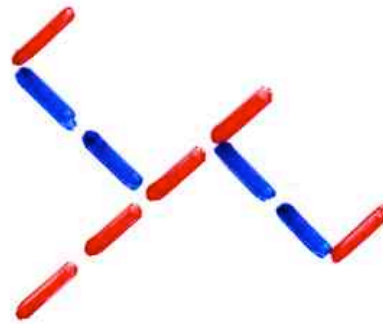
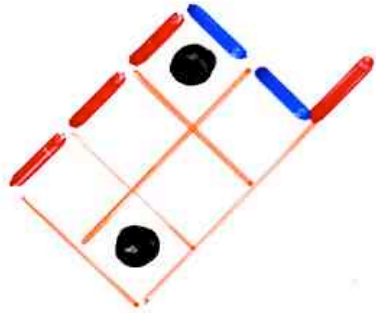


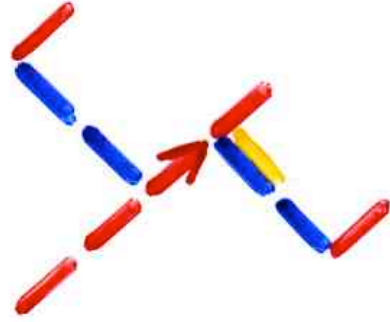


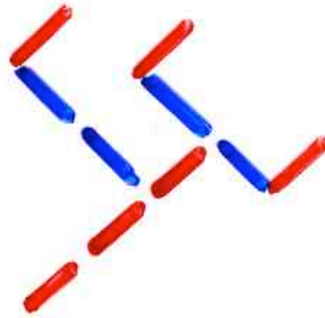
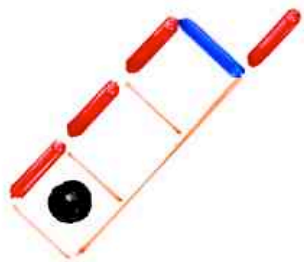


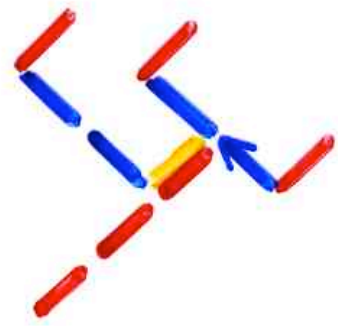


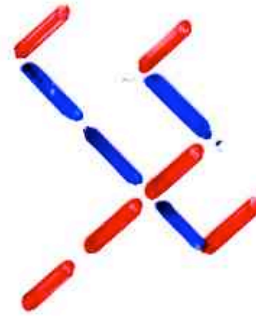


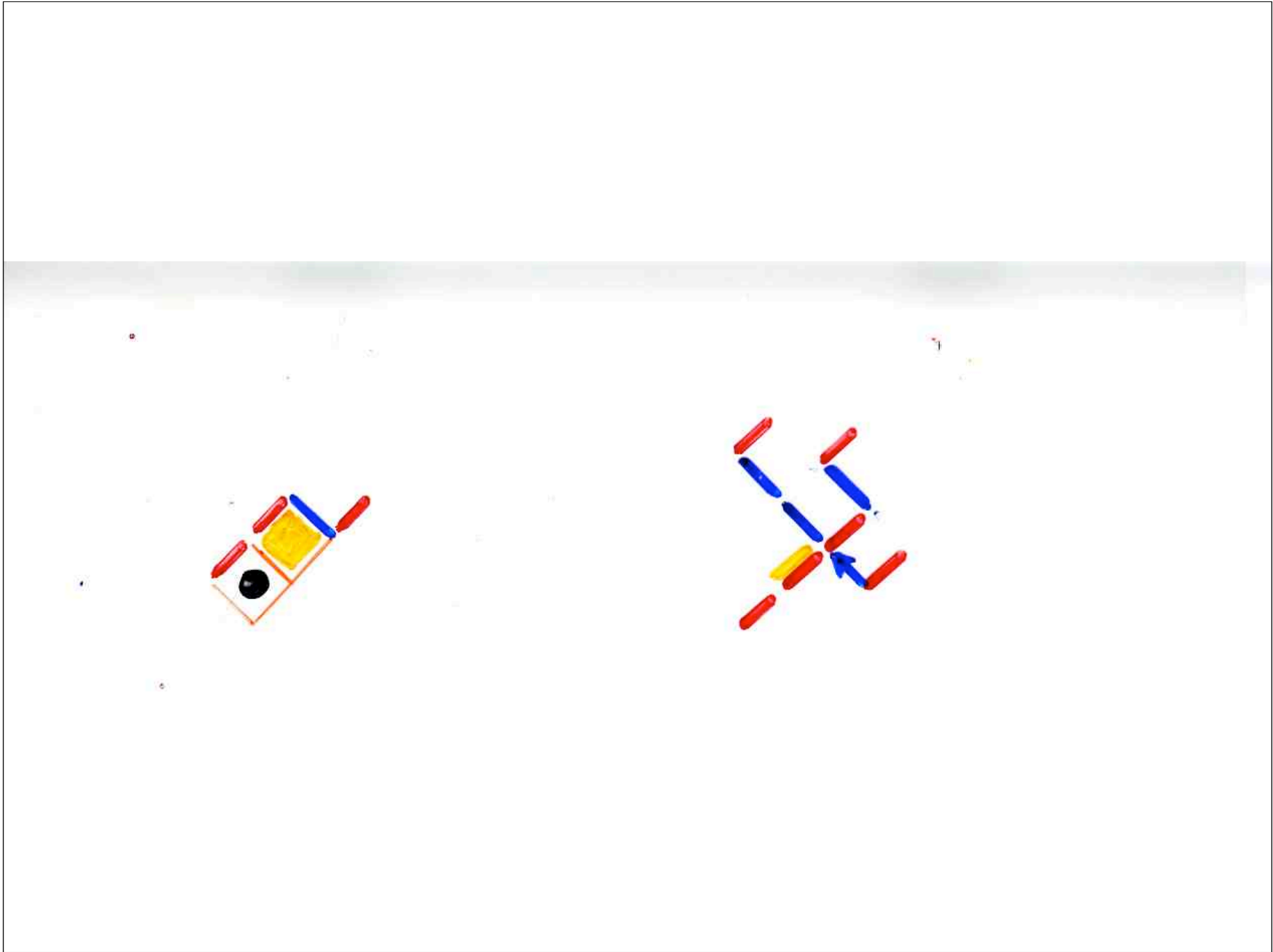


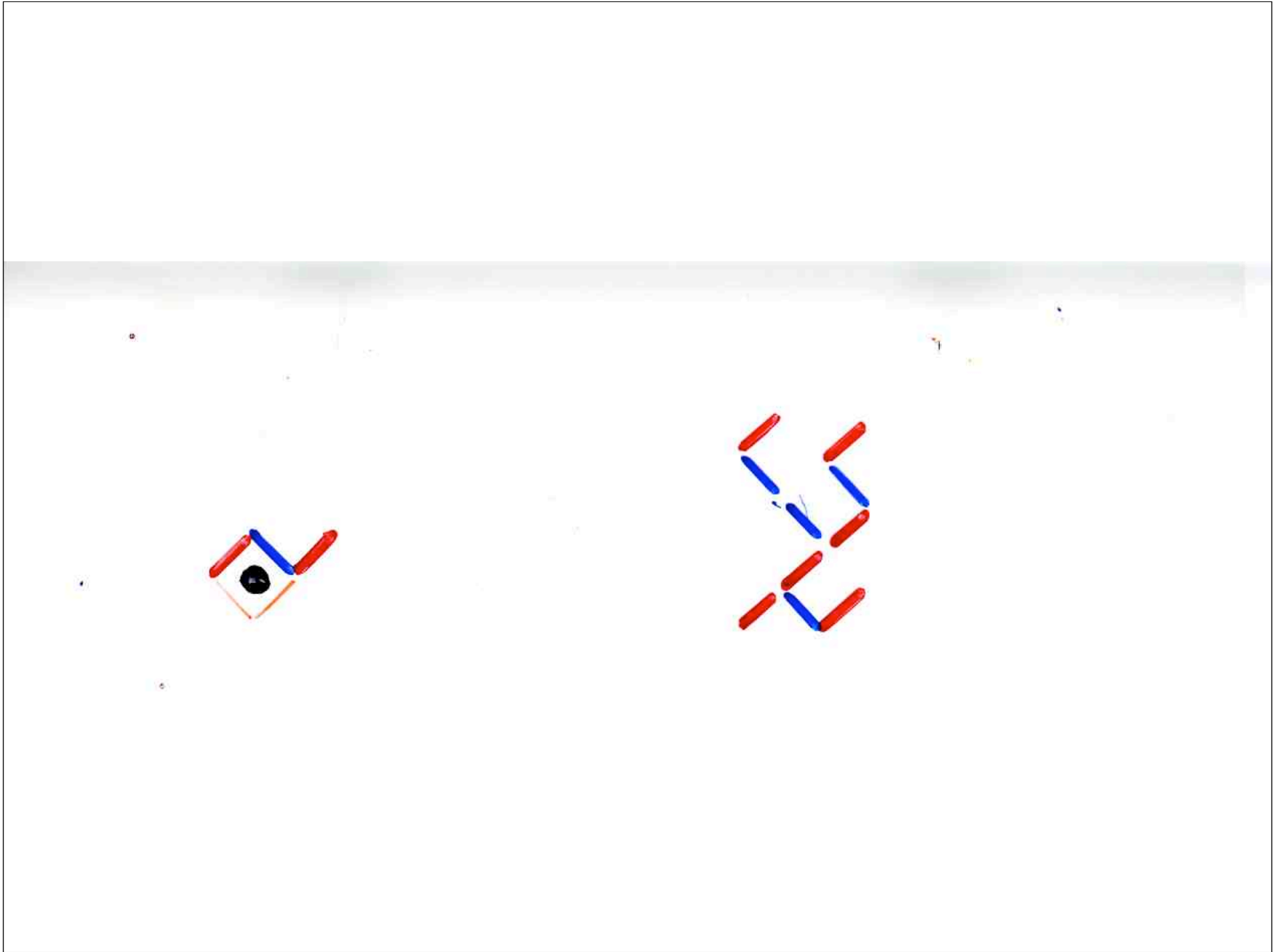




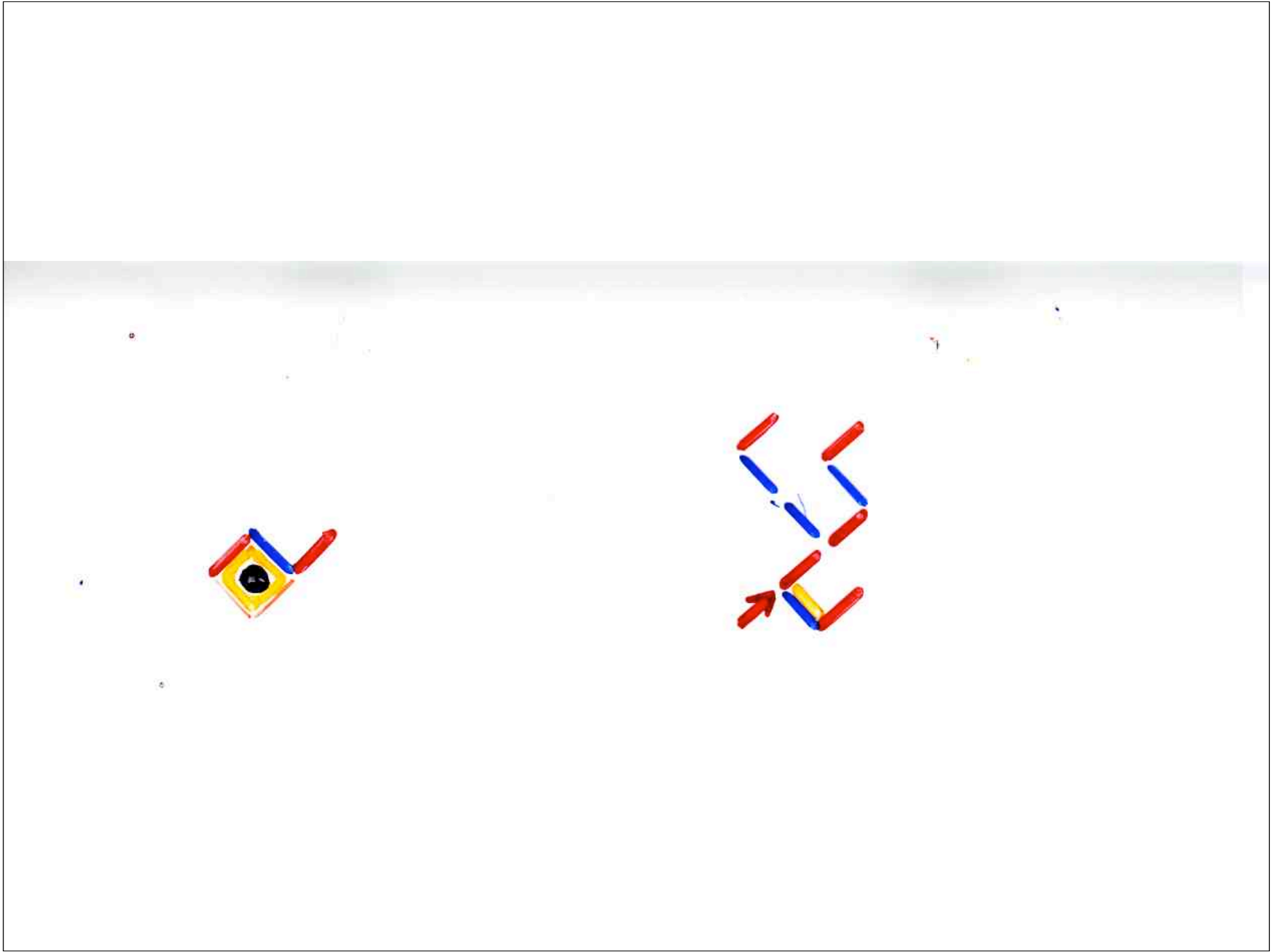


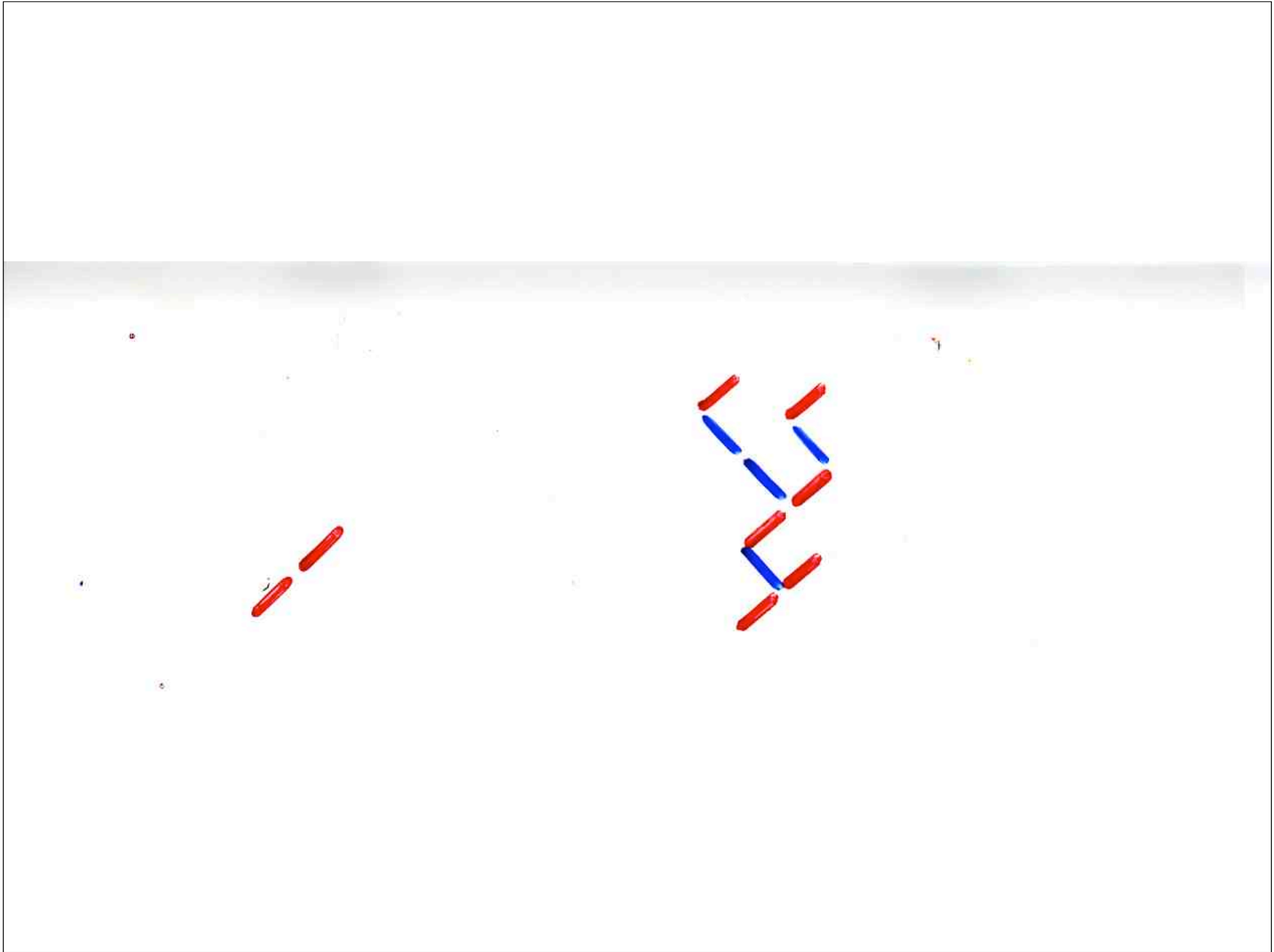


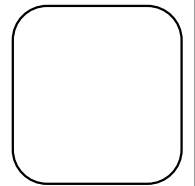
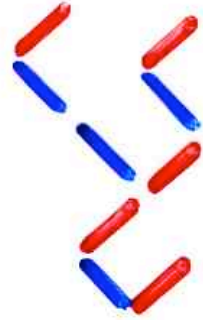












Representation of the  
operators  $D$  and  $E$

Data histories

Calcul du coût intégré  
d'une structure de données  
pour une séquence aléatoire  
d'opérations primitives

Françon, Flajolet, Vuillemin (1980, ...)  
connaissant le coût moyen  
d'une opération primitive.

24

17

10

8

24

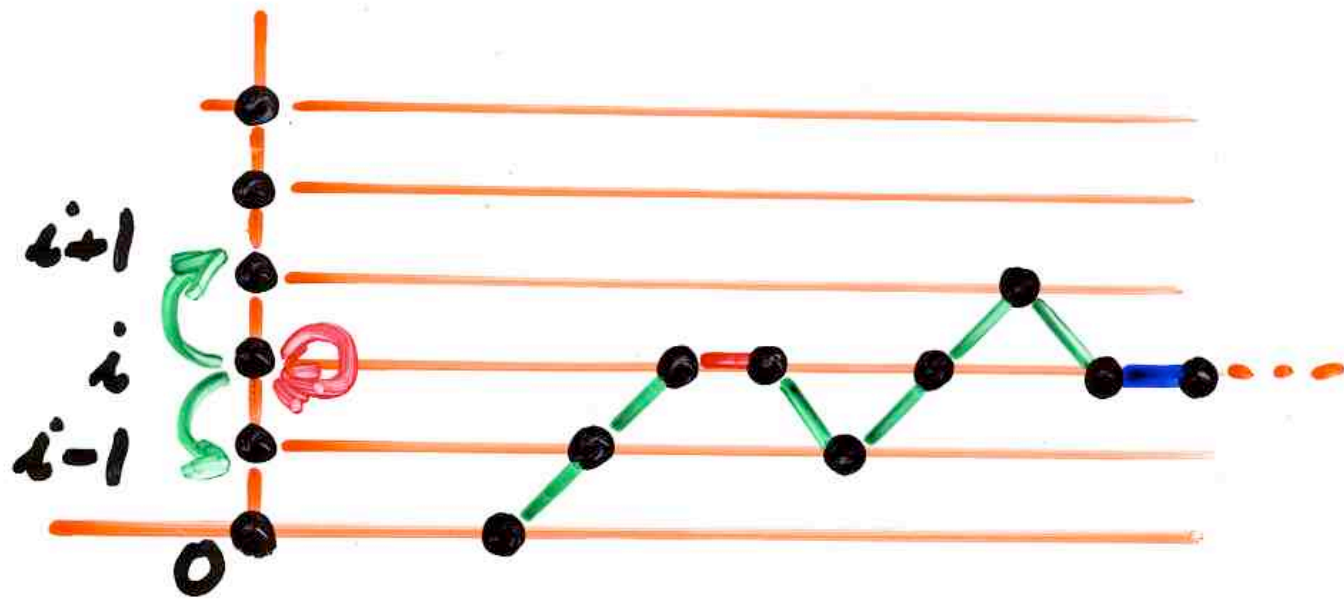
17

← 12

10

8





histoires de fichiers

Françon, (1976)

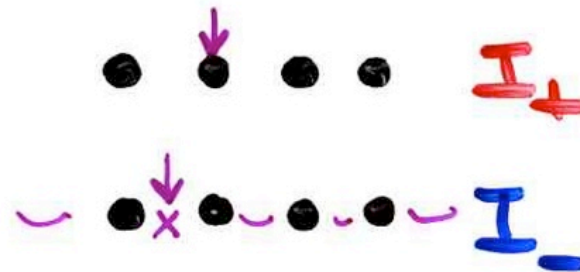
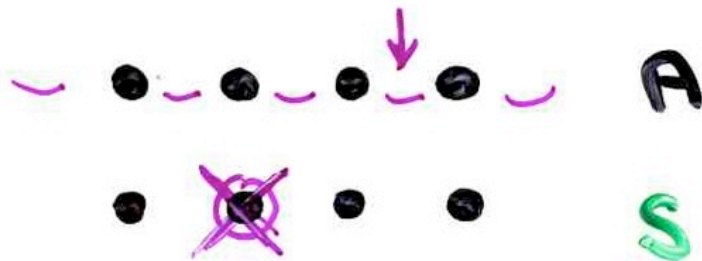


## Opérations primitives

A ajout

S suppression

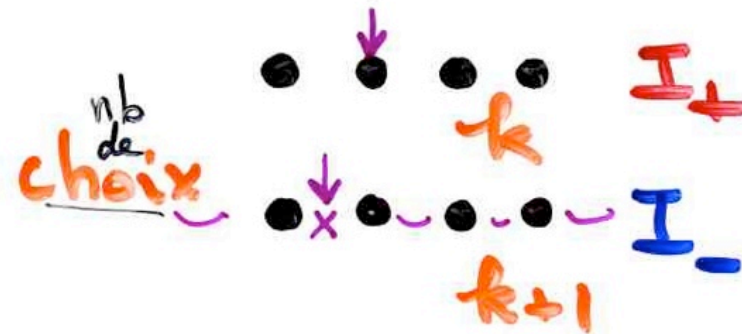
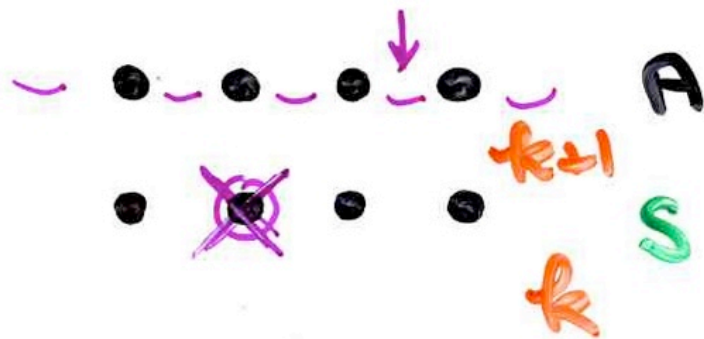
$I_+$  interrogation positive  
 $I_-$  interrogation négative



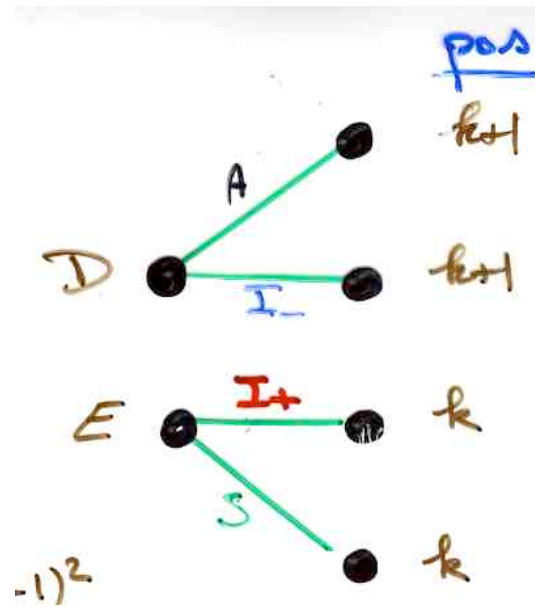
# Opérations primitives

**A** ajout  
**S** suppression

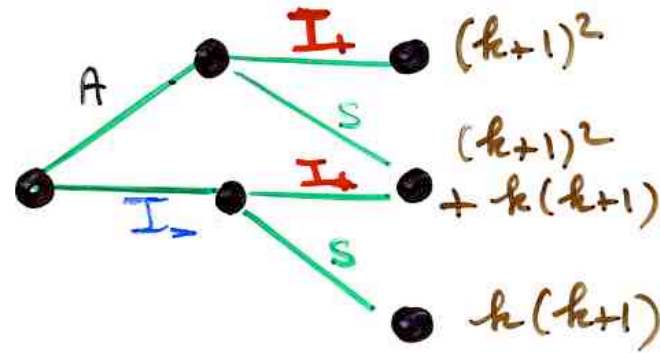
**I<sub>+</sub>** interrogation positive  
**I<sub>-</sub>** interrogation négative



$$\begin{cases} D = A + I_- \\ E = S + I_+ \end{cases}$$



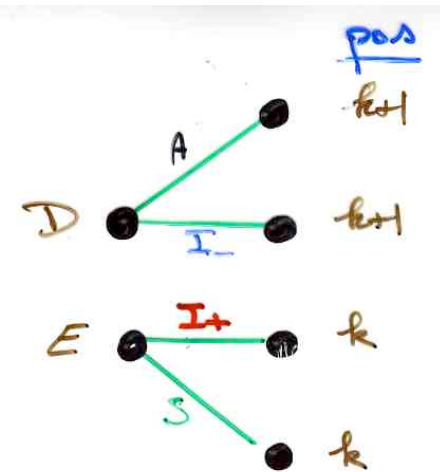
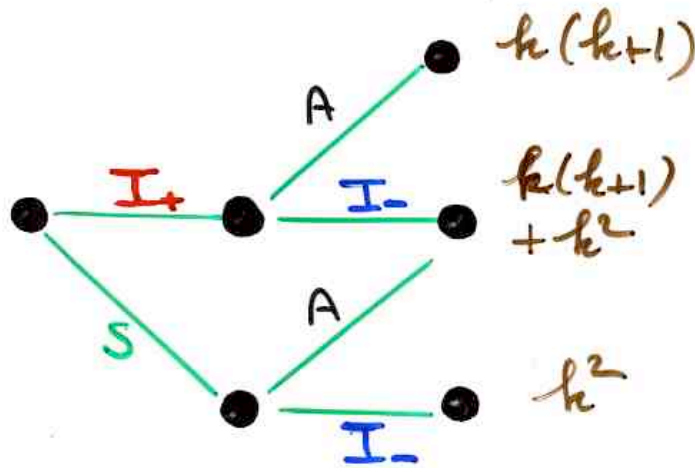
DE



DE - ED



ED

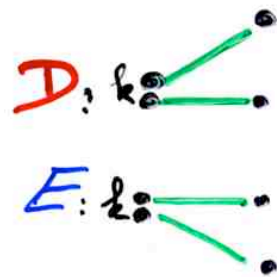


$$DE - ED = E + D$$

$$\begin{cases} D = A + I_+ \\ E = S + I_- \end{cases}$$

$$DE = ED + E + D$$

aussi:



(k+1) possibilités partout

(histoires de Laguerre)  
"larges"