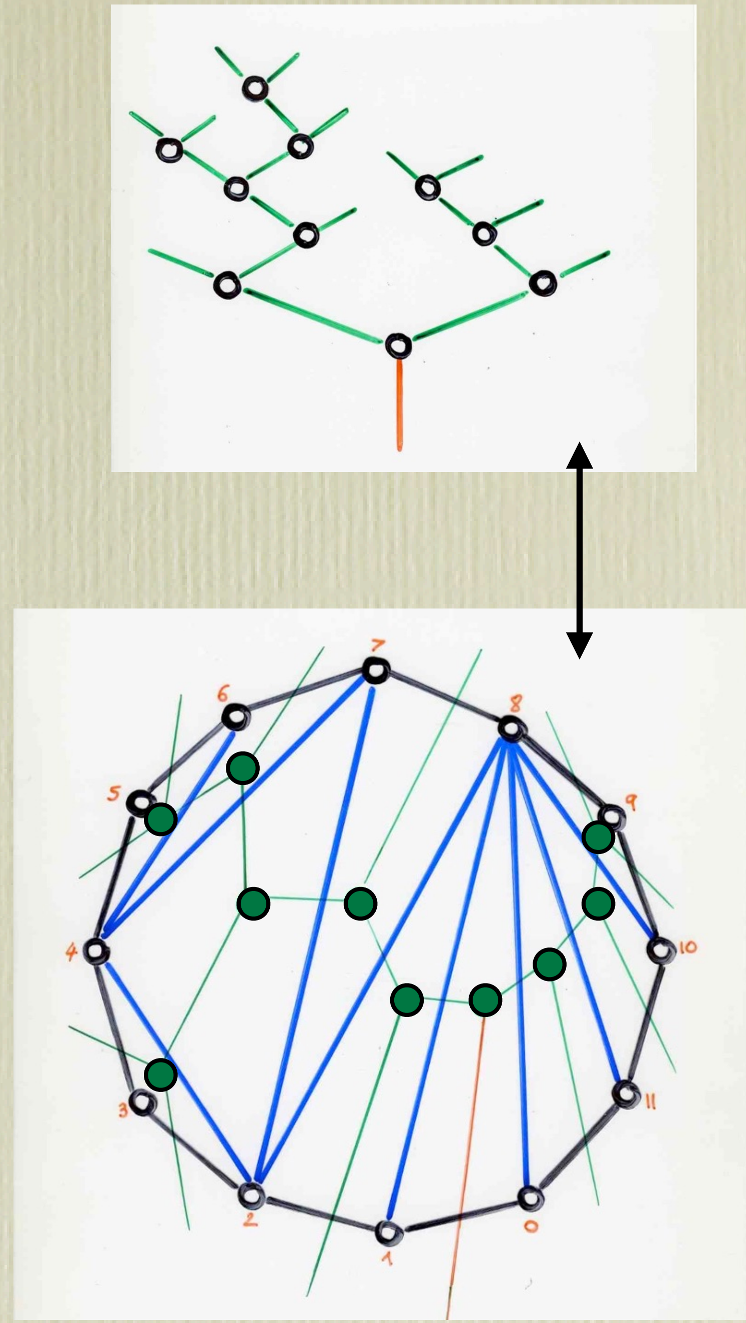
exercise 11

describe the reverse bijection

binary trees $\longrightarrow$ triangulations



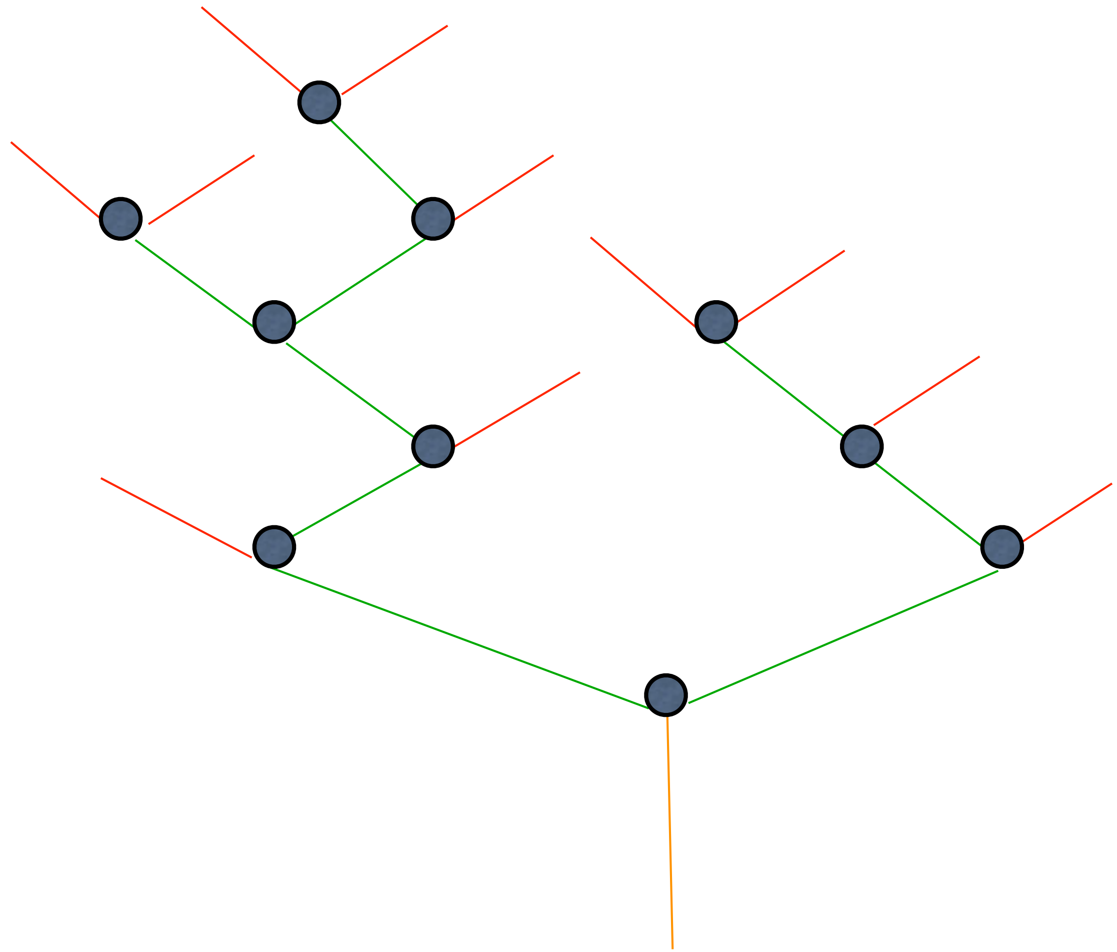
Let B be a binary tree with n (internal) vertices. First we label these vertices by the integers 1,2, …,n such that the labels are increasing when one goes from the root to any external vertex of B (i.e. we get an *increasing* binary tree, see exercise 3).

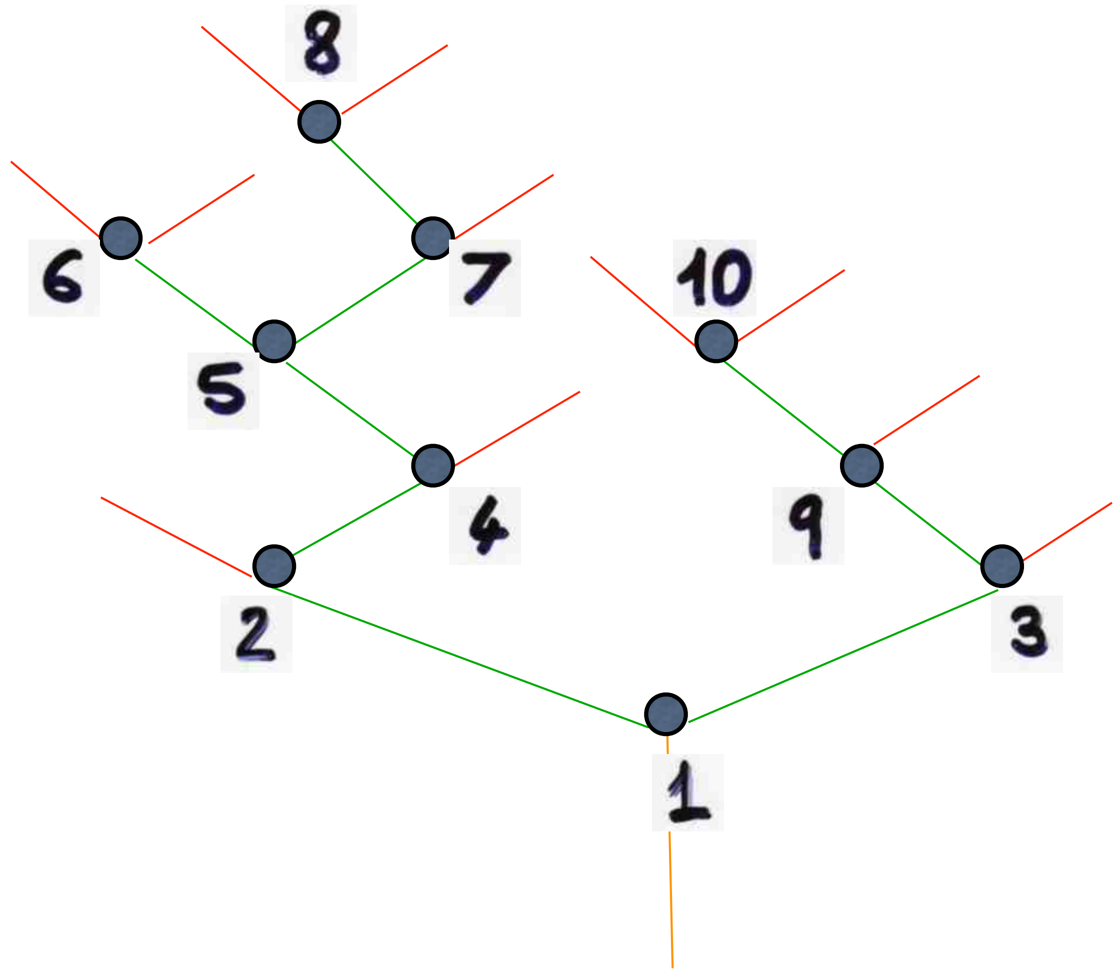
To the root of B (labelled « 1 ») we associate a triangle, labelled by 1, with one edge labelled « *inactive* » and called the « *root edge* » (coloured in orange on the figures). In the algorithmic construction, to each vertex of B we associate a triangle. One of the edge will get a label « *inactive* » (in black on the figures), the two other being labelled « *active* » (in blue on the figures). The triangles are embedded in a plane and we can define the *left edge* (resp. *right edge*) as being the first (resp. second) *active* edge when turning clockwise around the triangle, starting from the (unique) *inactive* edge. (see Figure below).

During the construction, after reading the vertices labelled 1,2,…,i of the binary tree B, we get a triangulation of a polygon with i+2 edges, all of them are active, except the root edge, the other edges of the triangles (i.e. the diagonals of the triangulation) are inactive. Each triangle is labelled by an integer j, $1 \leq j \leq i$.

Step (i+1). In the increasing binary tree B, the vertex labelled (i+1) is the left (resp. right) son of the unique vertex labelled j. On the triangle labelled j, we add a new triangle on the left (resp right) active edge. This edge become inactive. This triangle is added « outside » of the polygon and is labelled (i+1).

At the end after n steps, we get a triangulation of a (convex) polygon having (n+2) edges, one of them being distinguished as the root. The polygon is « labelled », we mean that it is not defined up to a rotation. This triangulation is independent of the increasing labelling ot the binary tree B.

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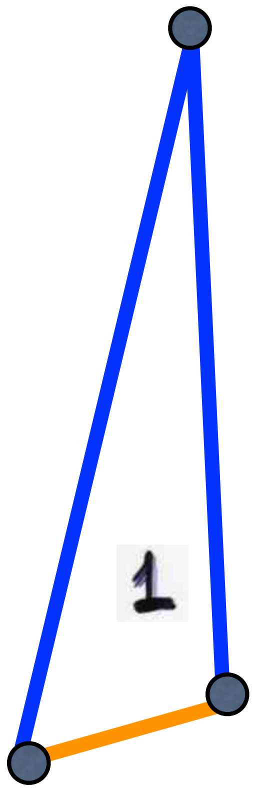
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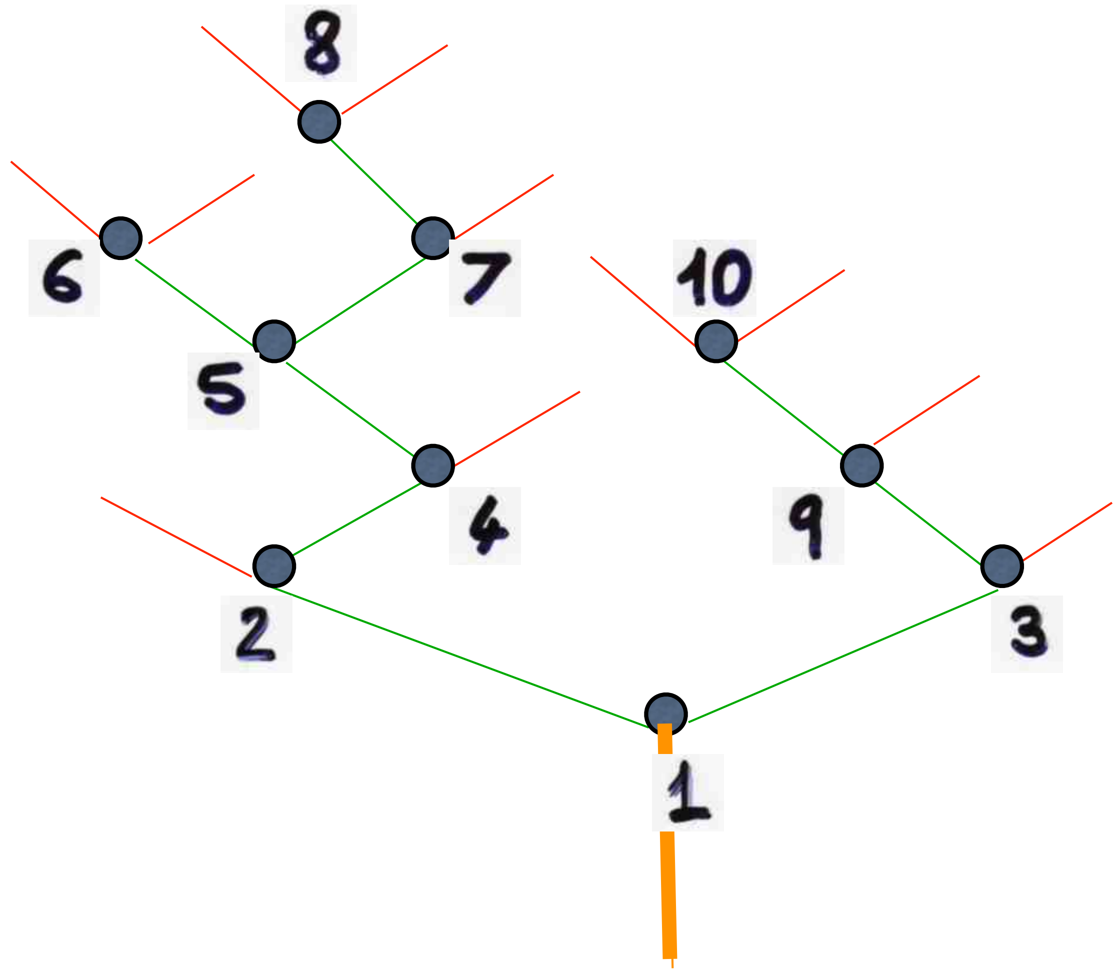
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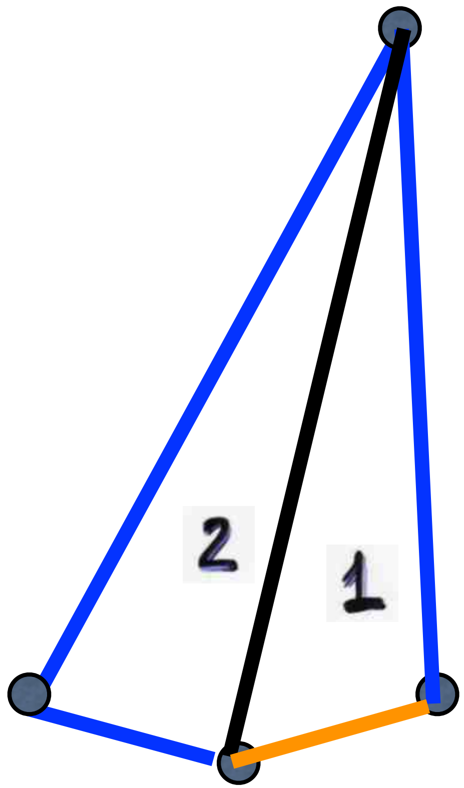
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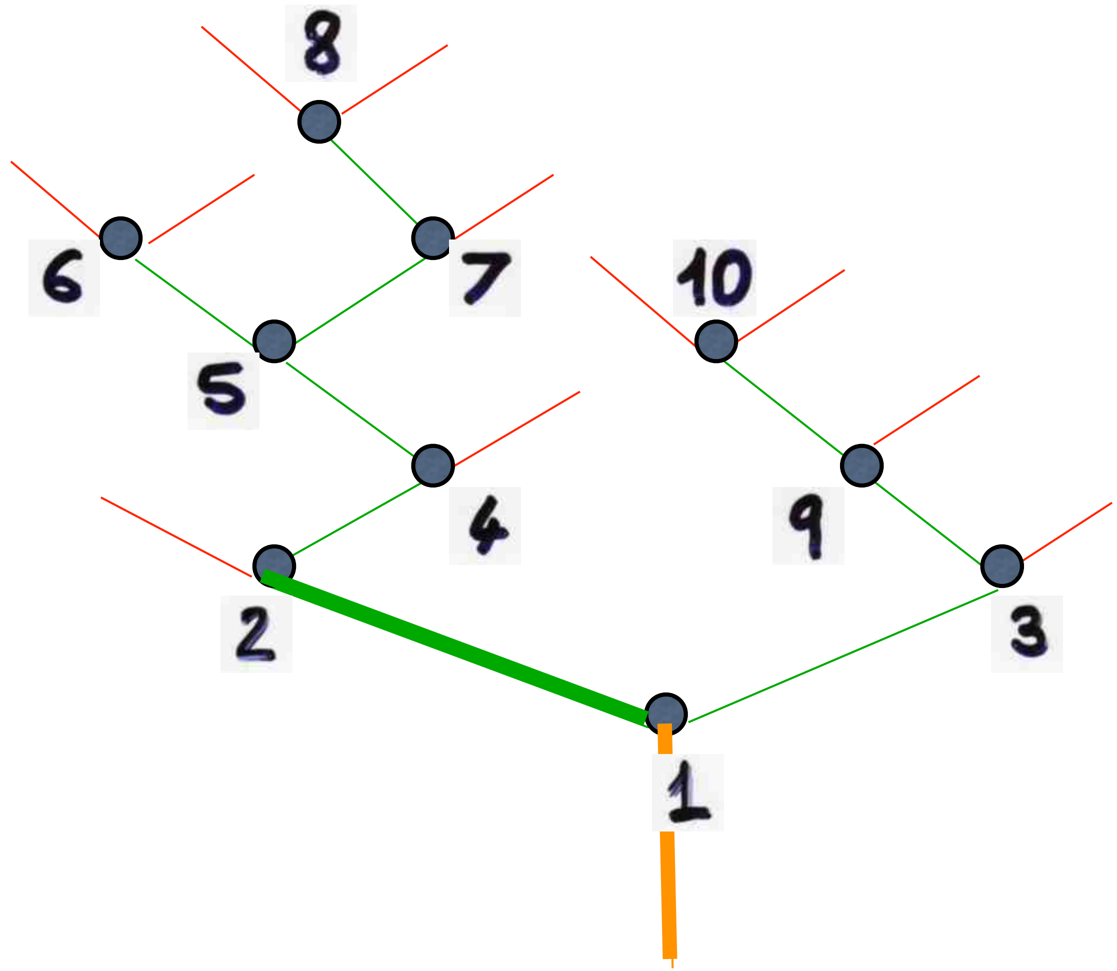
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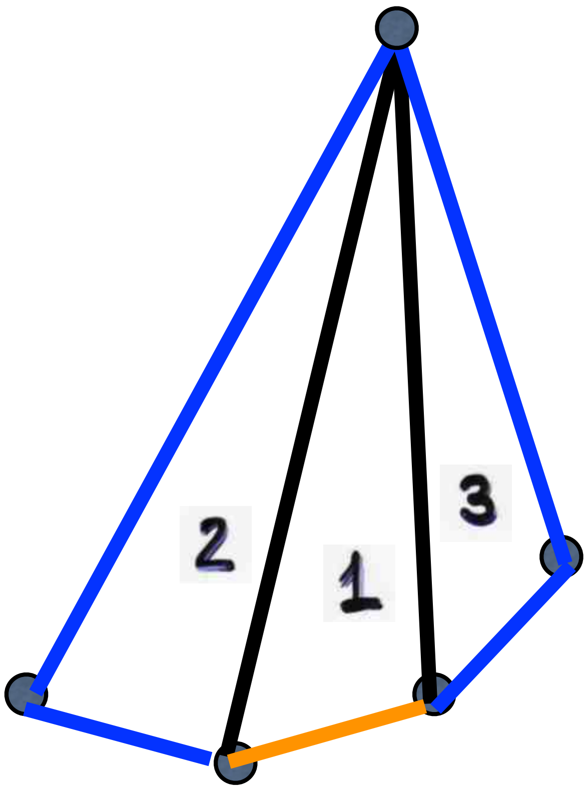
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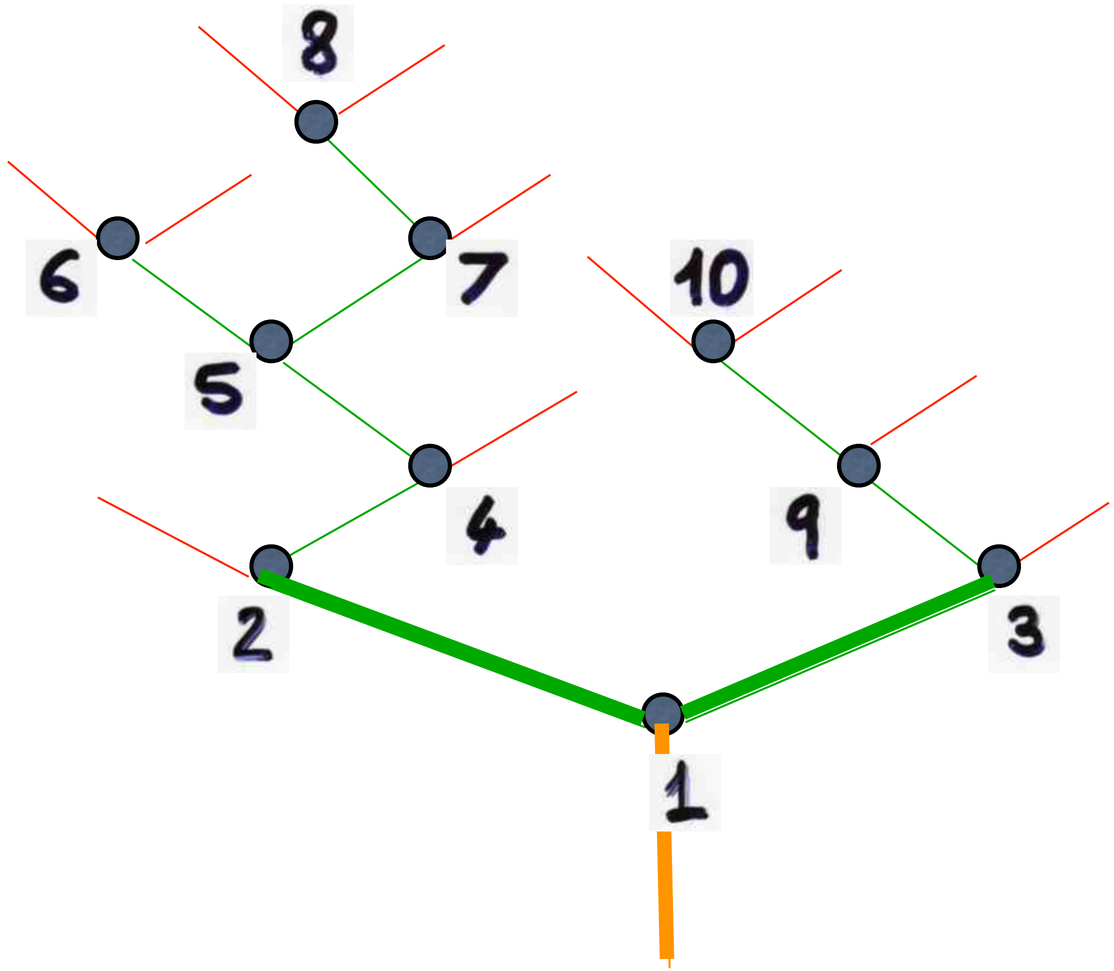
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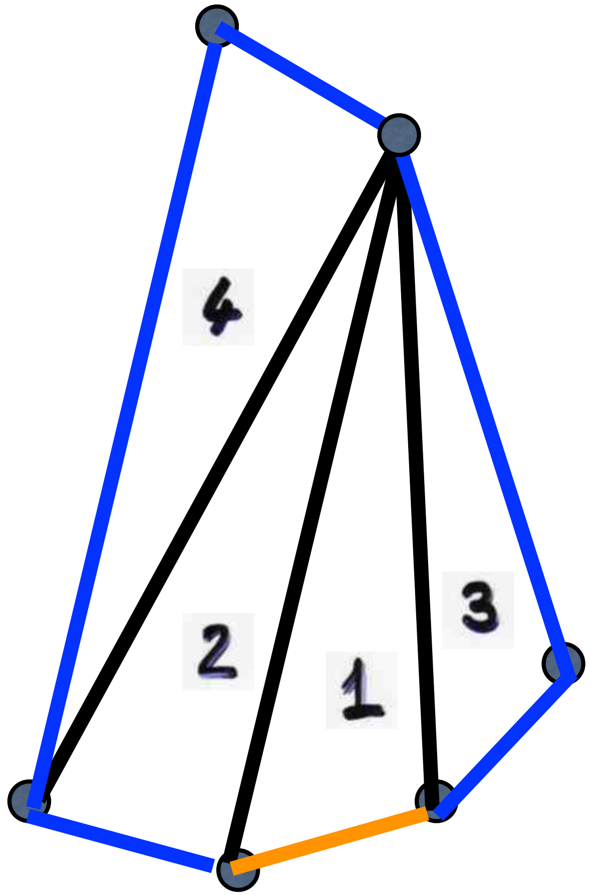
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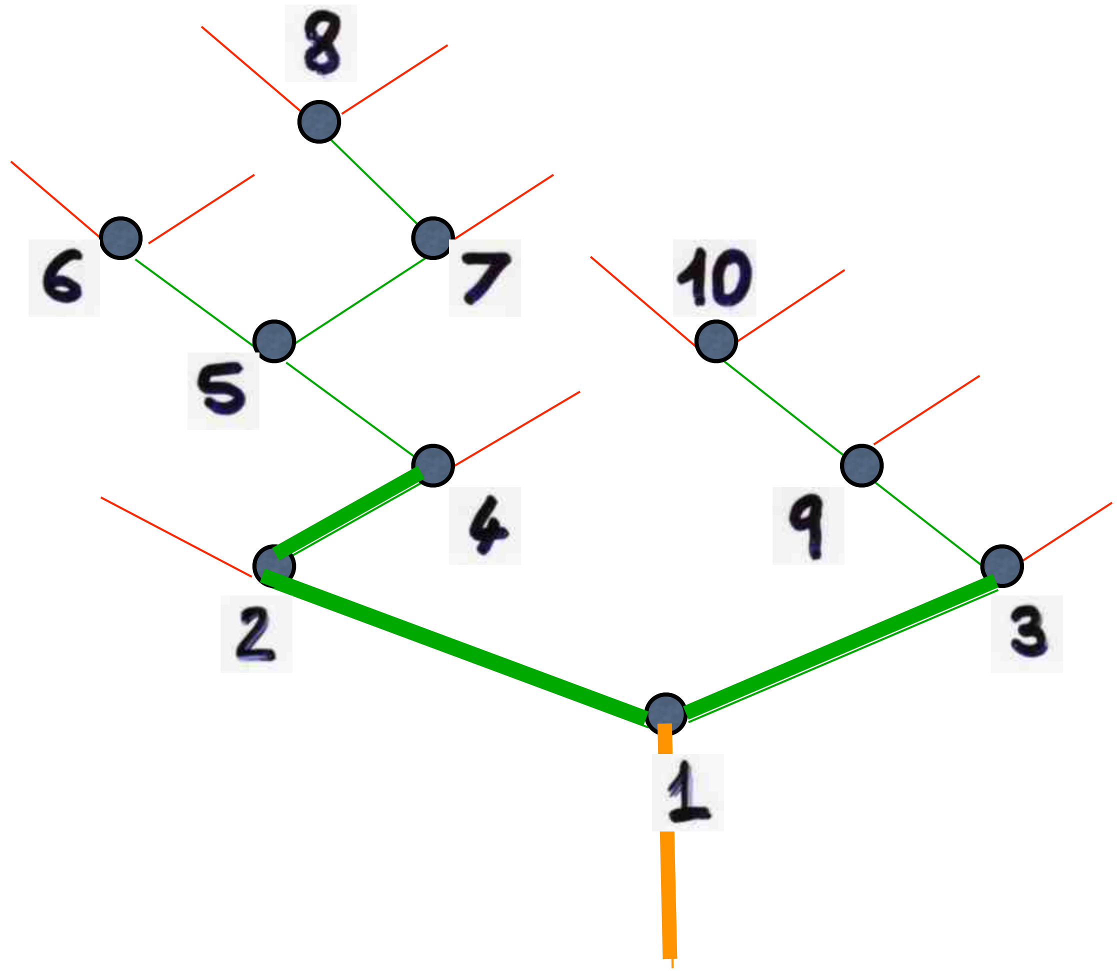
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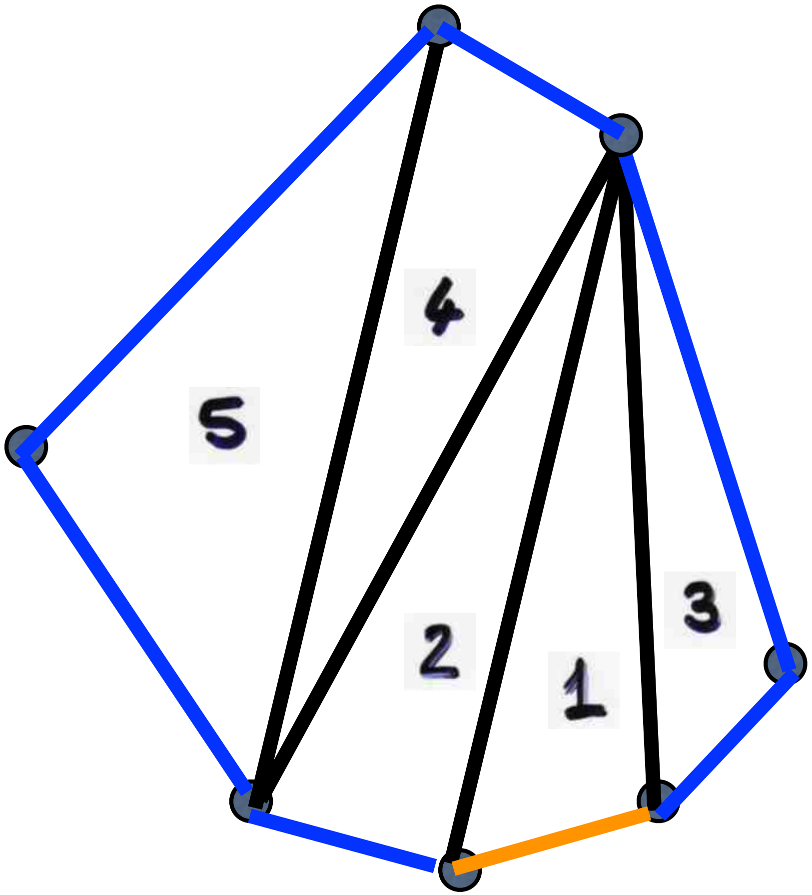
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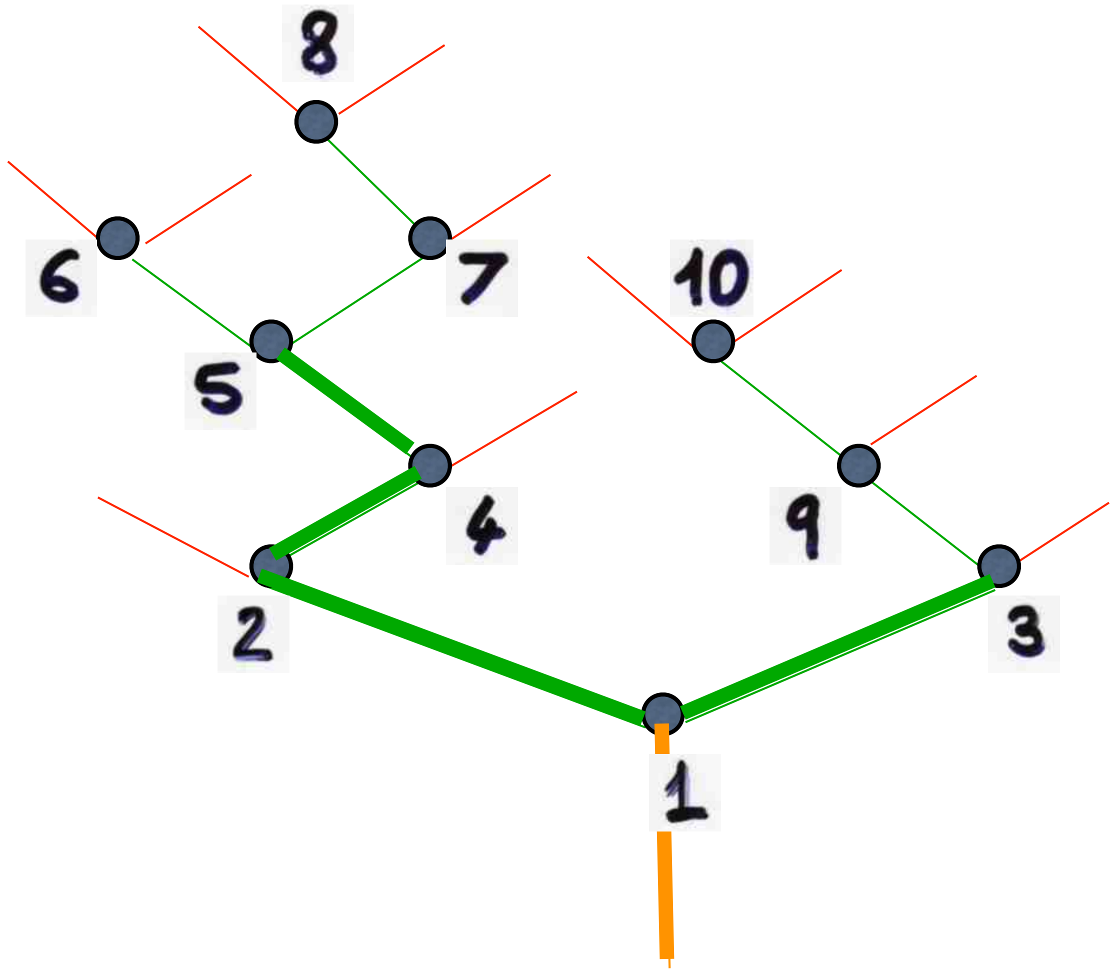
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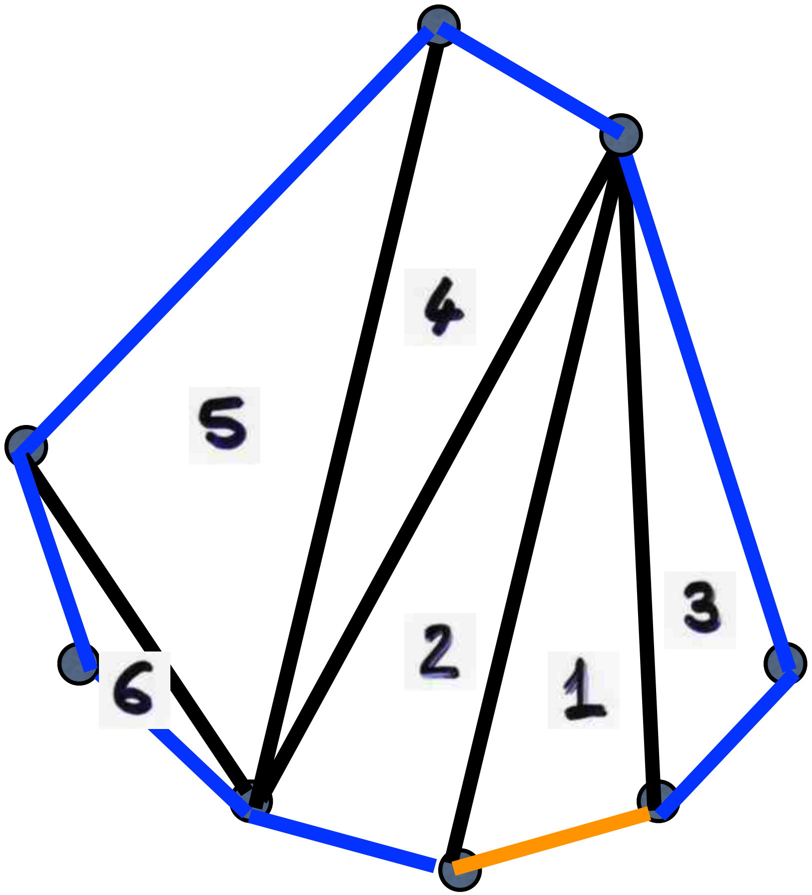

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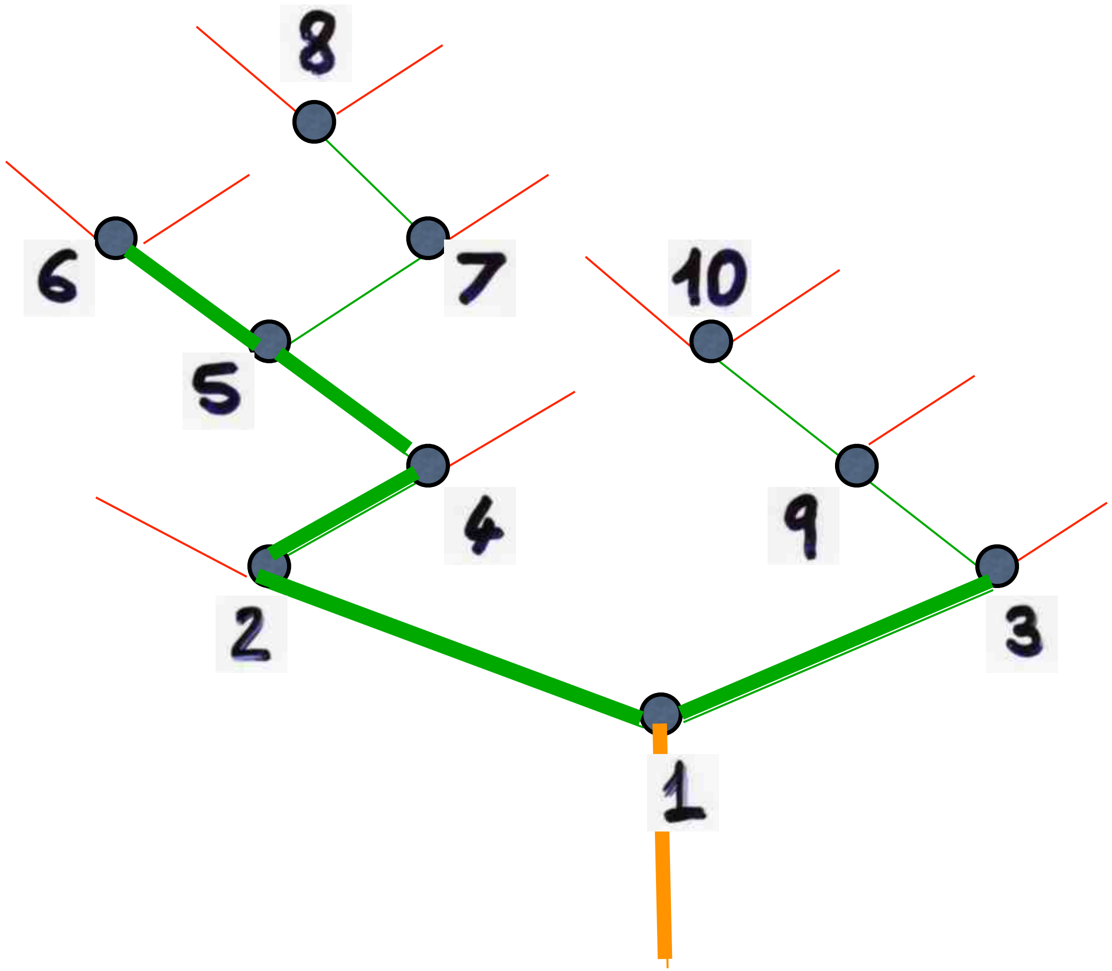

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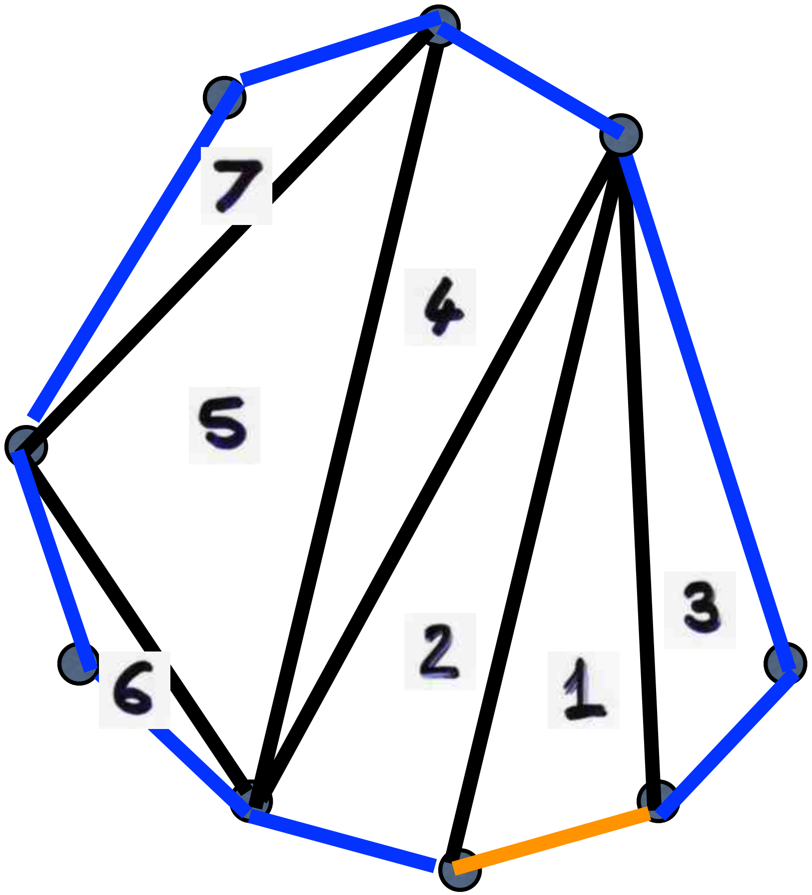
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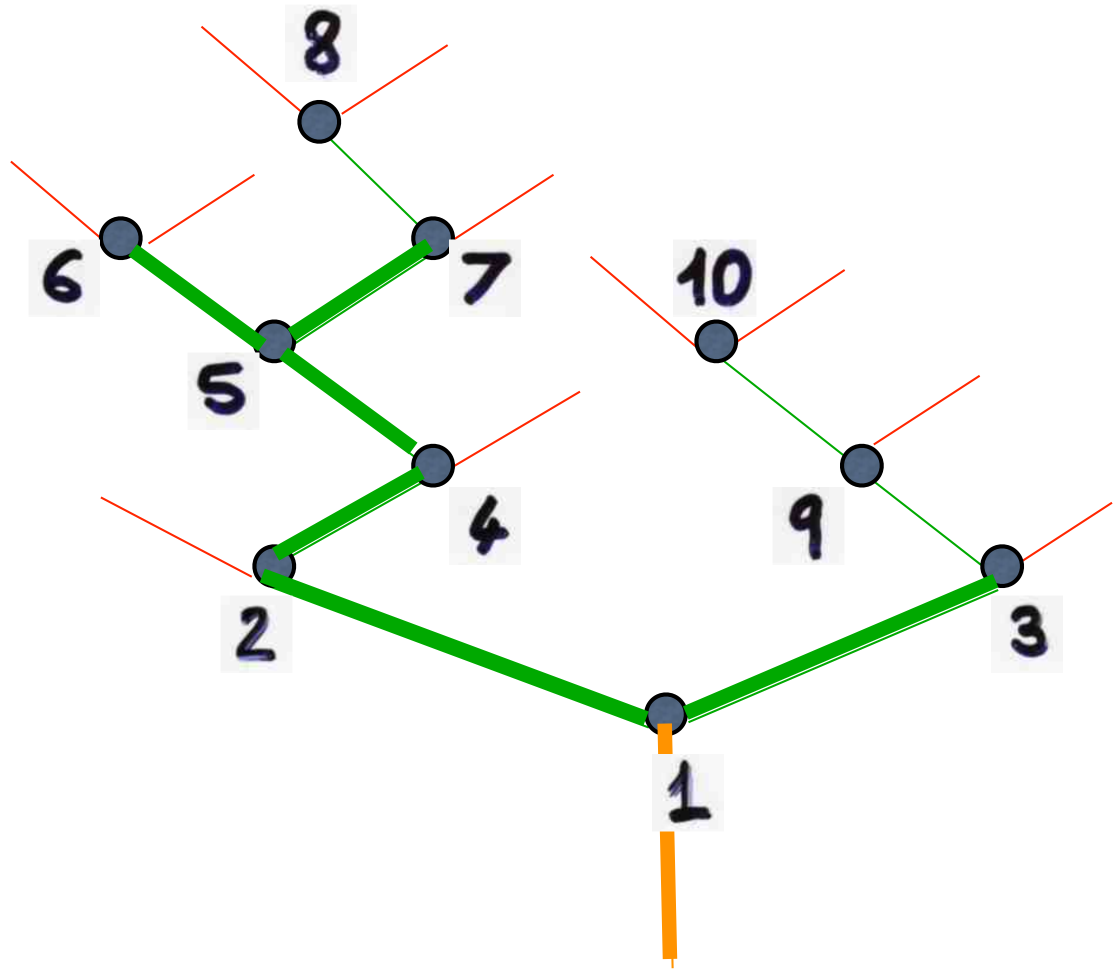
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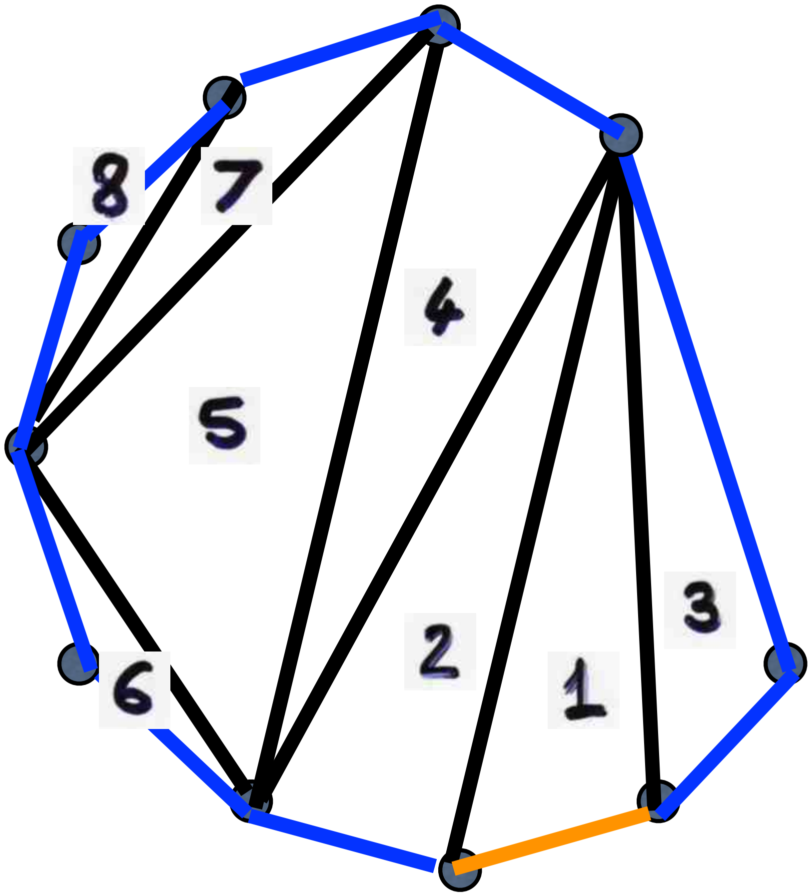
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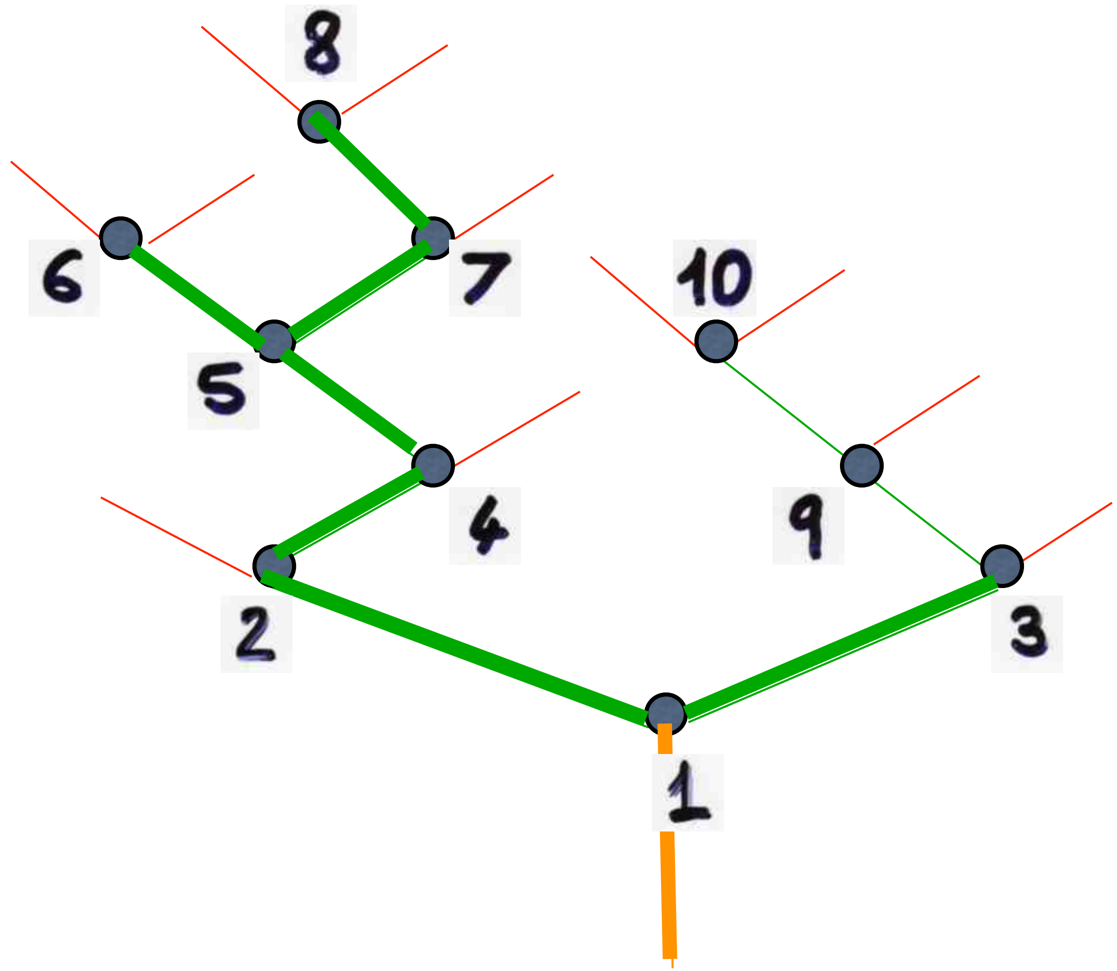
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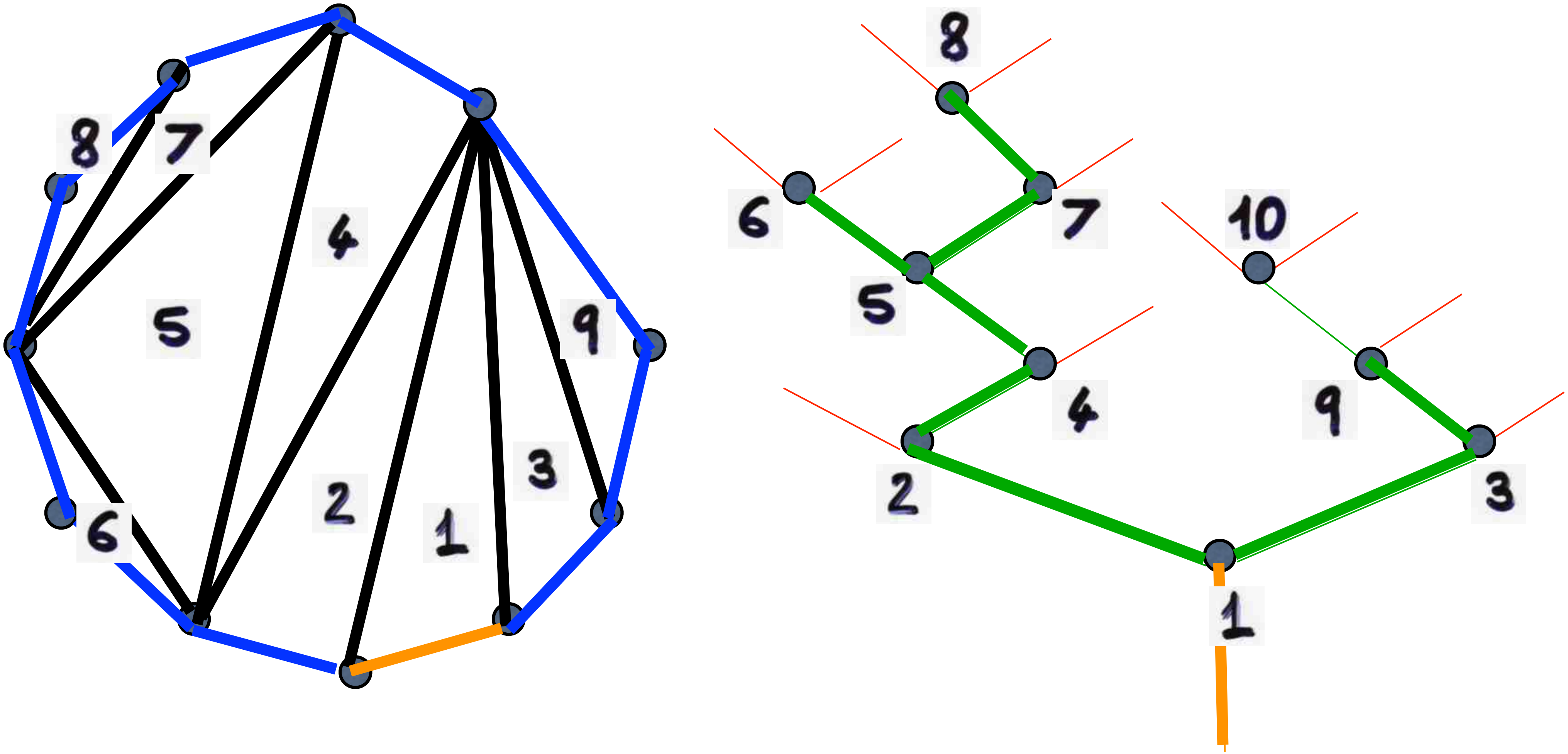

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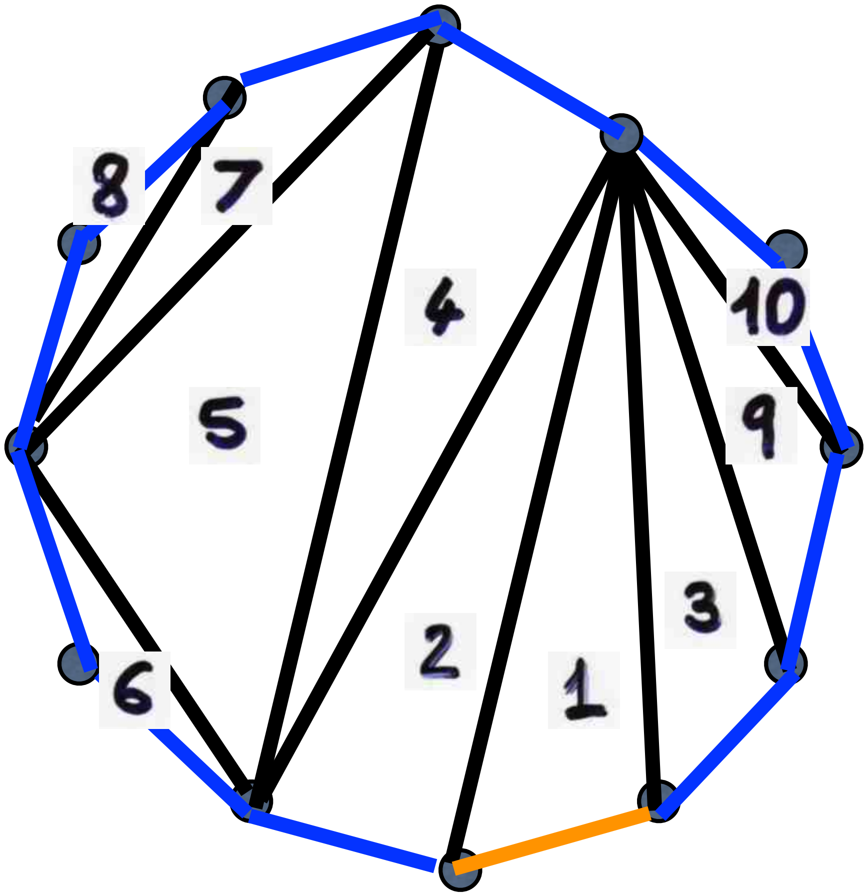
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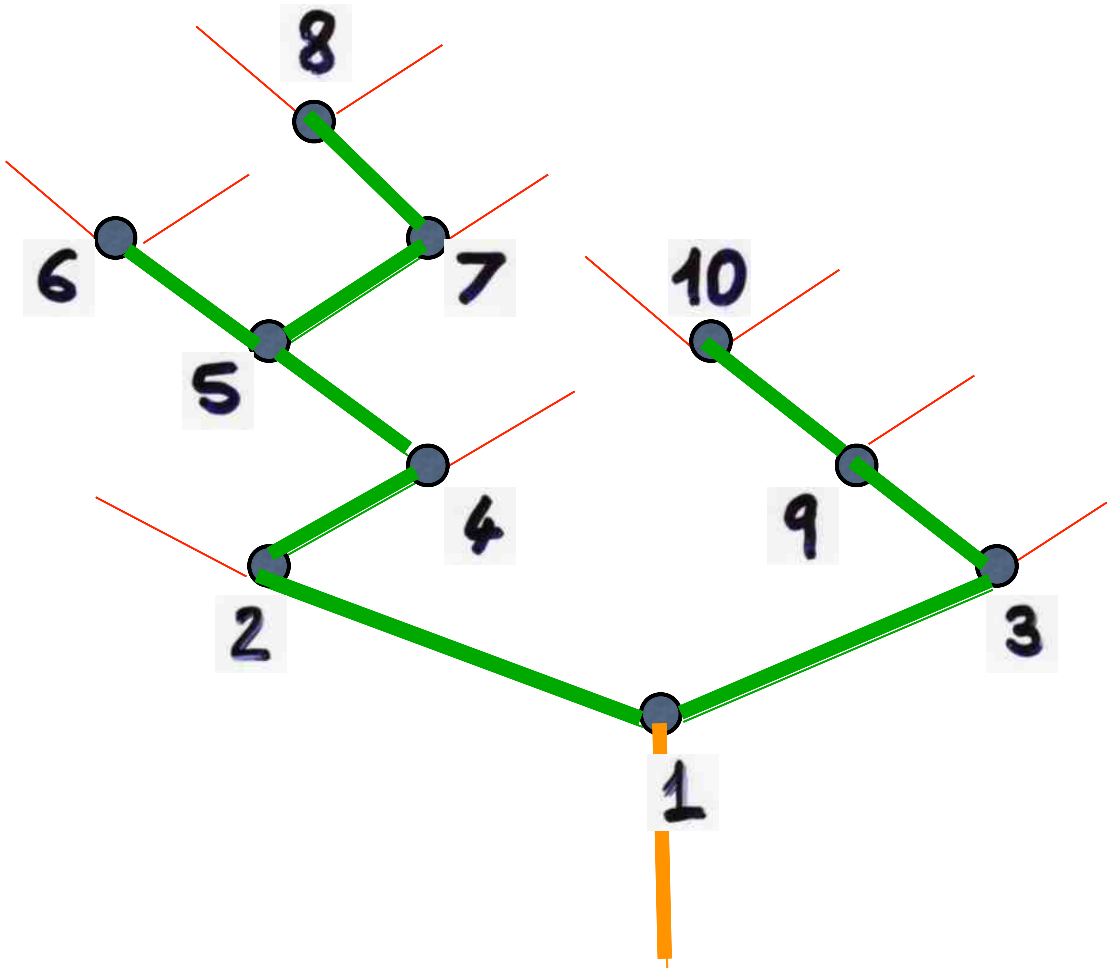
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