

“Des arbres dans les étoiles,
des arbres dans les grains de lumière”

Xavier Viennot

avec

Marcia Píg Lagos (textes)
Gérard H.E. Duchamp (violon)

Nancy

30 Mai 2013

BERNARD CLAVEL

Arbres de la nature,
des arbres partout

















CORAIL



DECHARGE ELECTRIQUE



DEPOTS ELECTROLYTIQUES

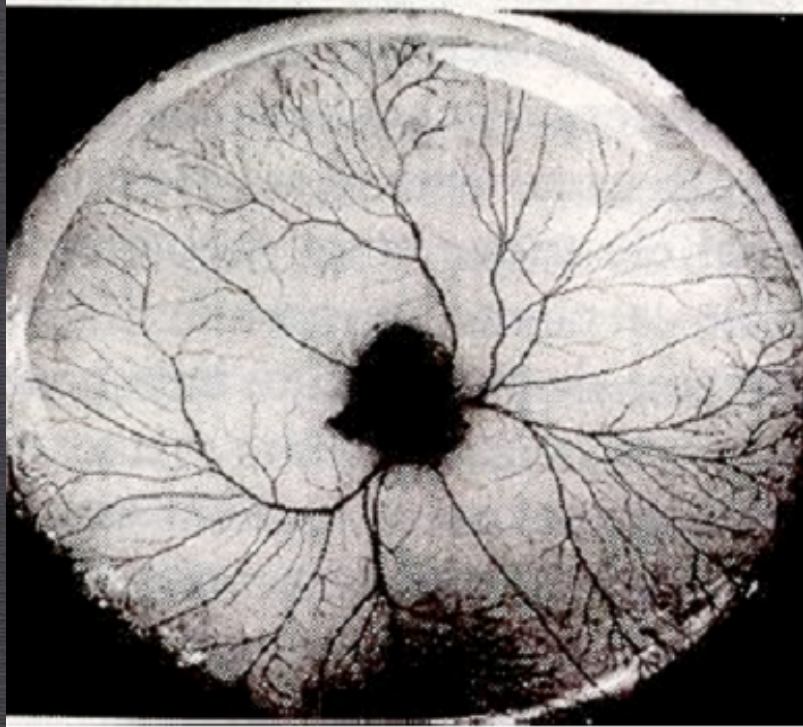
VINCENT FLEURY



DIGITATION VISQUEUSE

INJECTION D'HUILE ENTRE
DEUX PLAQUES

POUMON



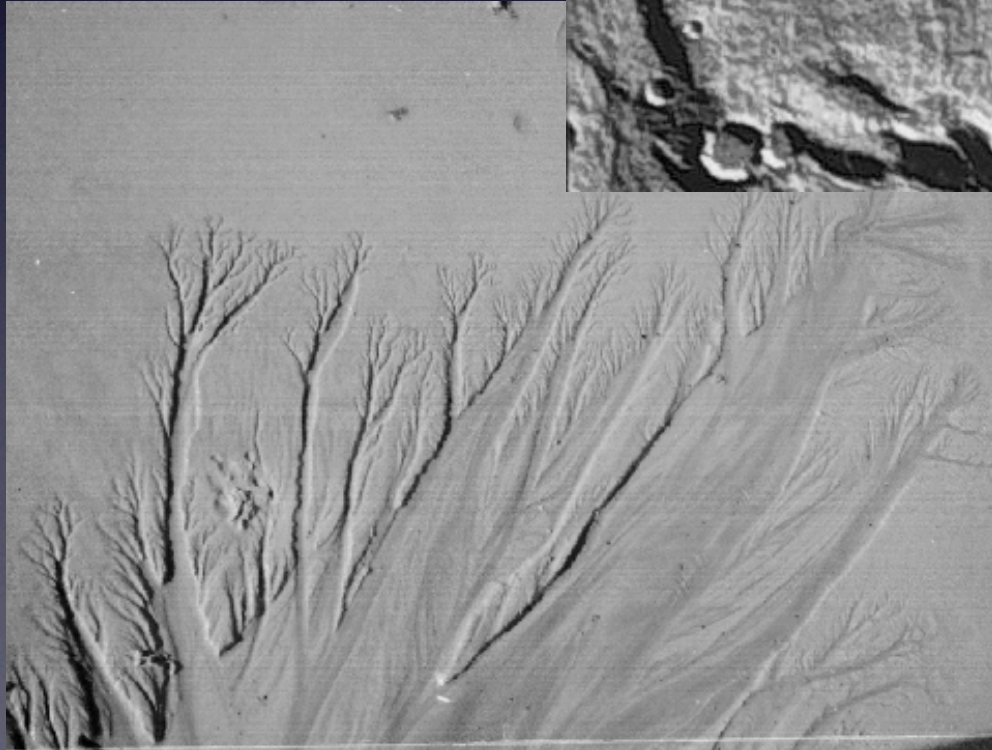
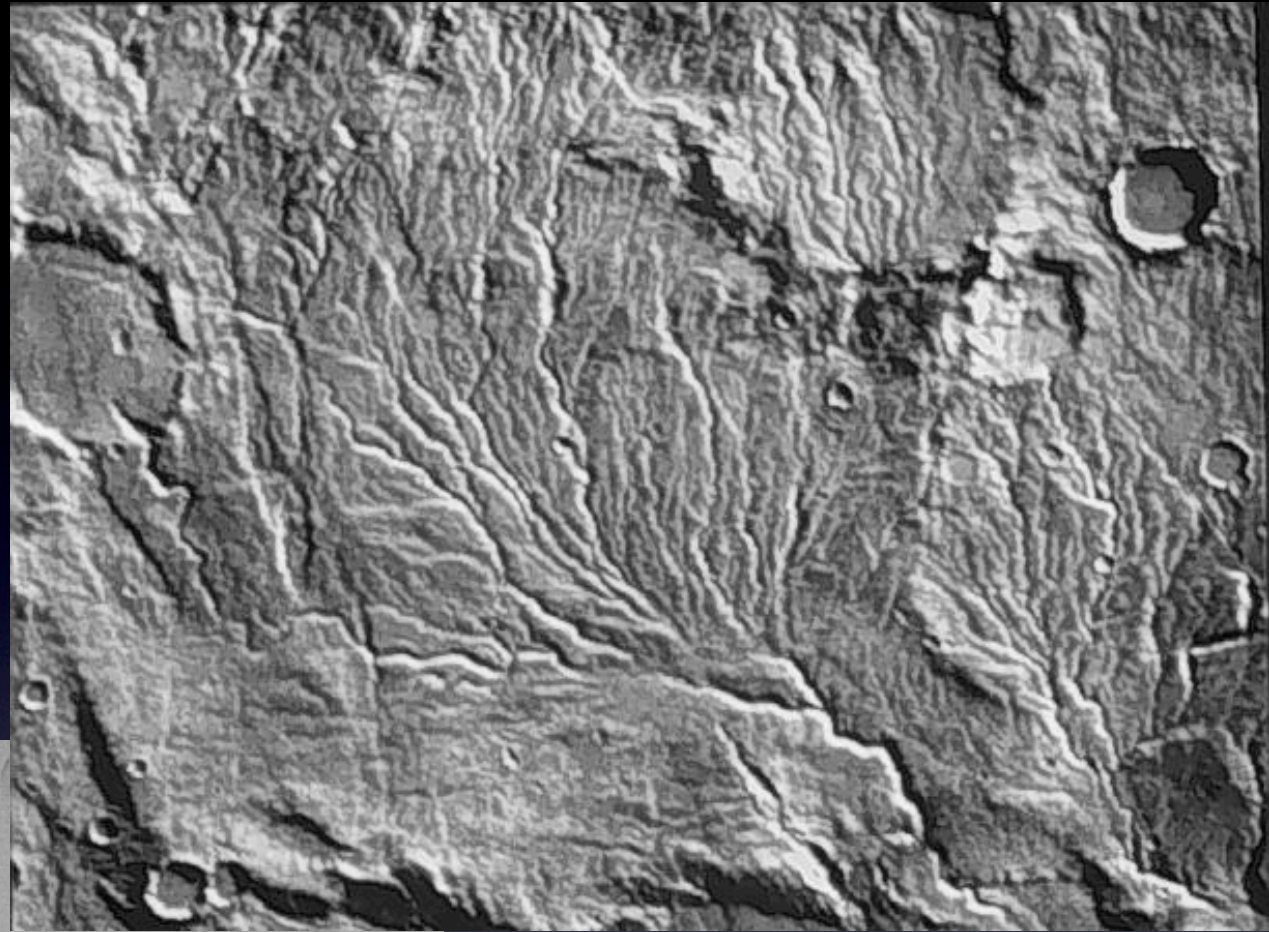
OEUF





NATIONAL GEOGRAPHIC

SUR MARS



SUR TERRE
SUR UNE PLAGE



DES ARBRES
DES ARBORESCENCES
PARTOUT

Des arbres dans les étoiles ?



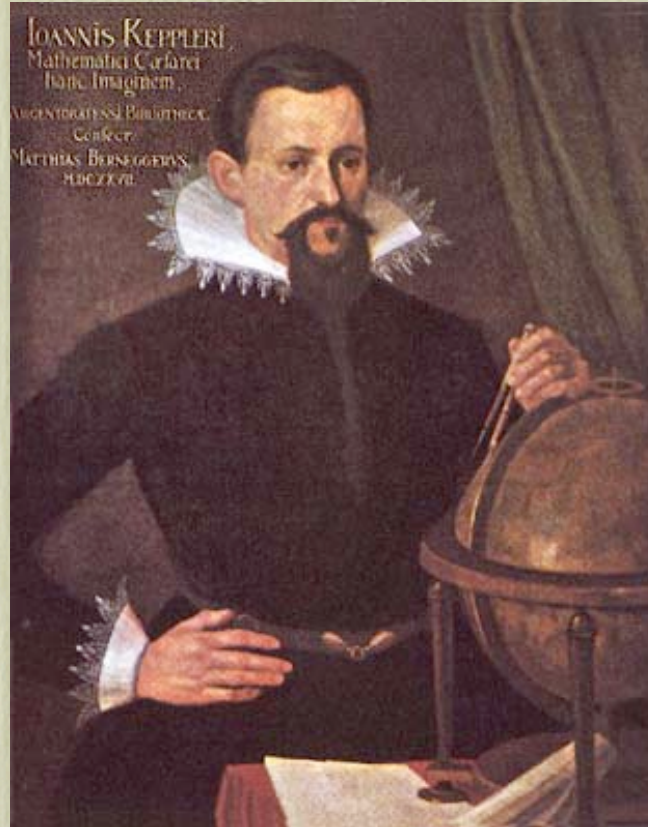
L'infiniment grand ...

Les étoiles, les planètes, les galaxies,
l'univers, sa naissance et son histoire,
espace, temps, matière

comprendre l'univers avec les mathématiques



Galileo Galilei
1564-1642



Johannes Kepler
1571 - 1630



Isaac Newton
1643-1727

géométrie
classique

mécanique
classique

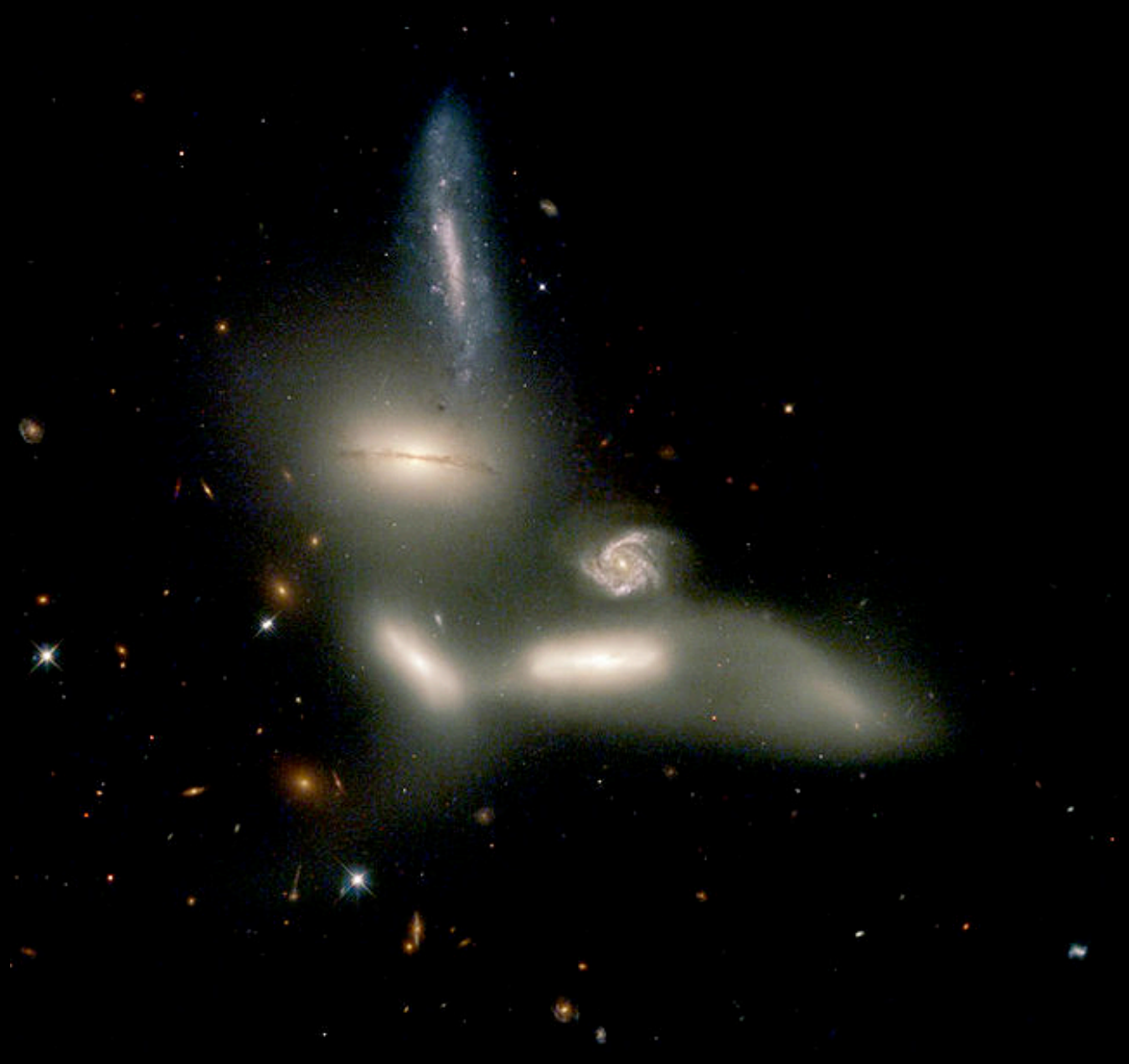
la géométrie euclidienne



Albert Einstein
1879-1955

Théorie de la relativité restreinte générale

gravitation





des arbres dans les
grains de lumière ?





collégiale Notre-Dame Vernon



Daniel B. Holeman

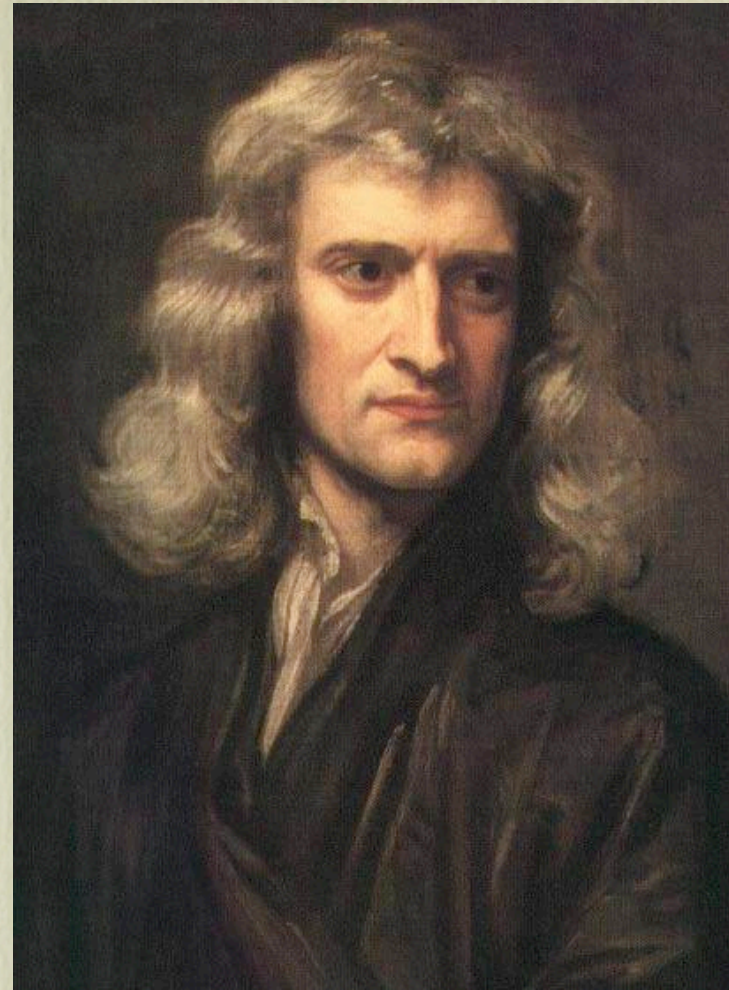
L'infiniment petit ...

les atomes, les électrons,
les particules de matière, de lumière,
les photons, ...





Christian Huygens
1629-1695

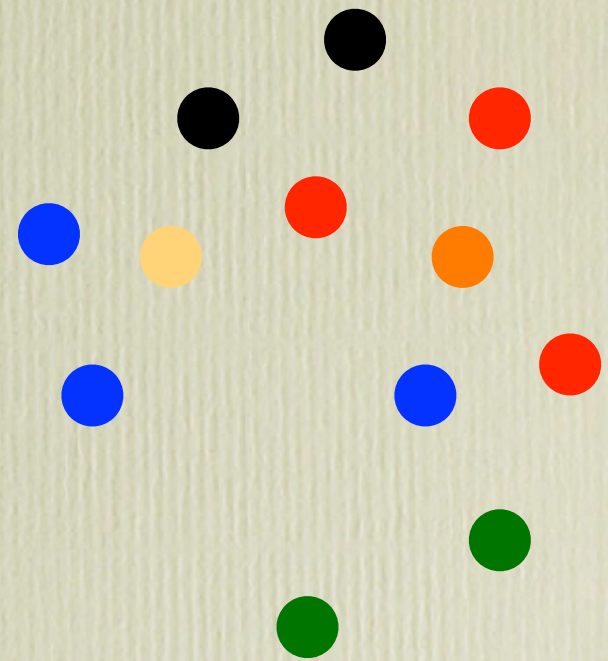


Isaac Newton
1643-1727

la lumière:

vibration ?

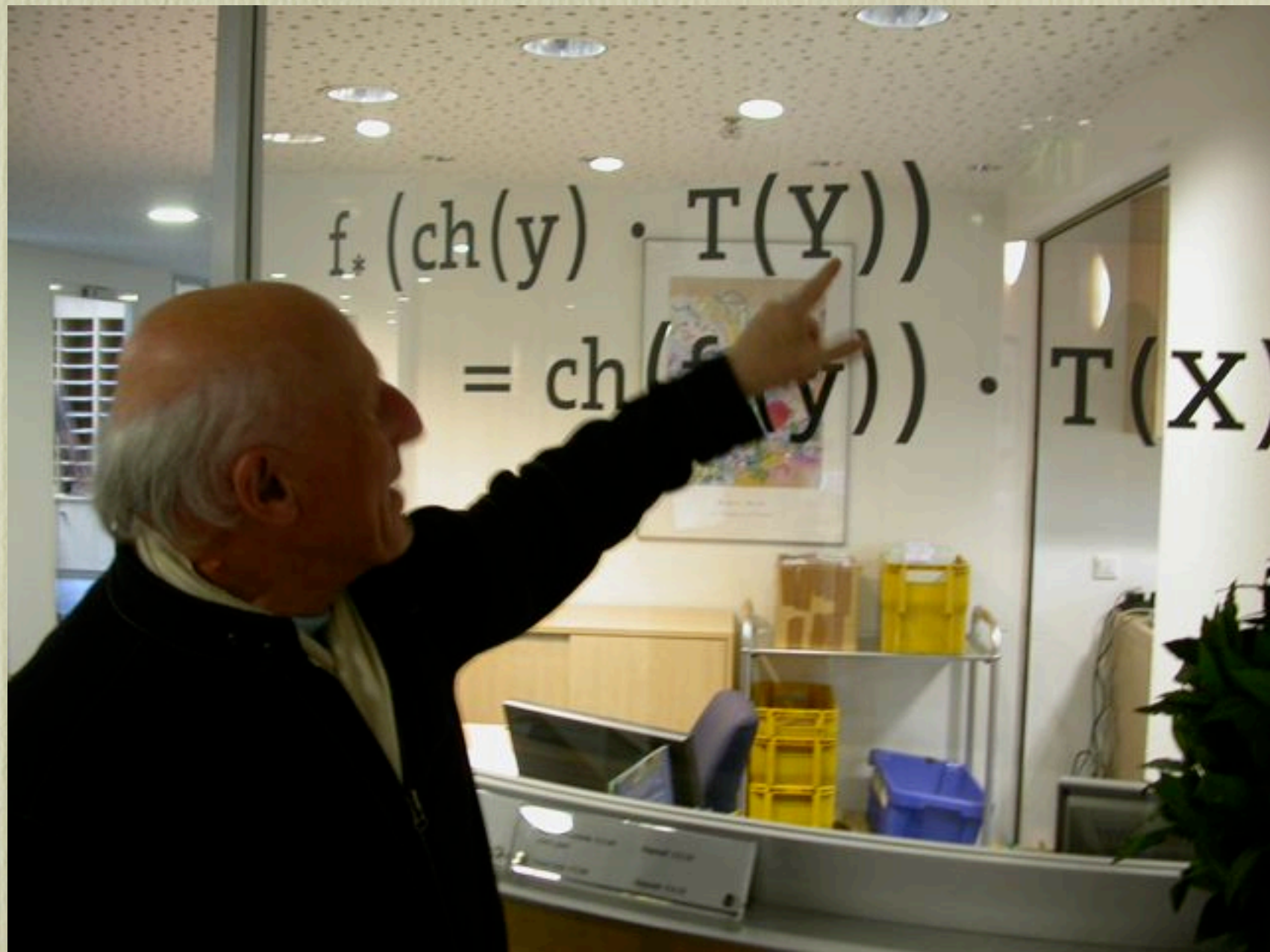
ou particules de matière ?







Si vous êtes perdus dans la forêt
des mathématiques, détendez
vous et regardez les images



regardez une formule mathématique
comme une oeuvre d'art abstrait

Rogers - Ramanujan identities

$$R_I \quad \sum_{n \geq 0} \frac{q^{n^2}}{(1-q)(1-q^2)\dots(1-q^n)} = \prod_{\substack{i \equiv 1, 4 \\ \text{mod } 5}} \frac{1}{(1-q^i)}$$

$$R_{II} \quad \sum_{n \geq 0} \frac{q^{n^2+n}}{(1-q)(1-q^2)\dots(1-q^n)} = \prod_{\substack{i \equiv 2, 3 \\ \text{mod } 5}} \frac{1}{(1-q^i)}$$

Srinivasan
Ramanujan
(1887-1920)



Le langage des mathématiques, c'est comme
le solfège utilisé pour écrire de la musique

mais les mathématiques sont la musique !

Habituellement, à l'école vous apprenez seulement comment
écrire des mathématiques, mais il est difficile d'entendre la
beauté des mathématiques ...

Paris



Un exemple d'objet mathématique:
les arbres binaires ou arbres mathématiques

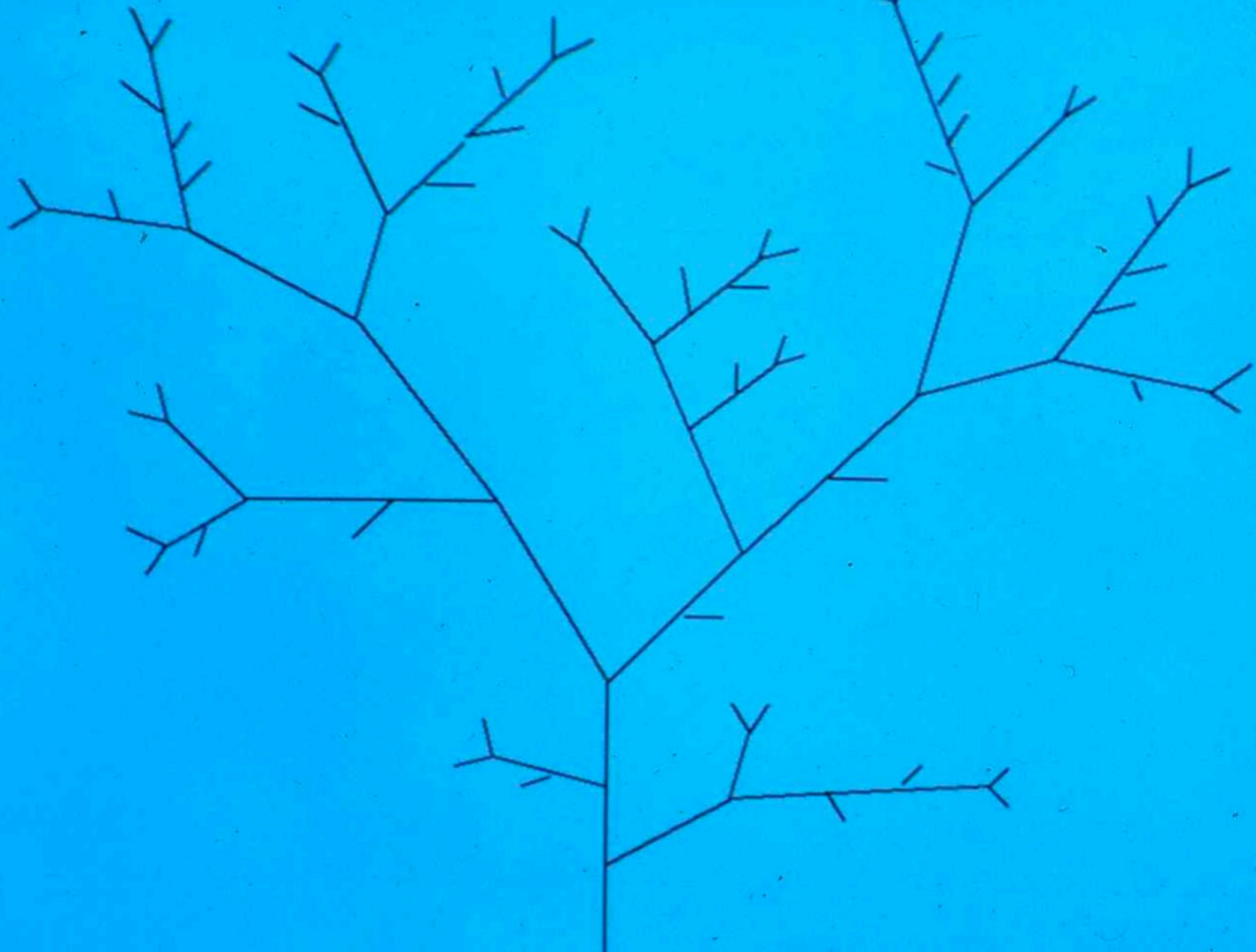
donner une abstraction des arbres
dans le monde qui nous entoure

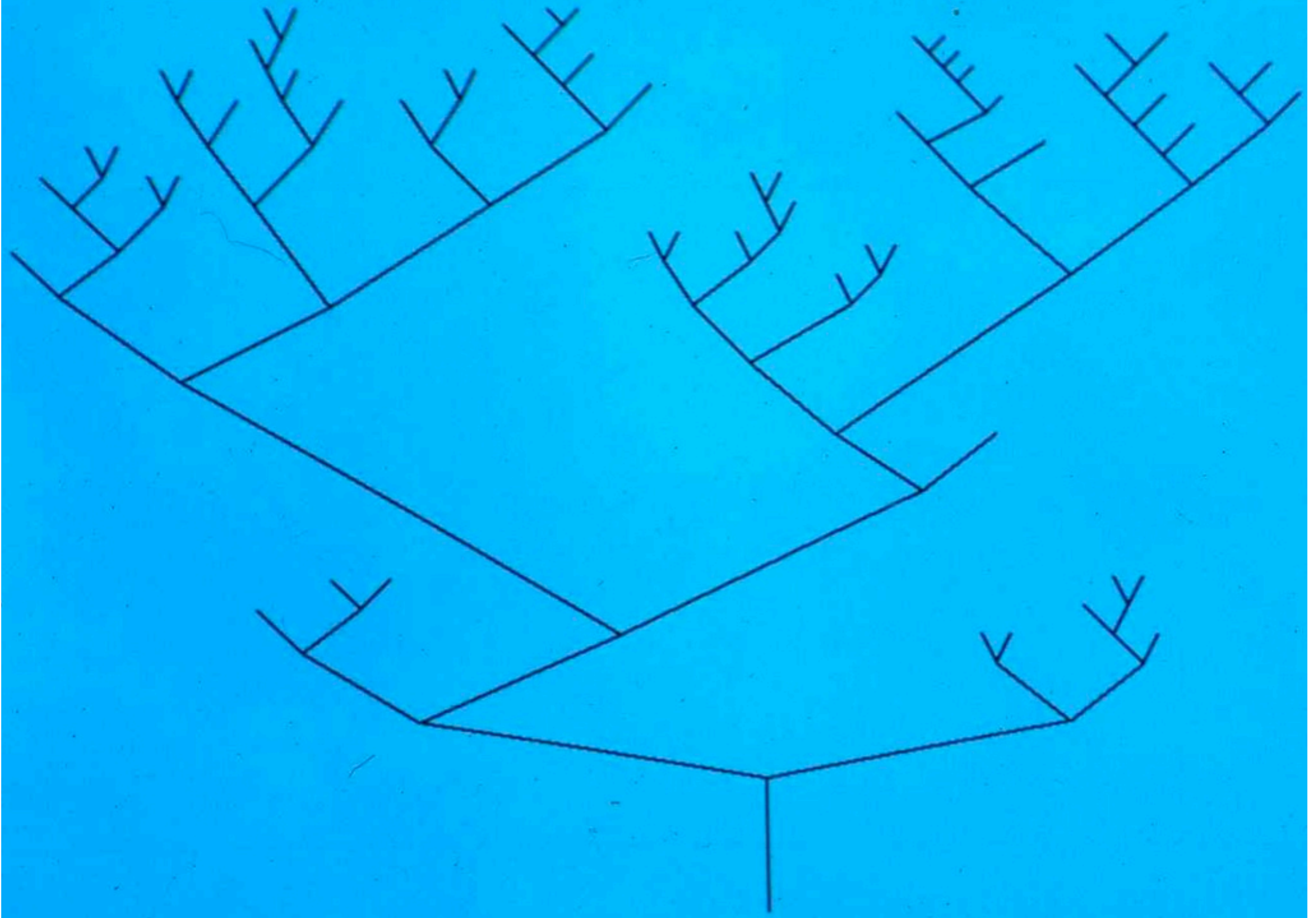
Des arbres naturels ...
aux arbres mathématiques

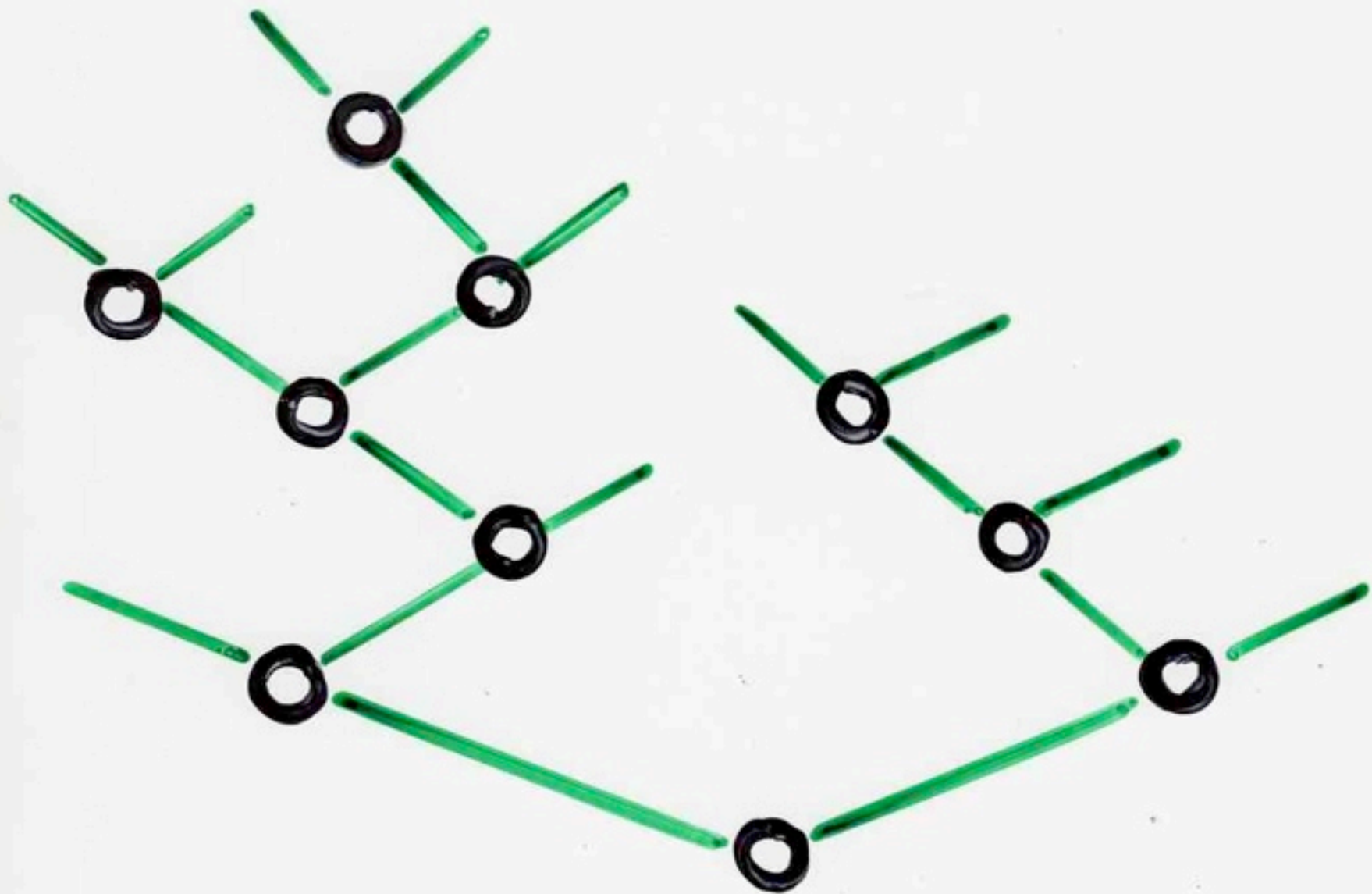










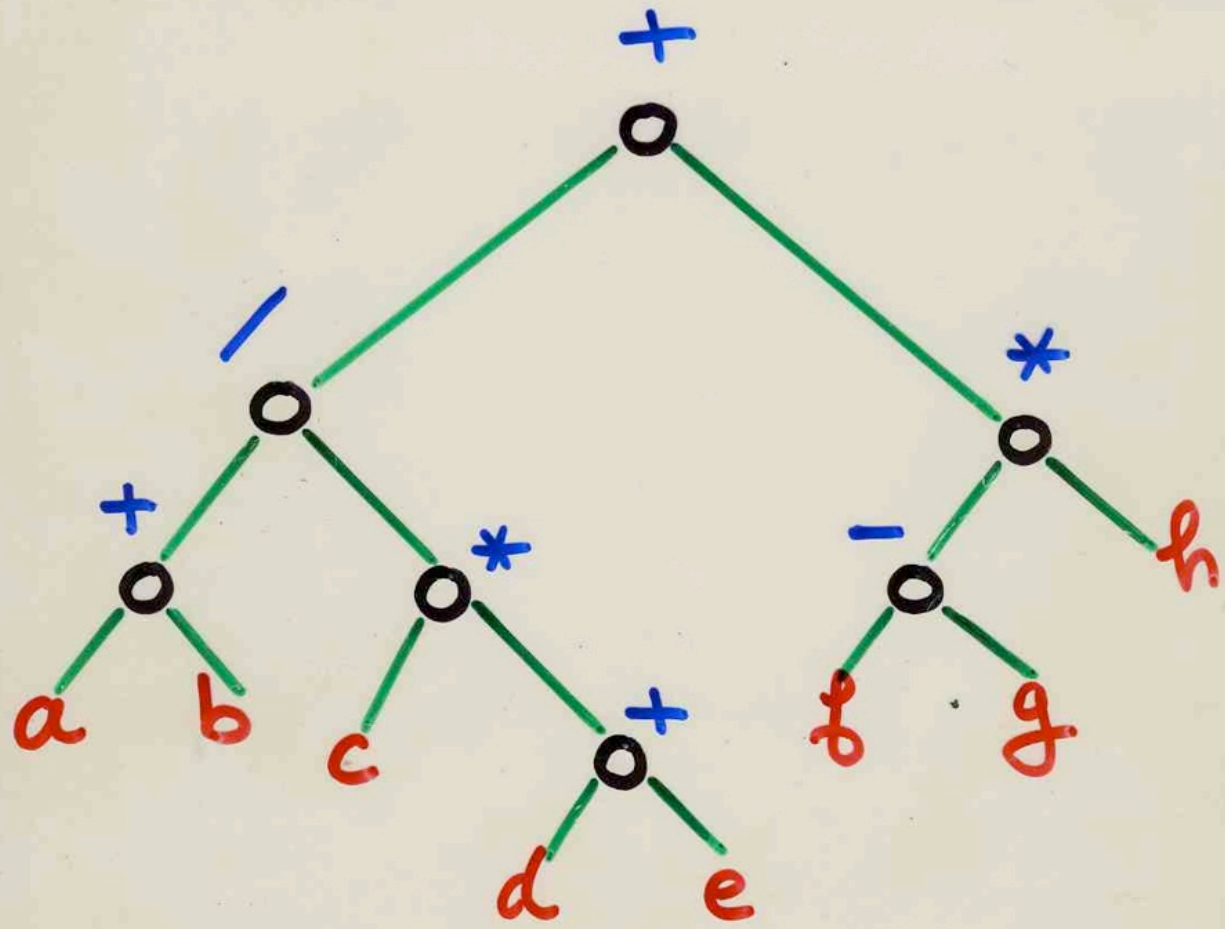


VICTOR HUGO

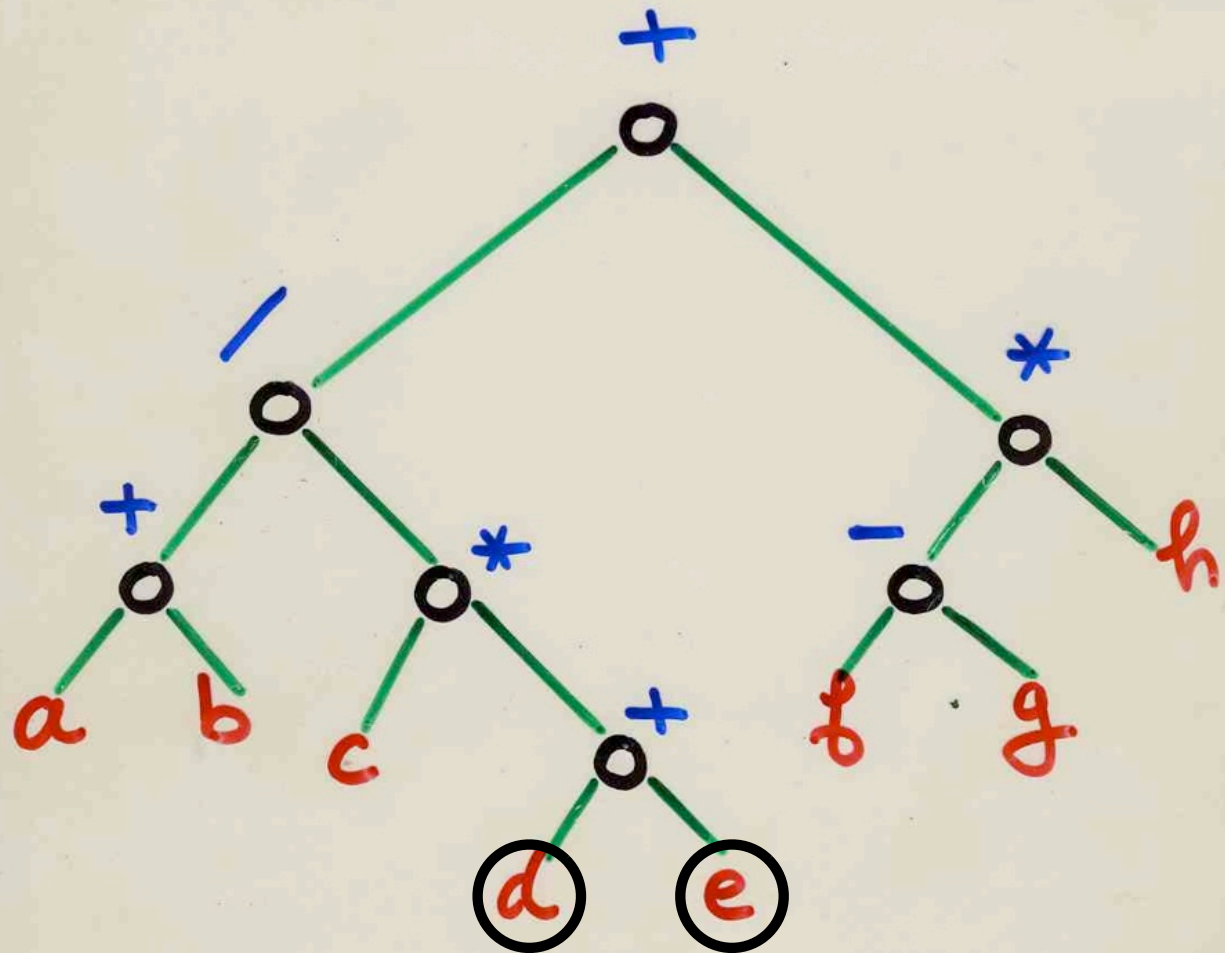
des arbres dans les ordinateurs ...

calcul d'une expression arithmétique

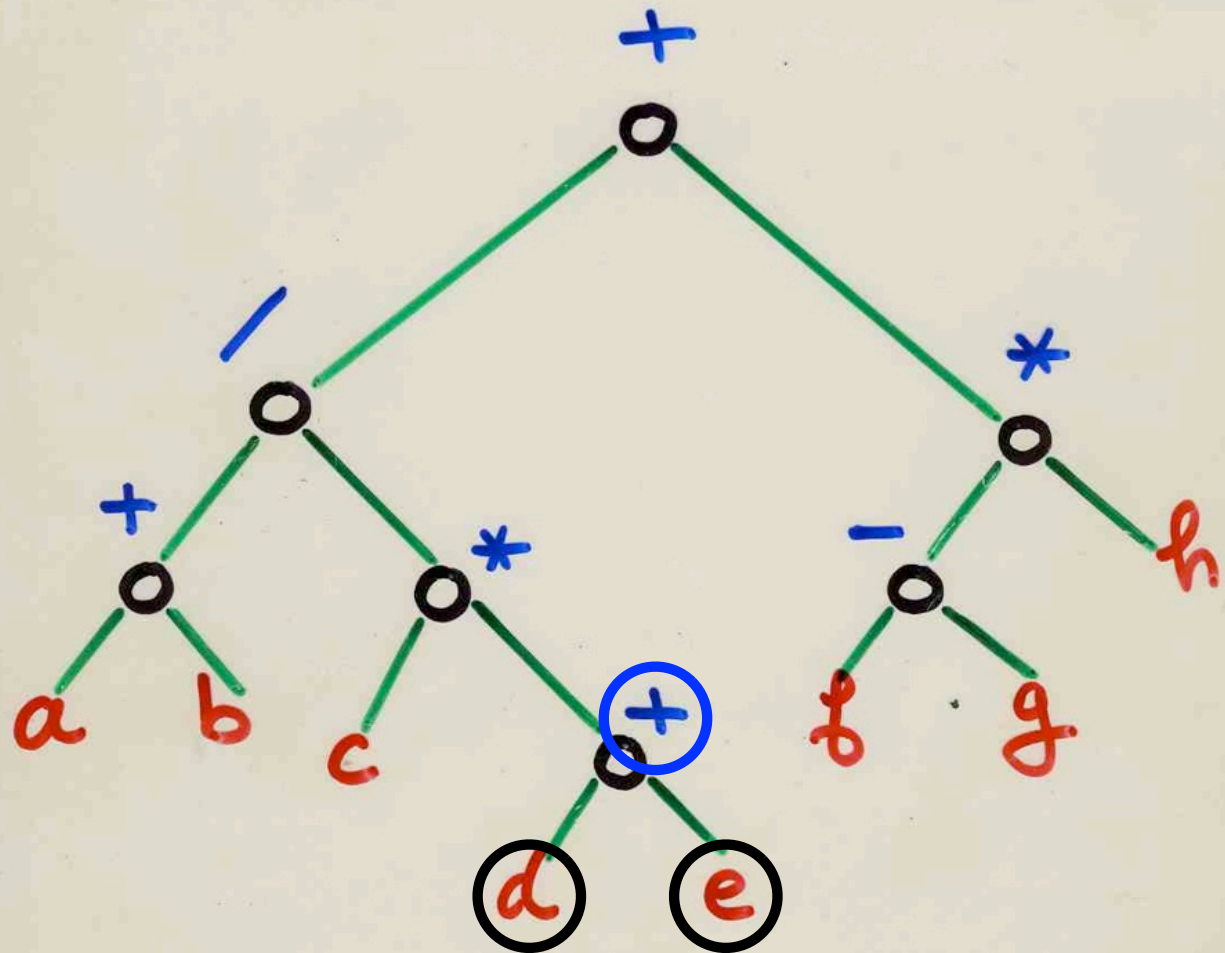
$$\frac{(a+b)}{c(d+e)} + (f-g)h$$



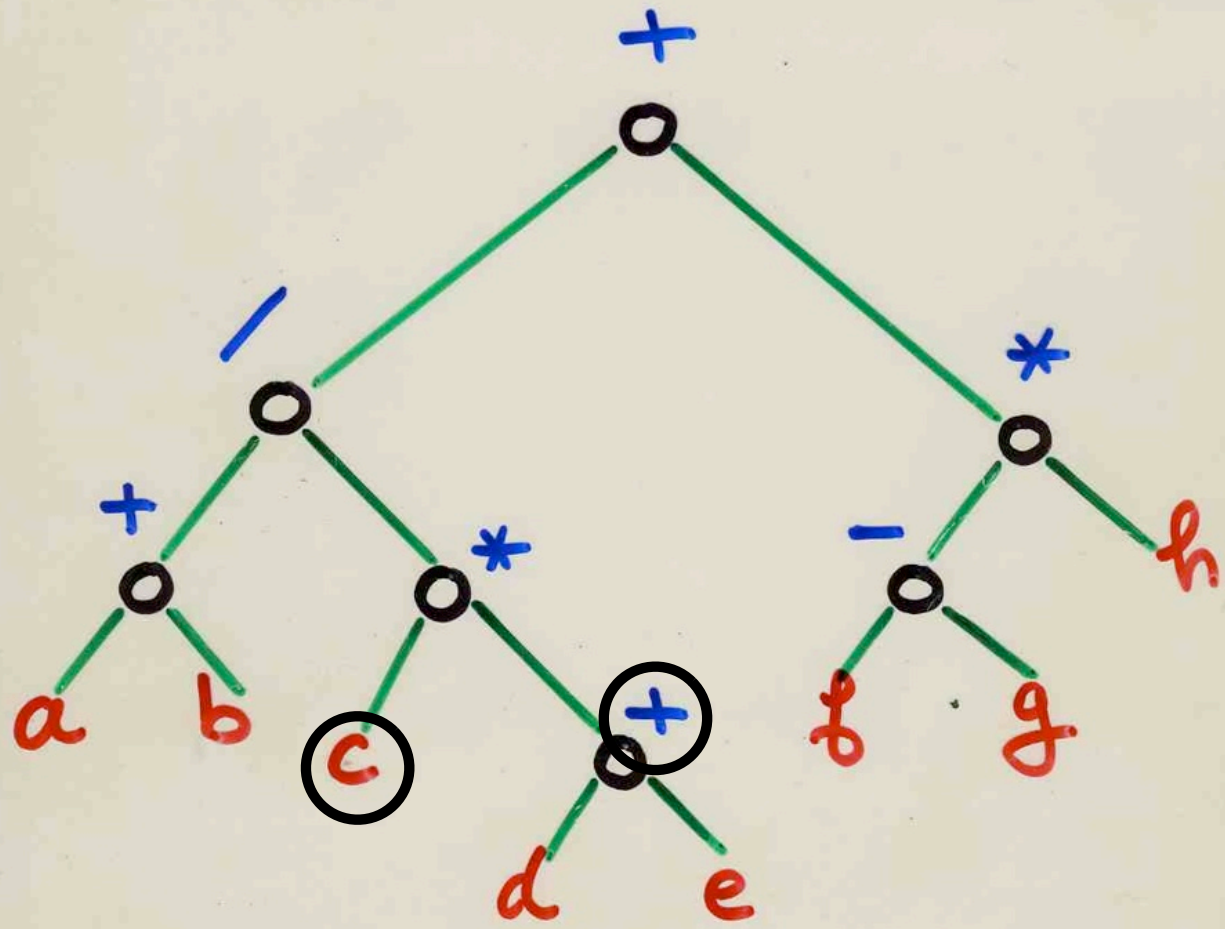
$$\frac{(a+b)}{c(d+e)} + (f-g)h$$



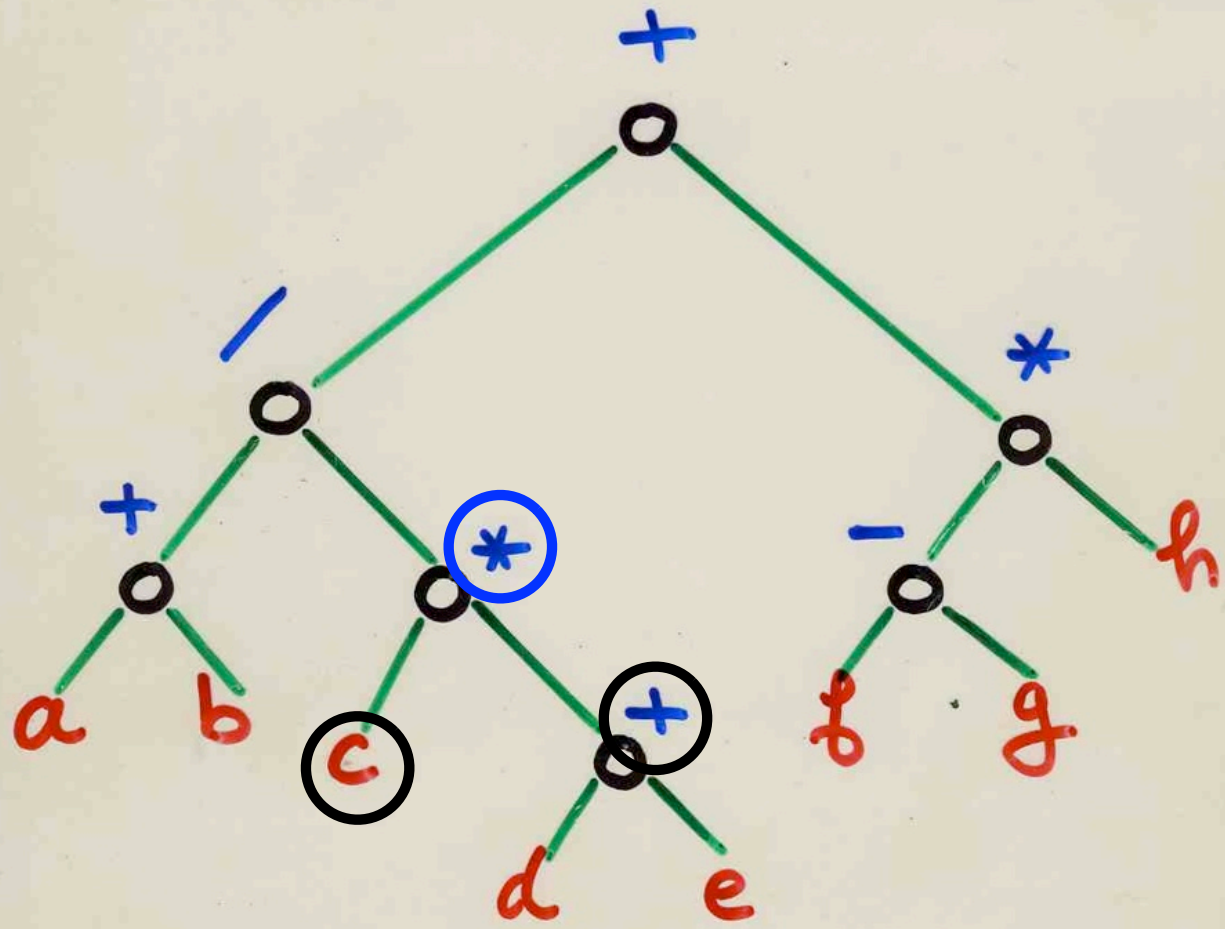
$$\frac{(a+b)}{c(d+e)} + (f-g)h$$



$$\frac{(a+b)}{c(d+e)} + (f-g)h$$



$$\frac{(a+b)}{c(d+e)} + (f-g)h$$



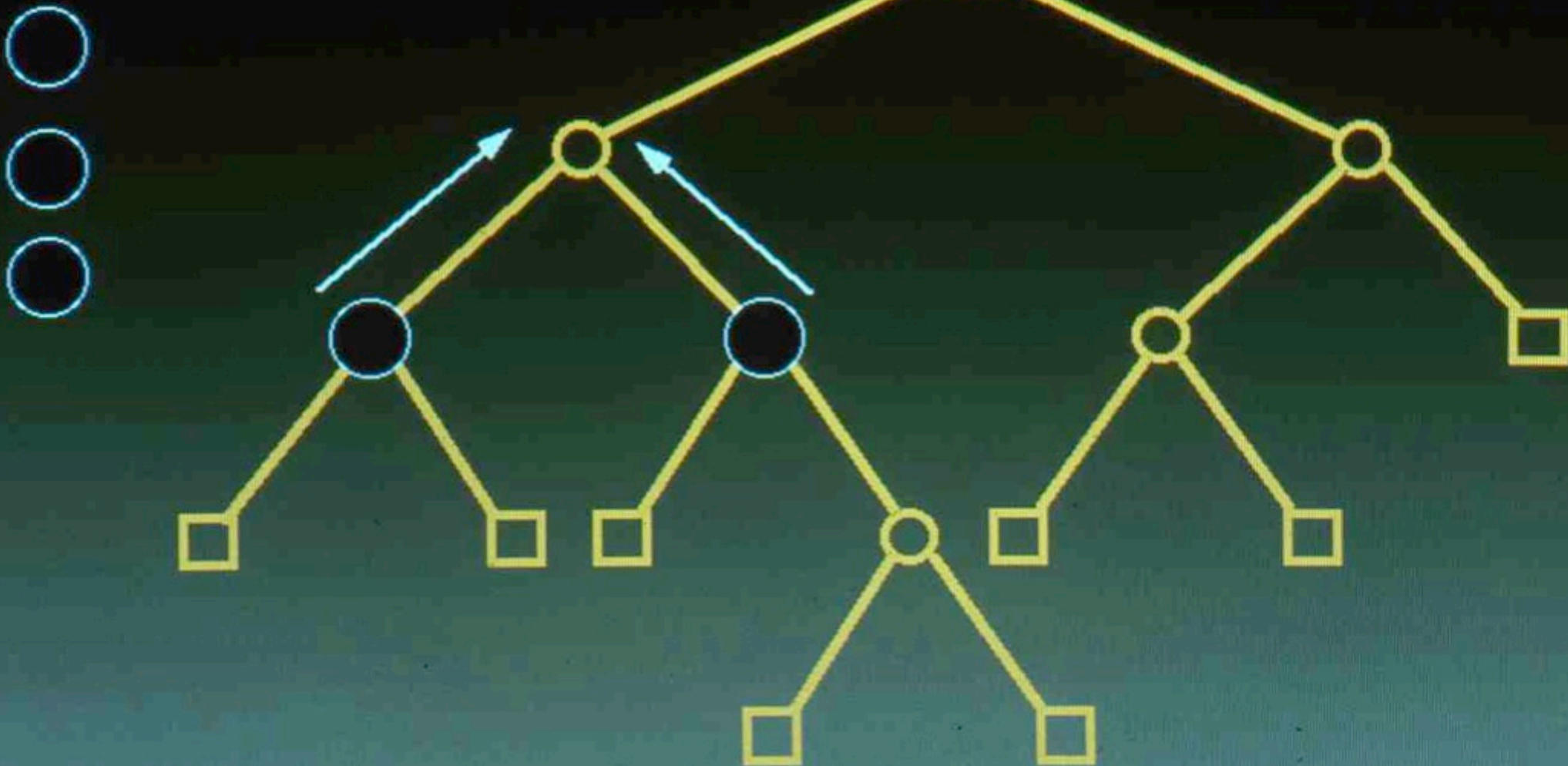
$$\frac{(a+b)}{c(d+e)} + (f-g)h$$

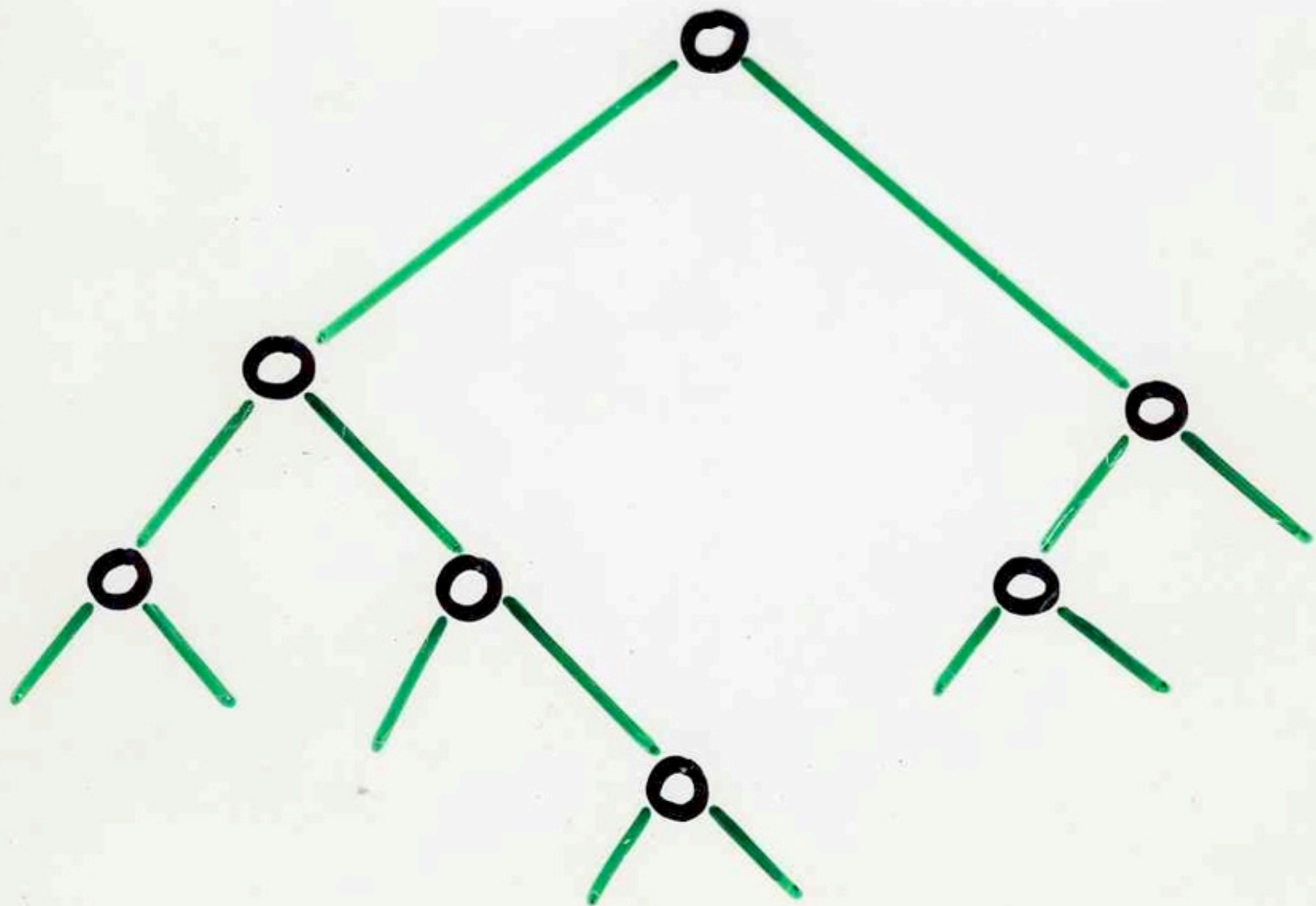
nombre
de
nécessaires
d'une

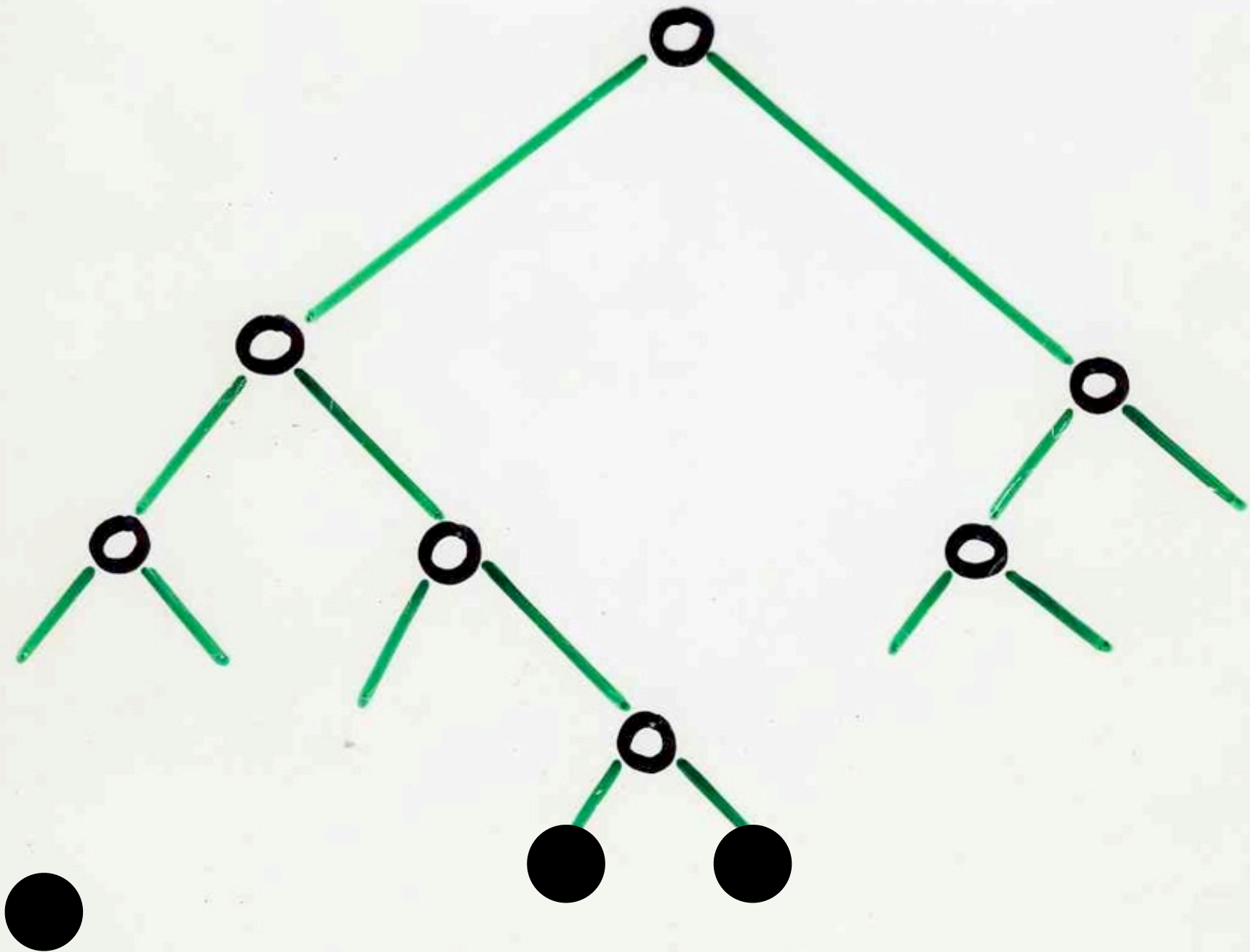
minimum
registres

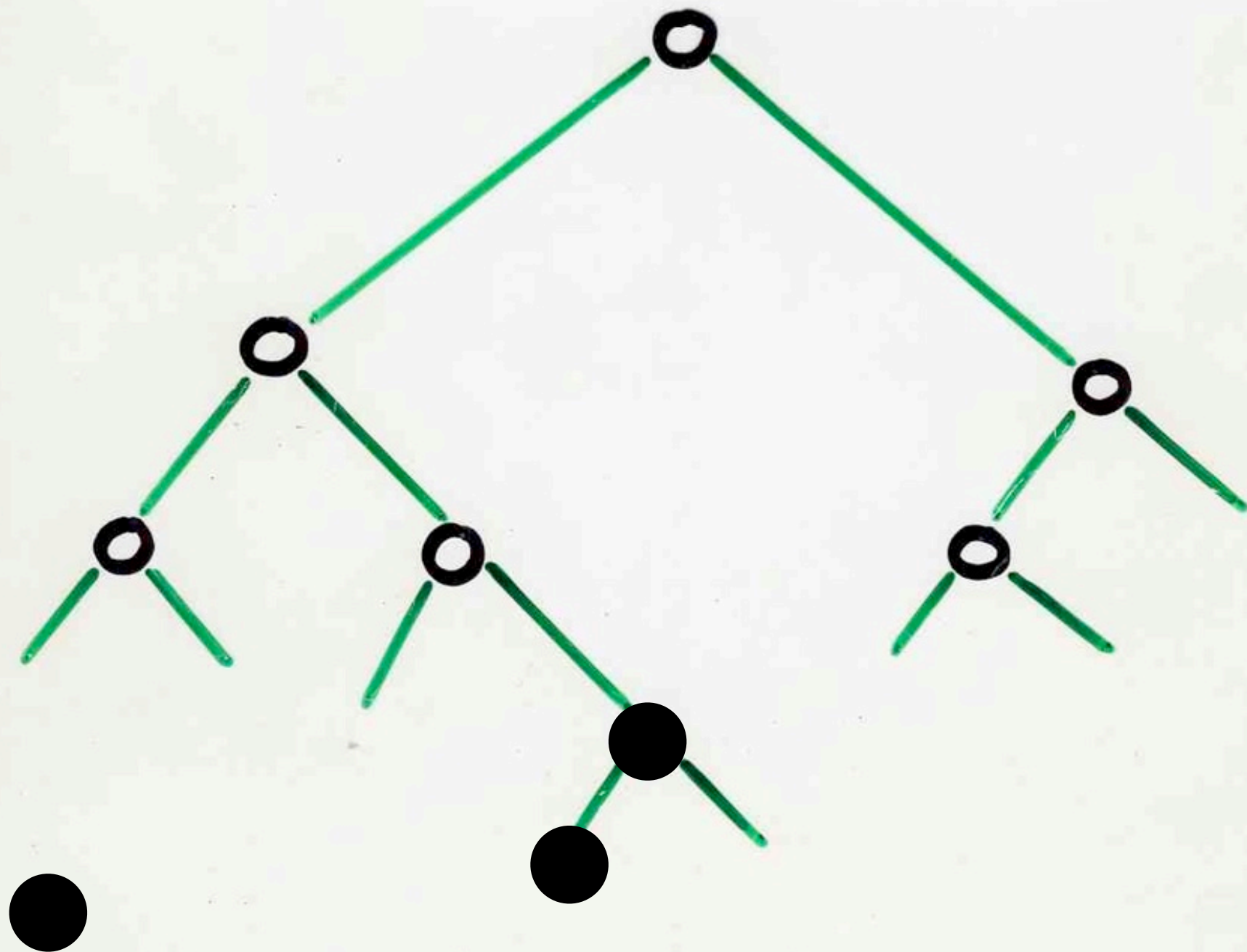
pour le calcul
expression arithmétique

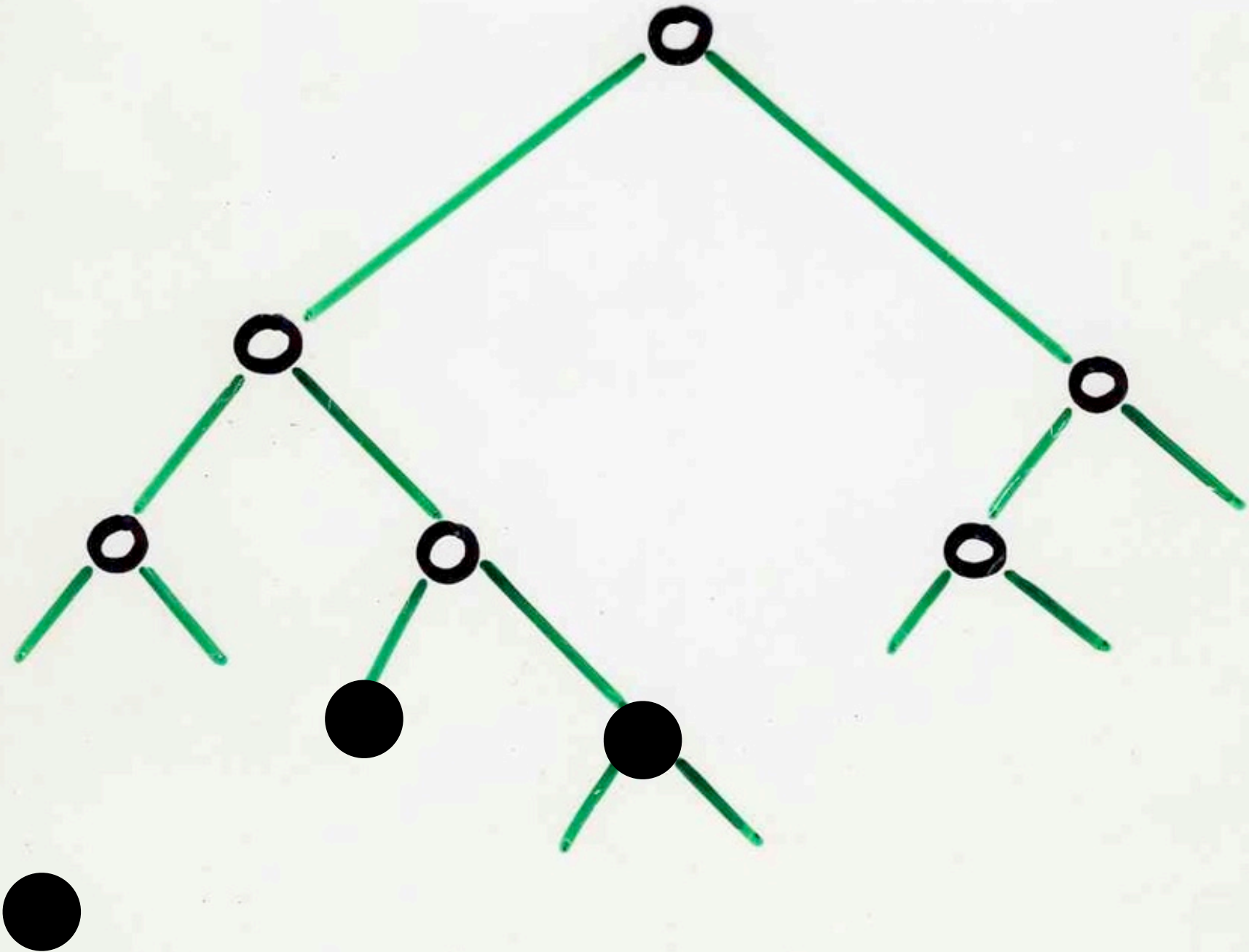
Pebbles problem

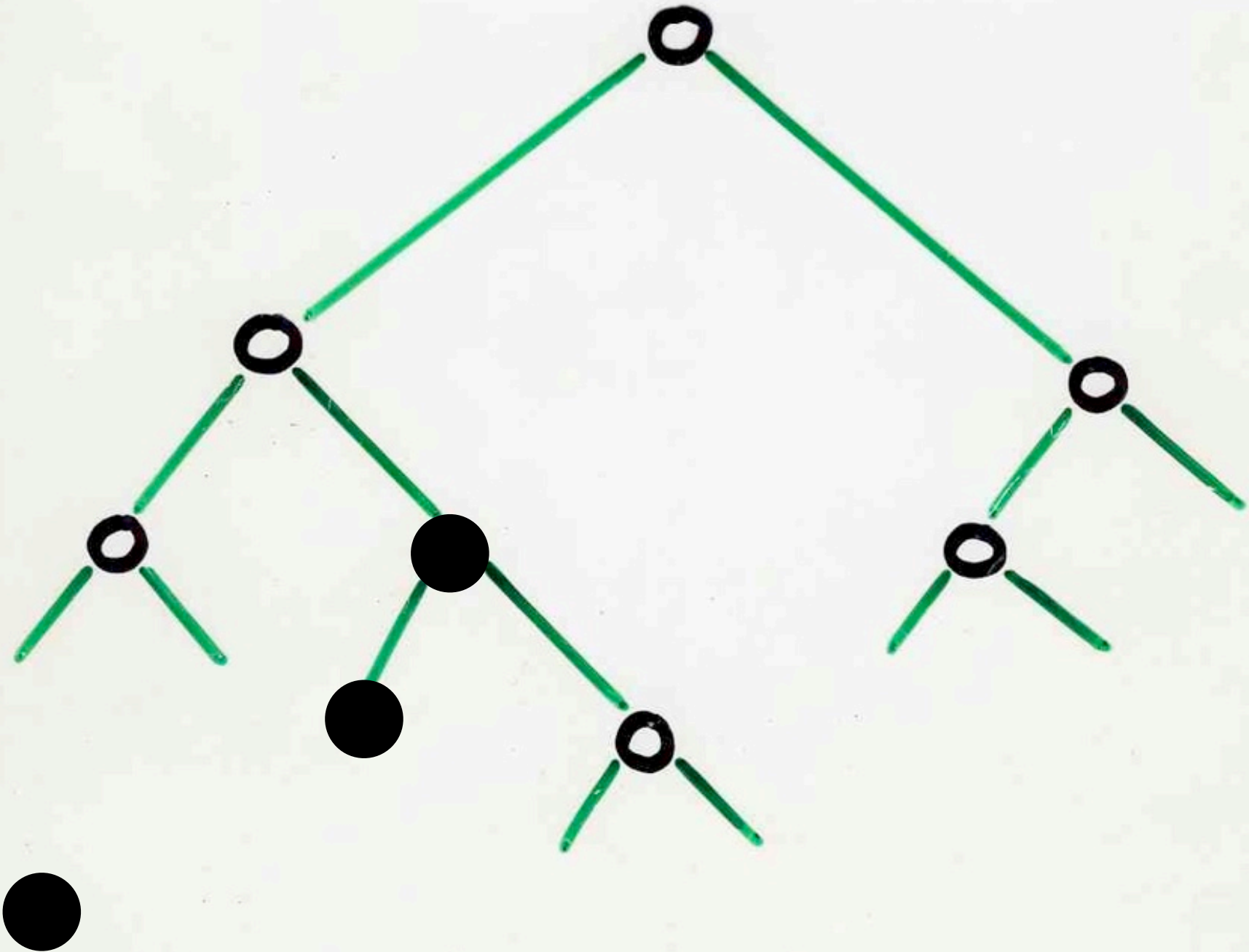


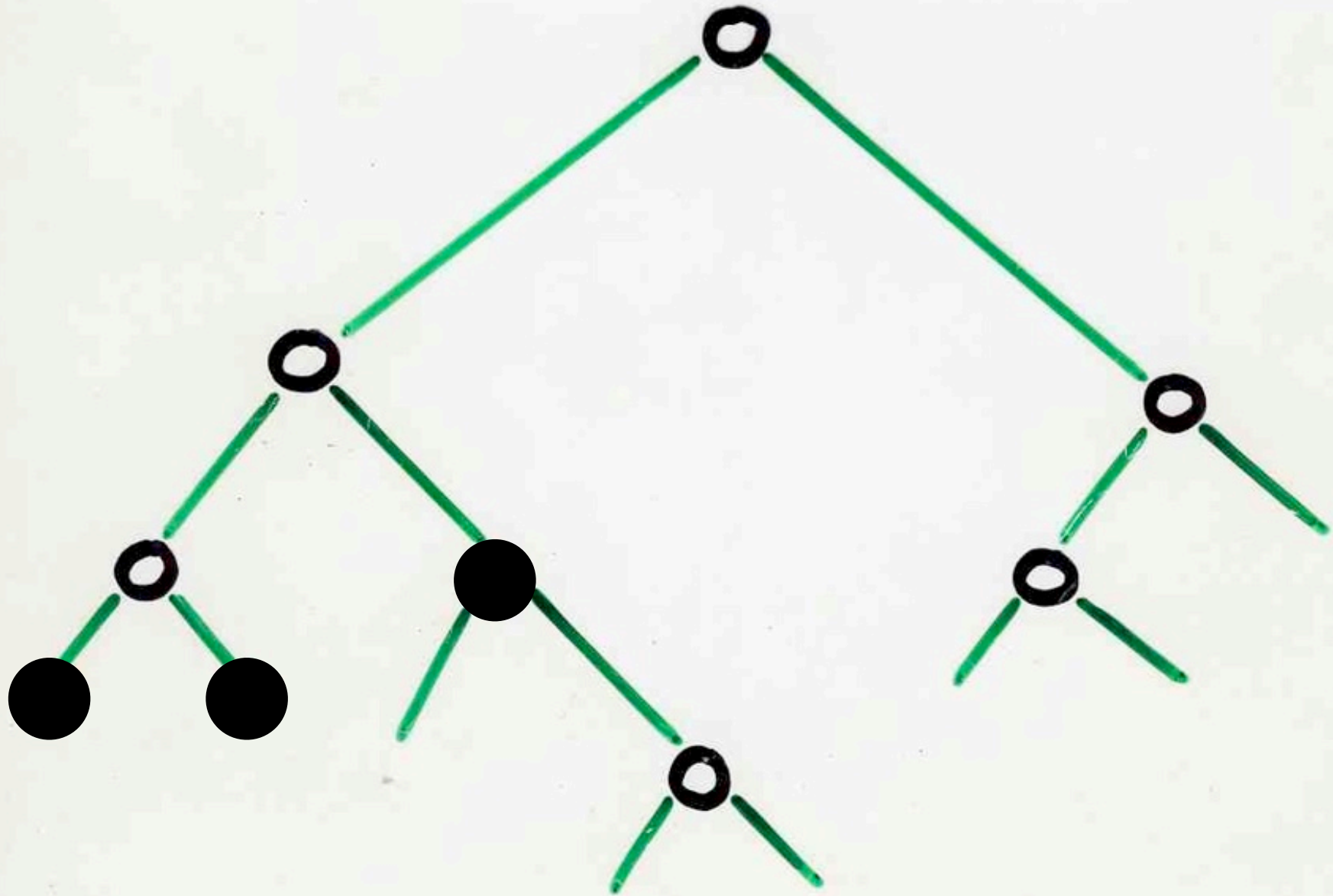


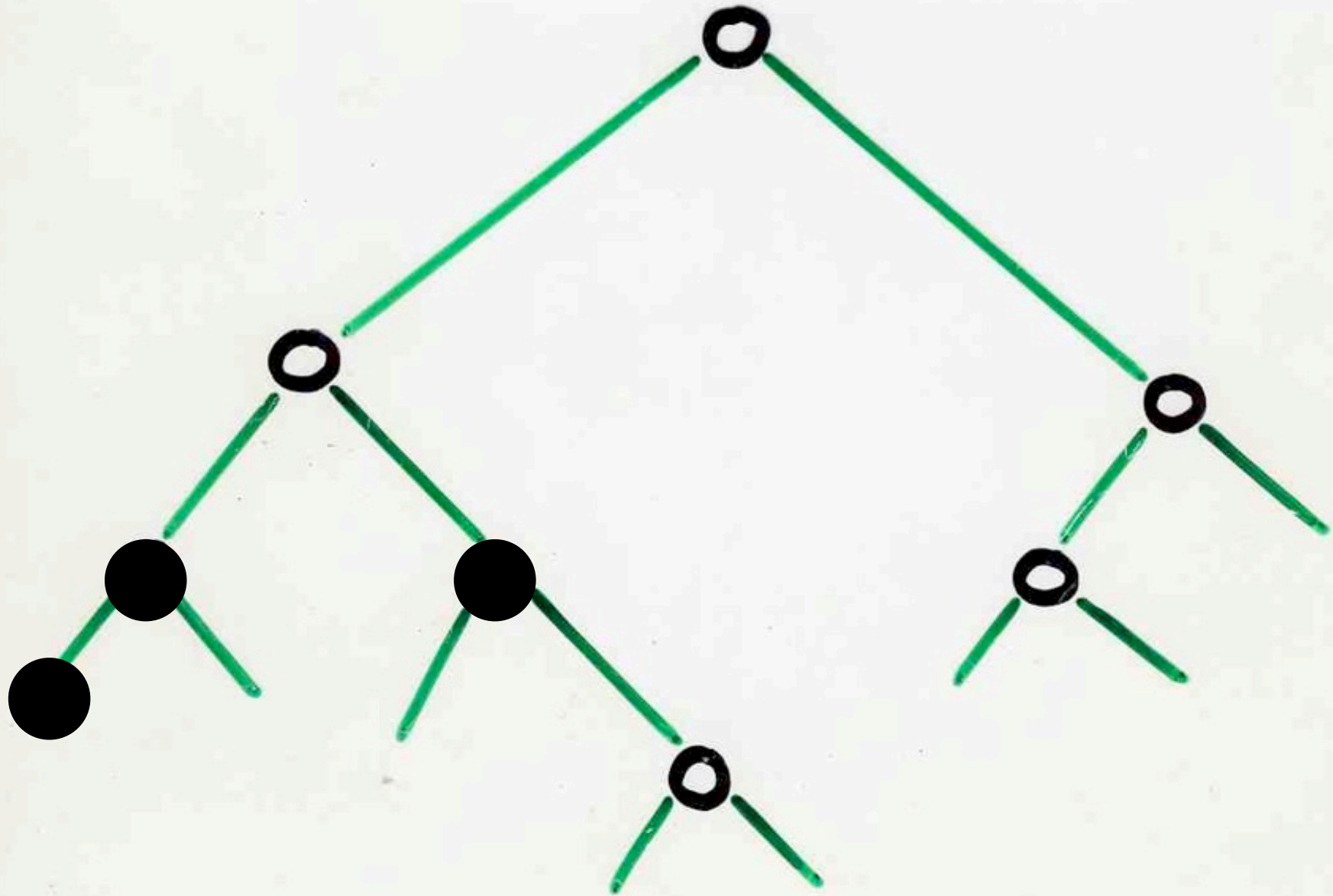


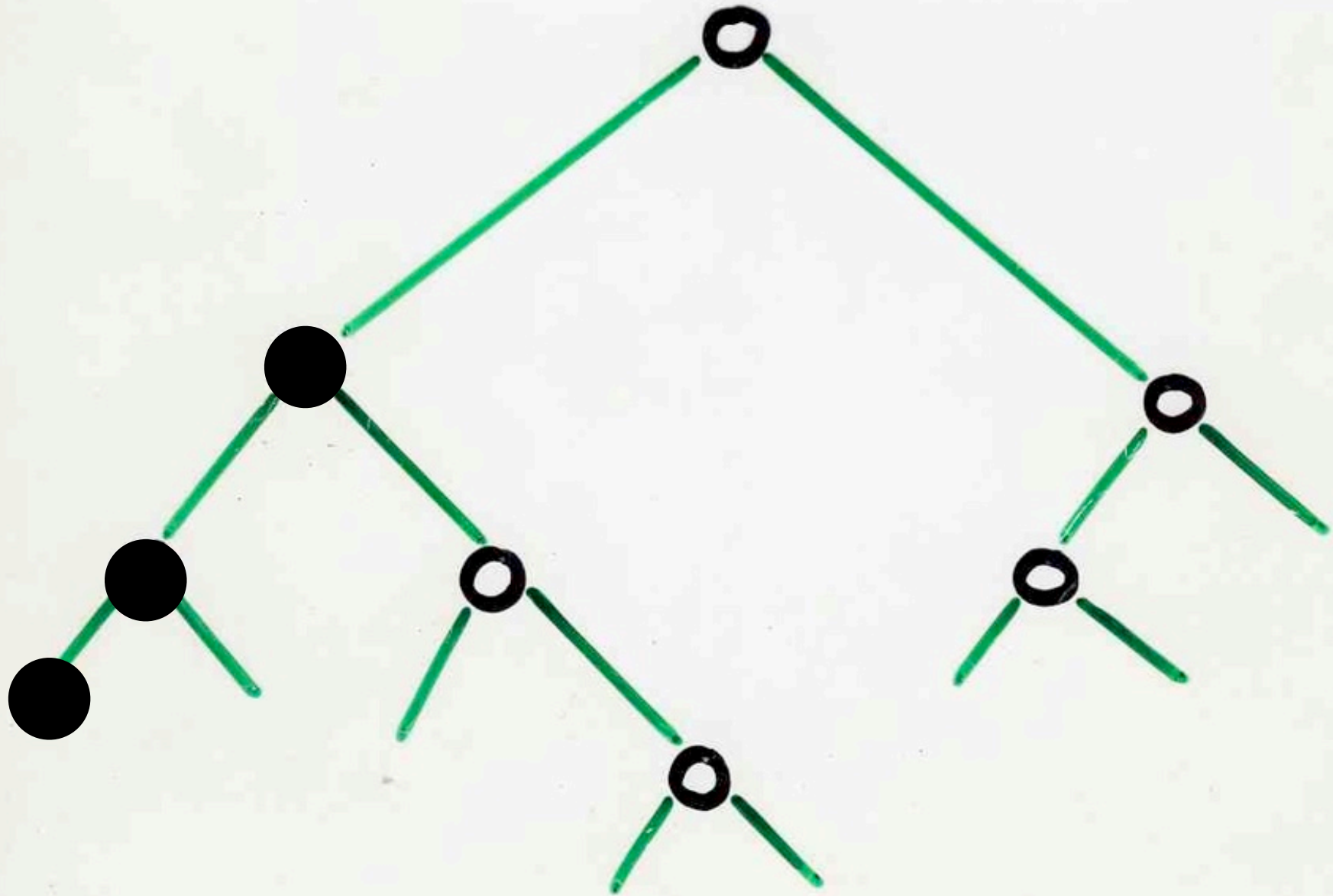


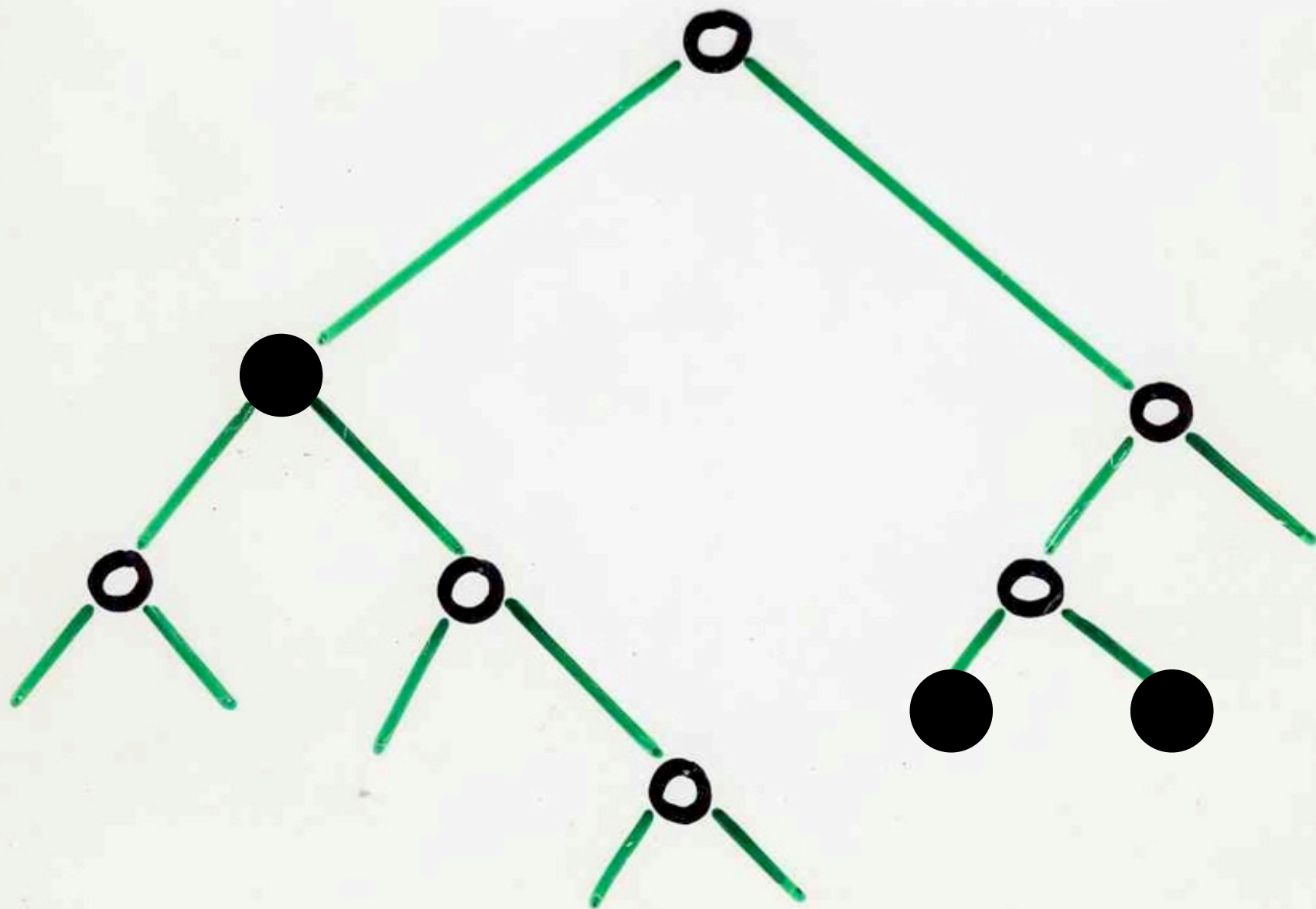


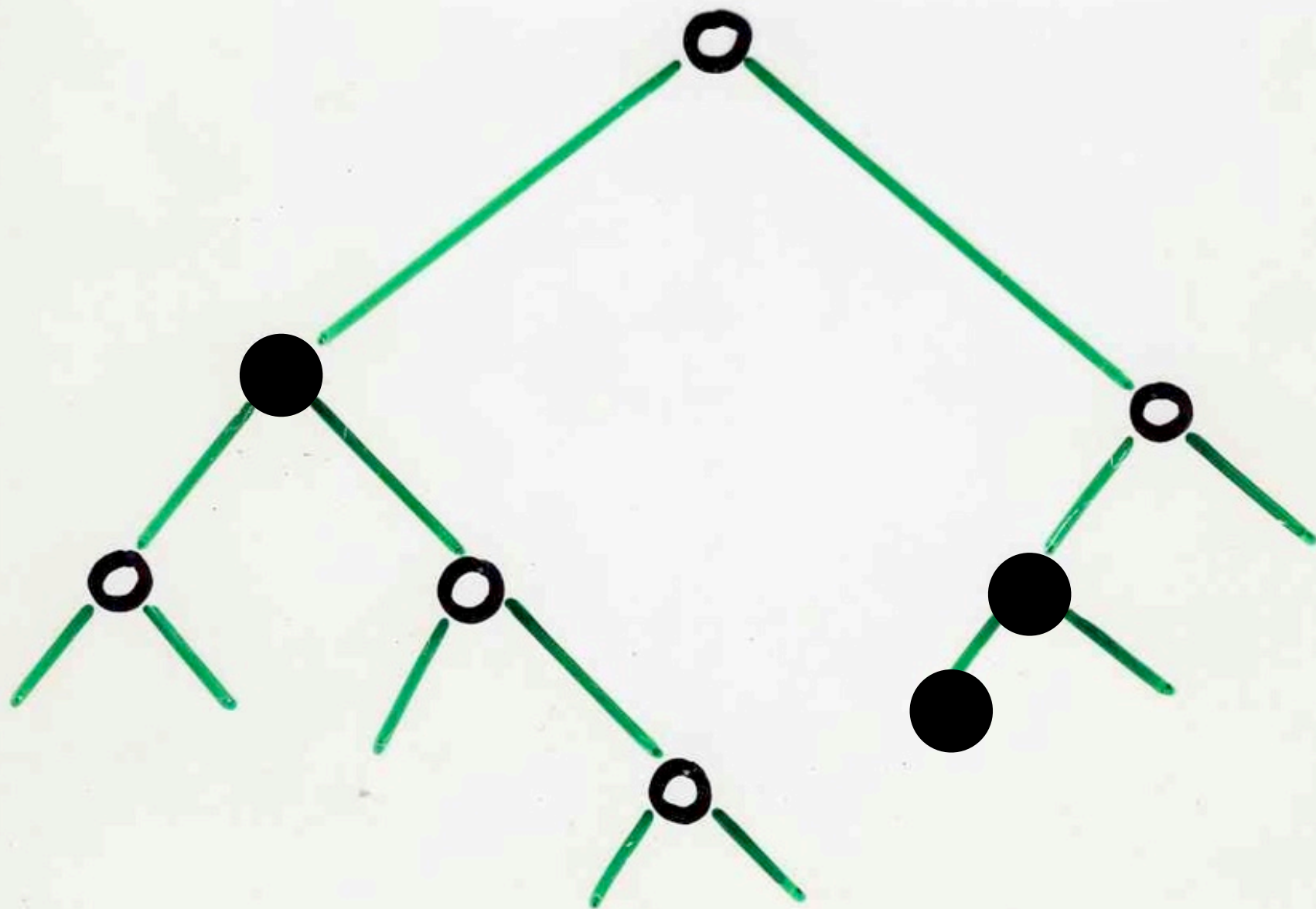


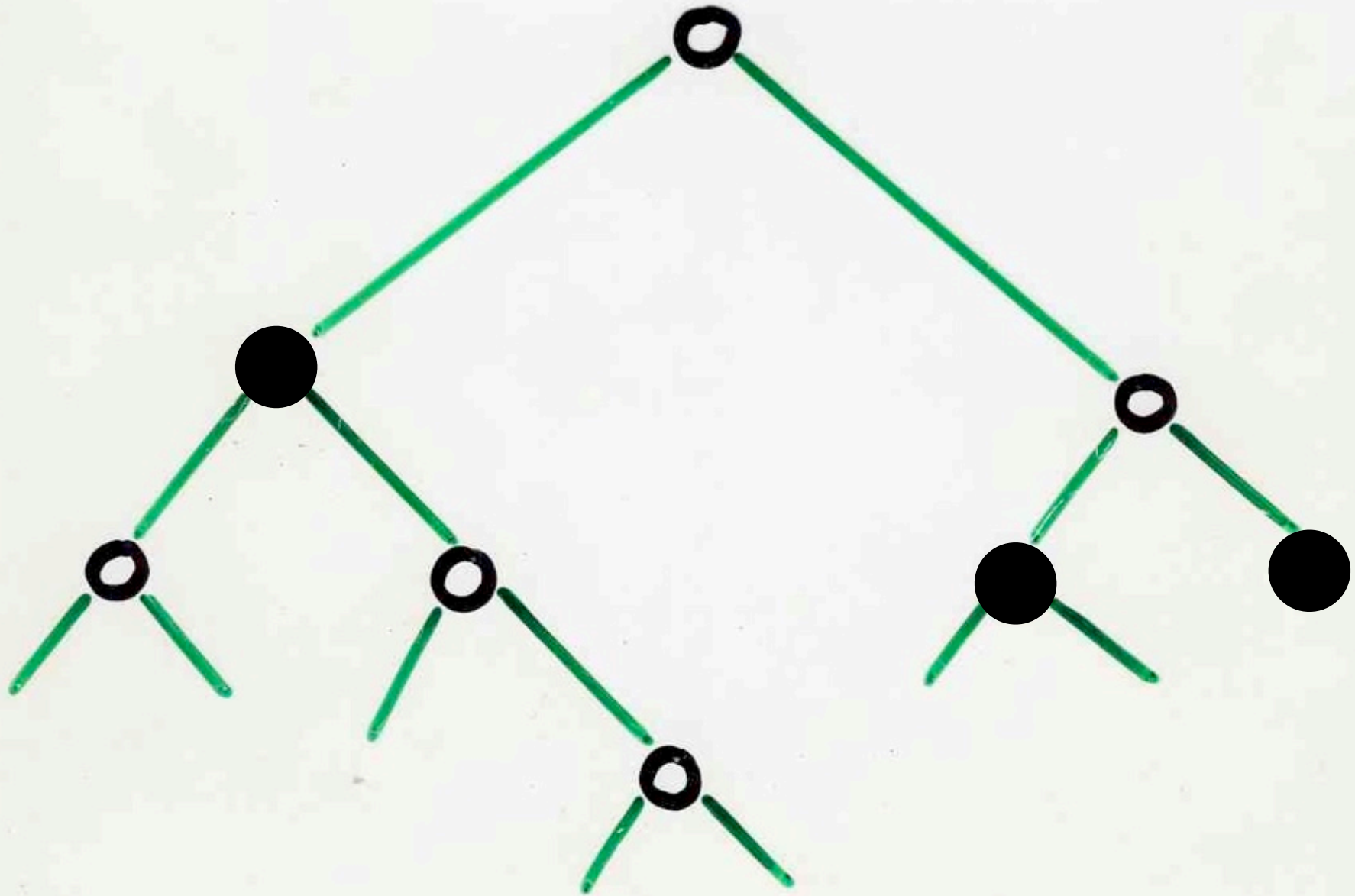


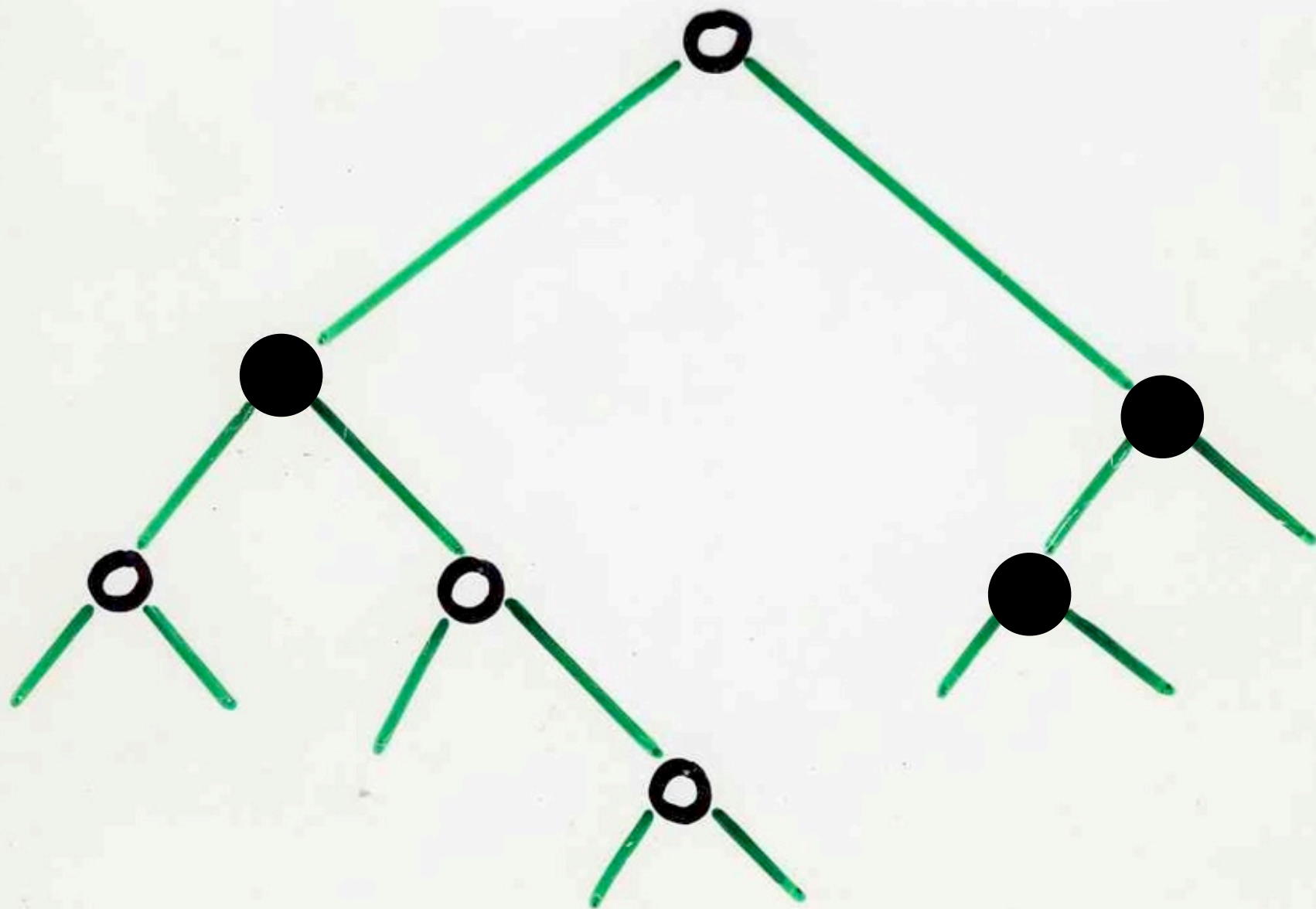


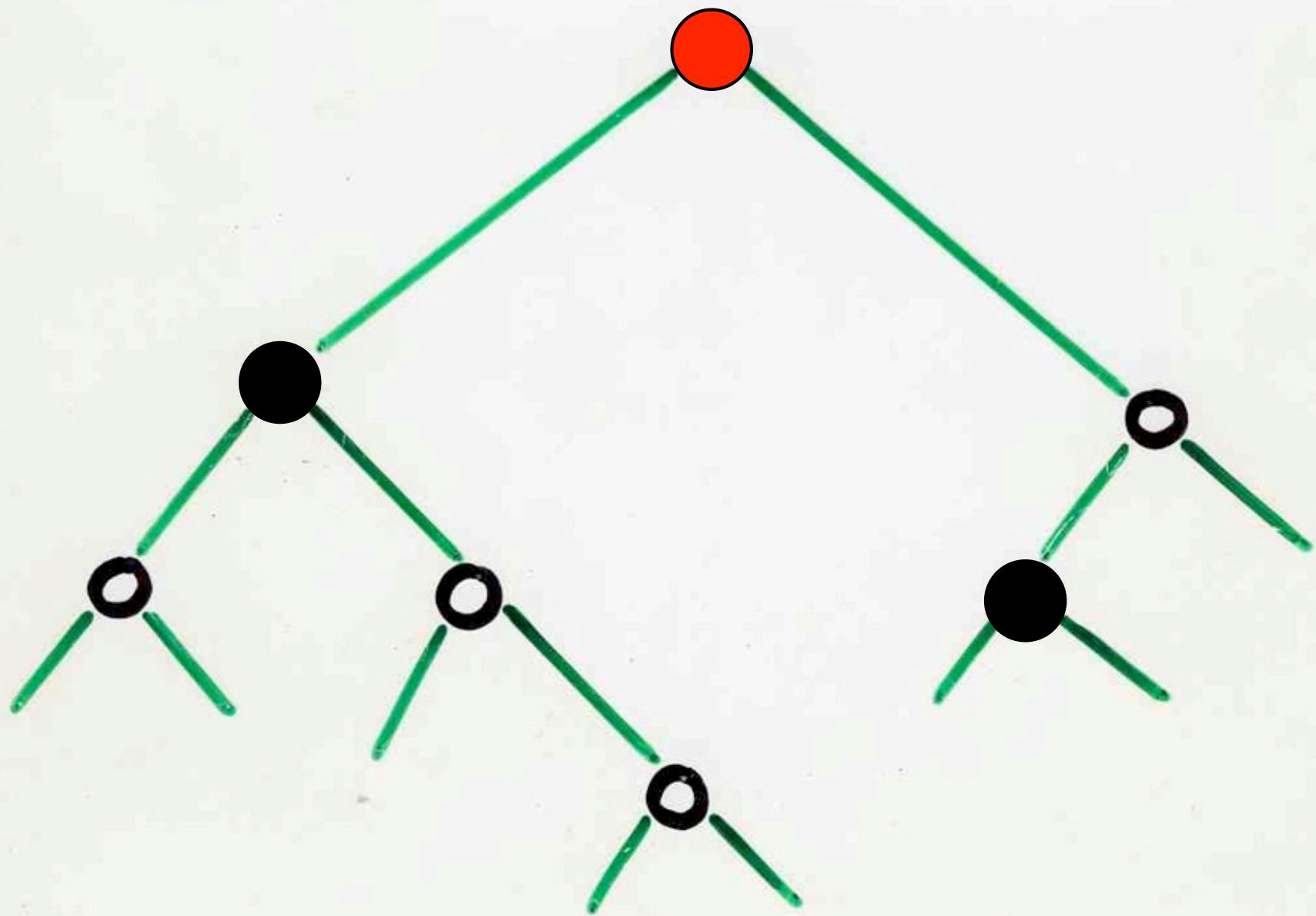












Des arbres aux rivières ...





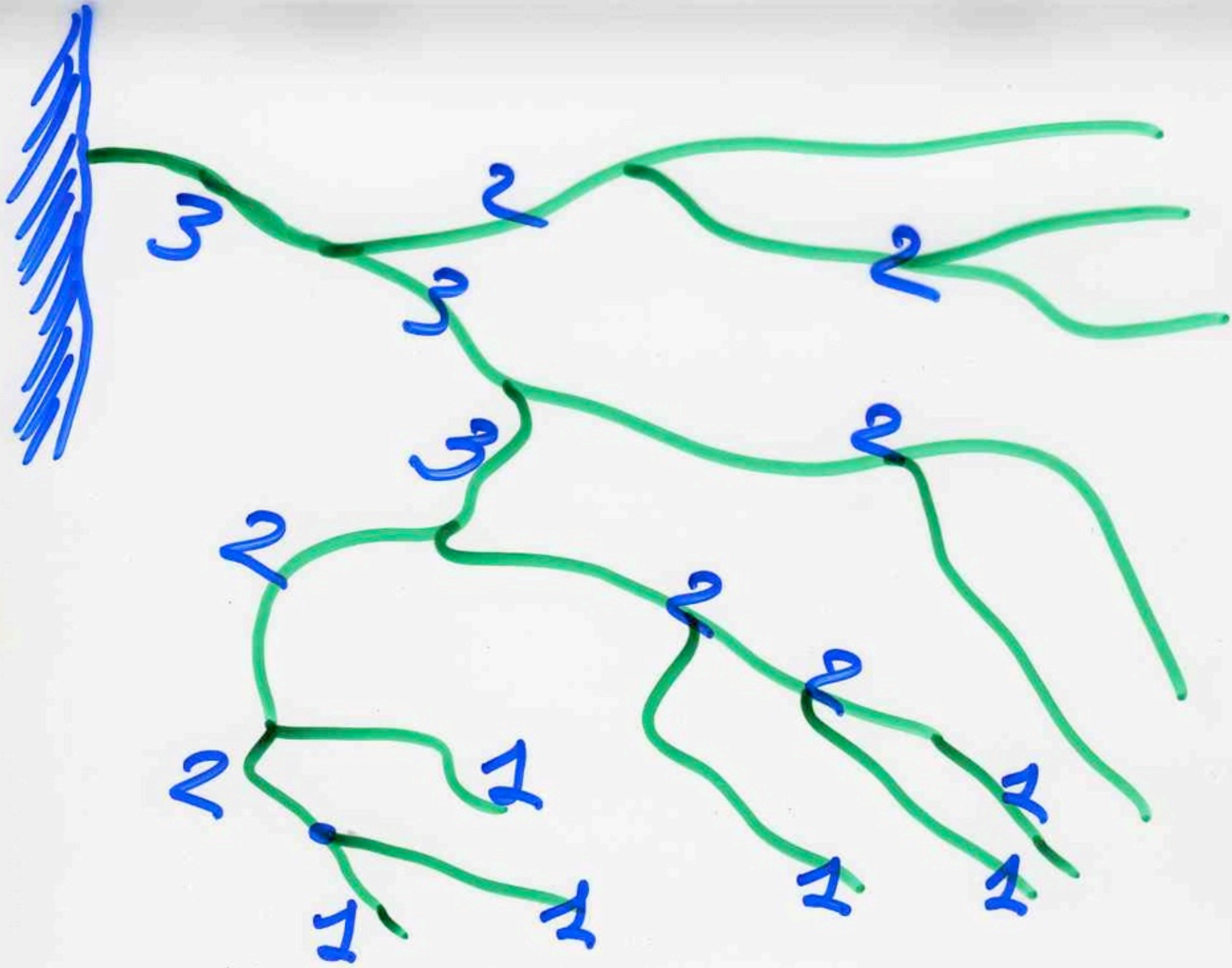
Horton (1945)

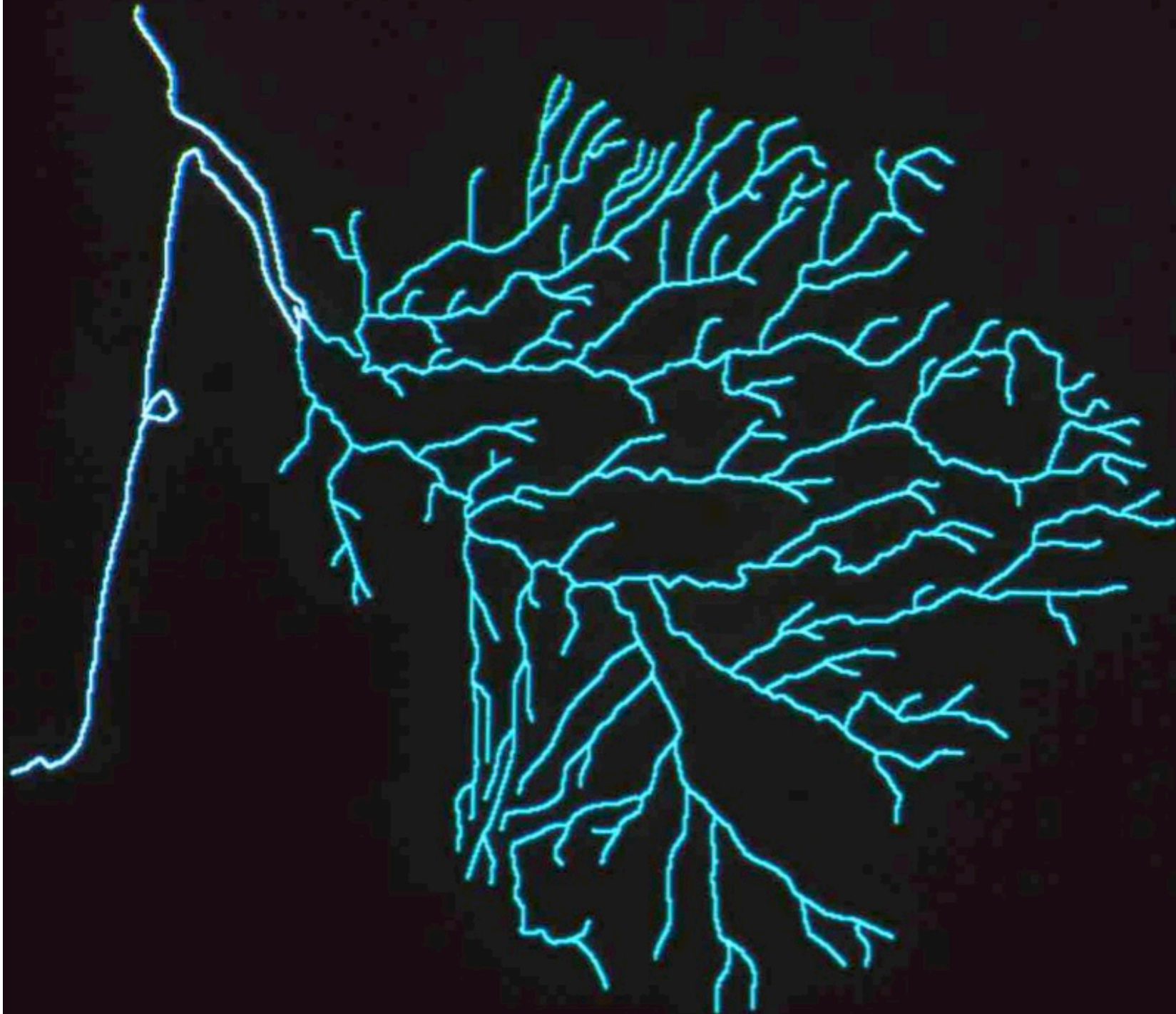
Strahler (1952)

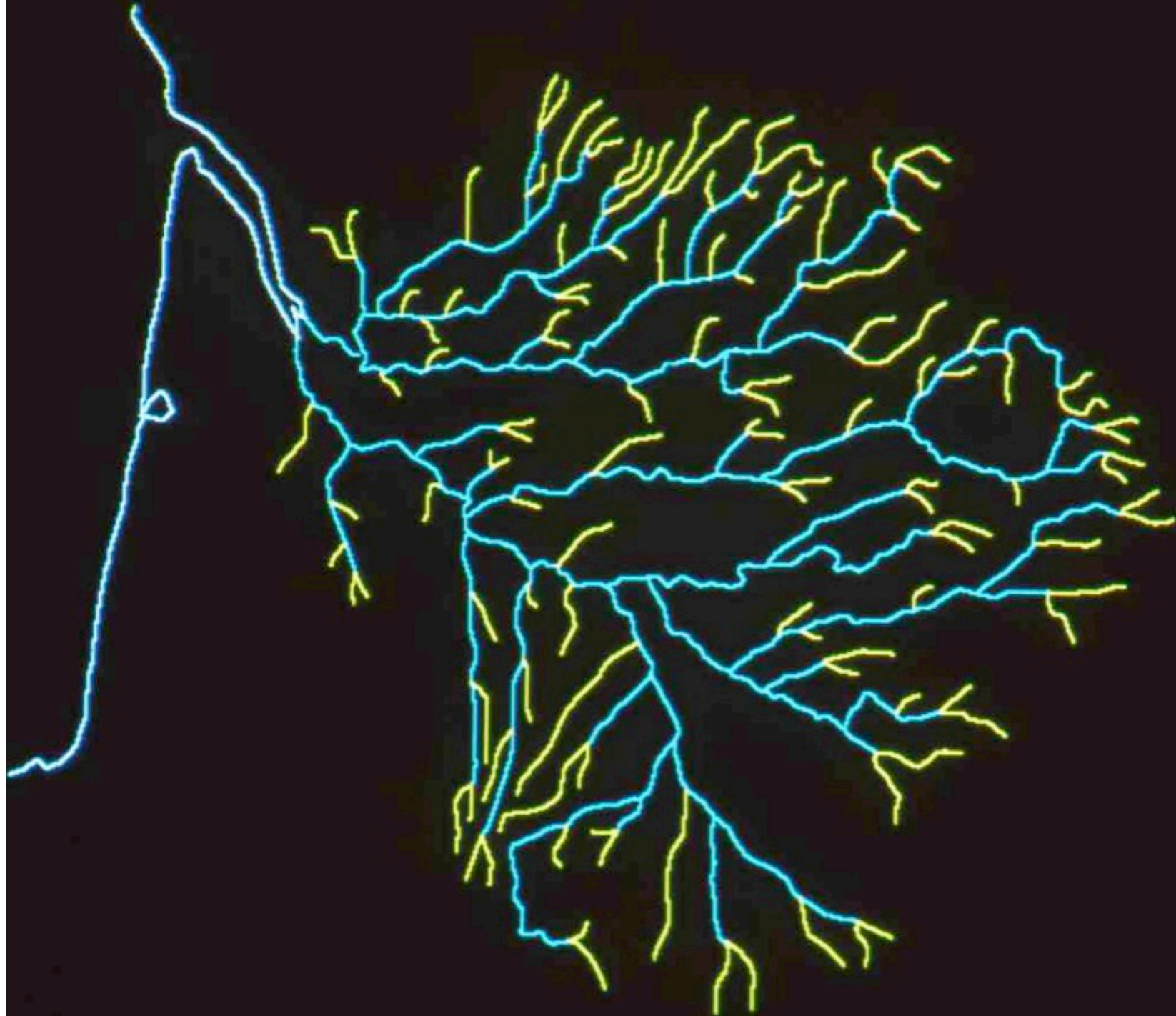
Hydrogéologie

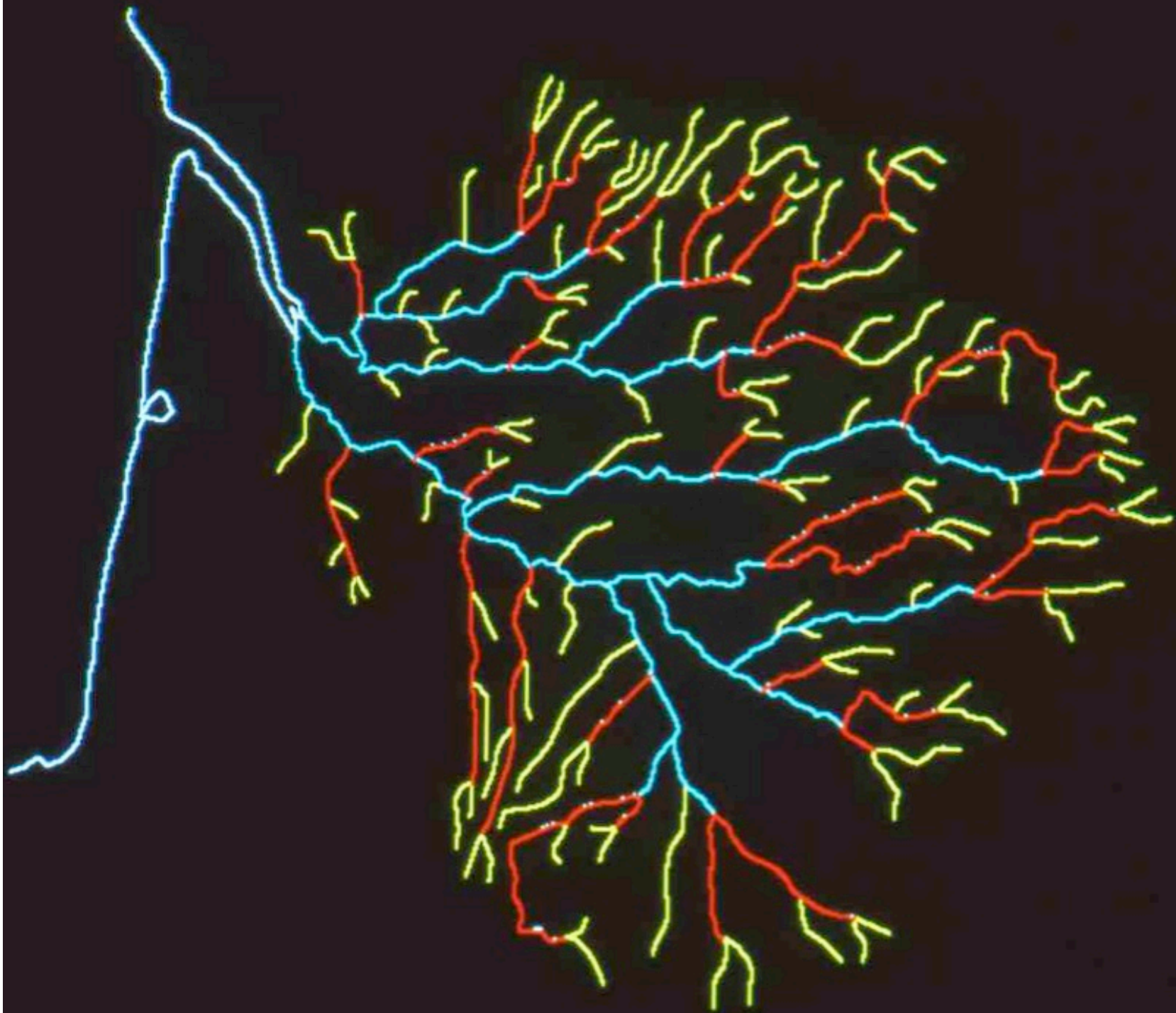
morphologie des
bassins fluviaux

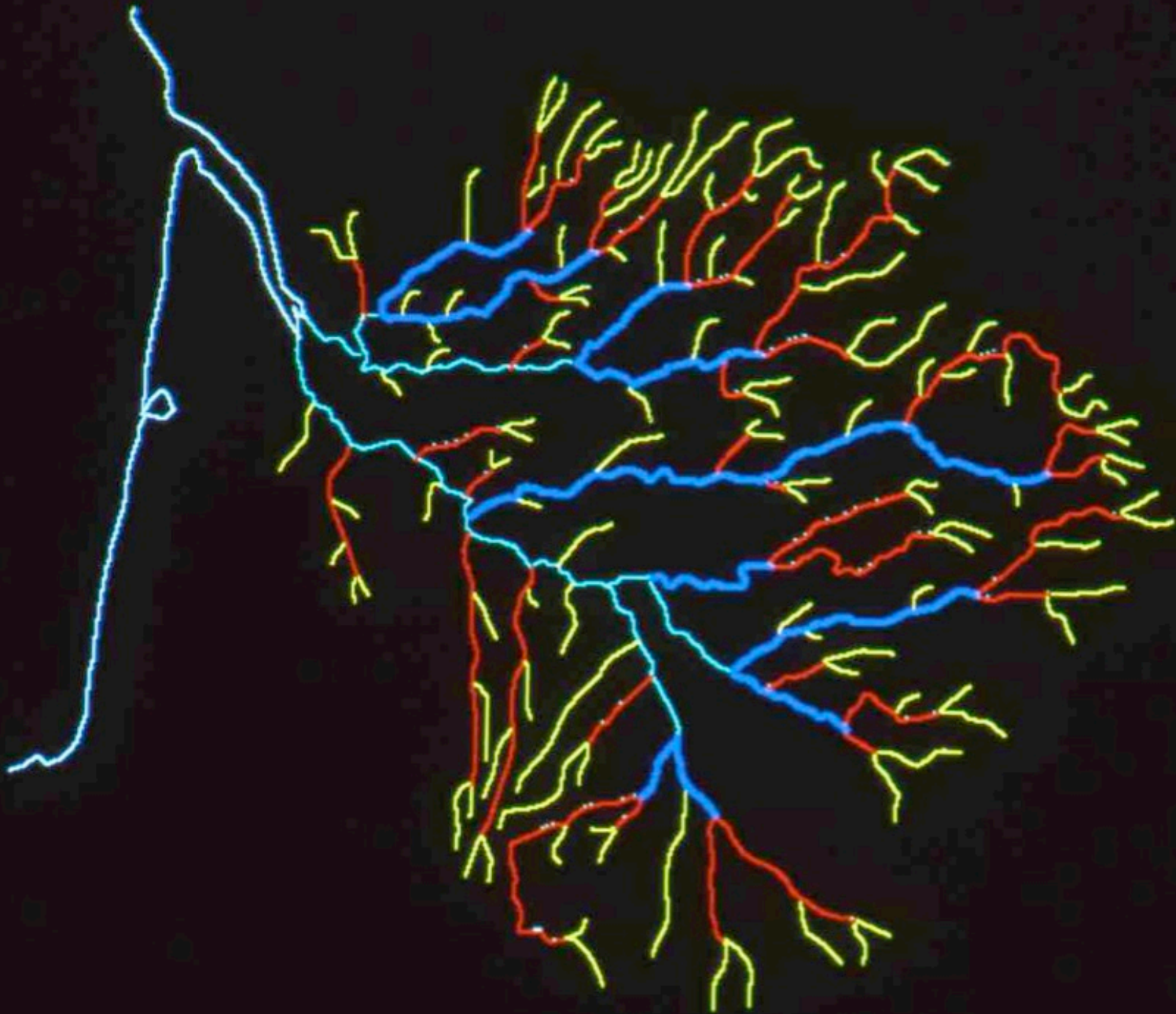
Ordre d'une rivière

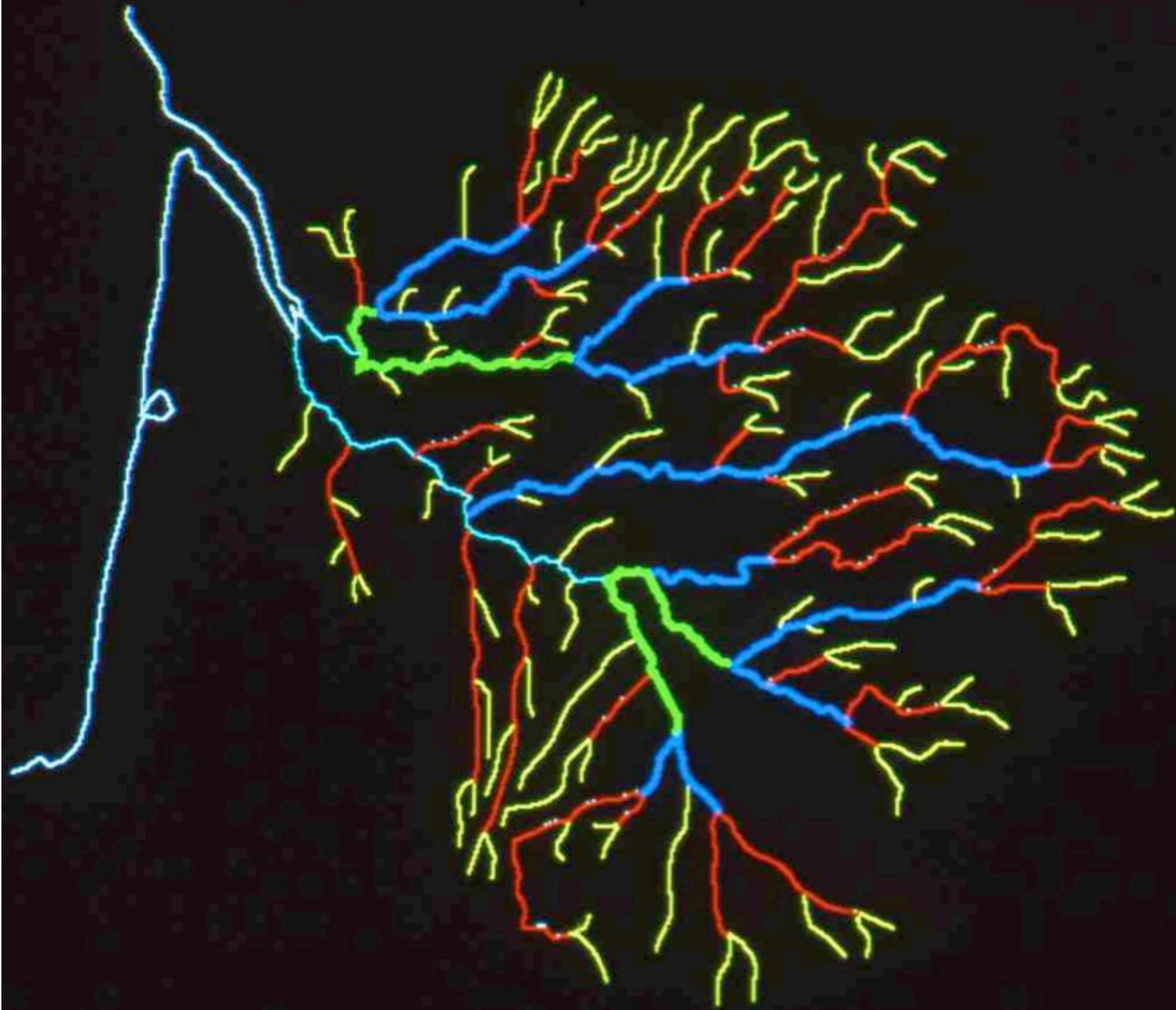


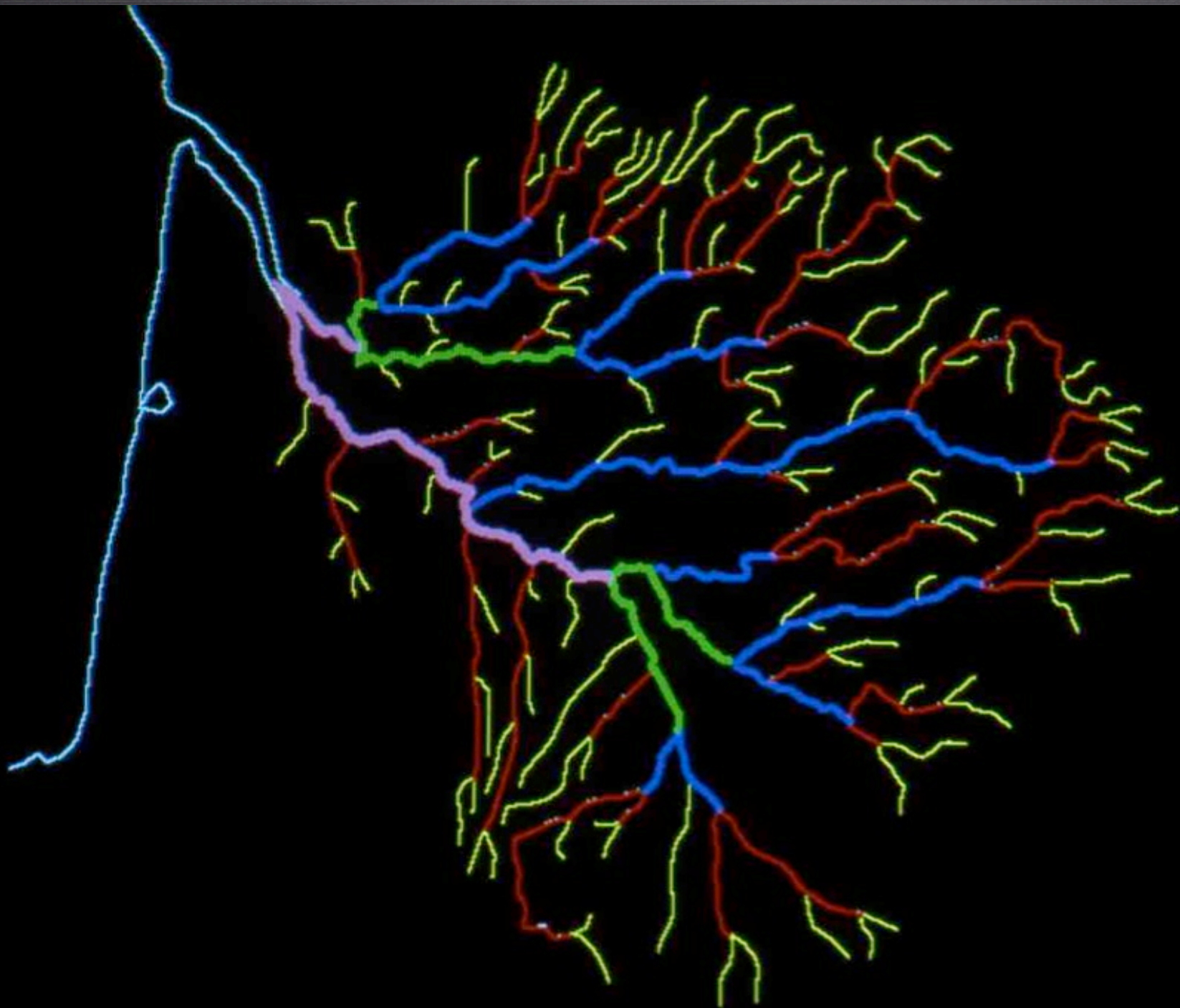


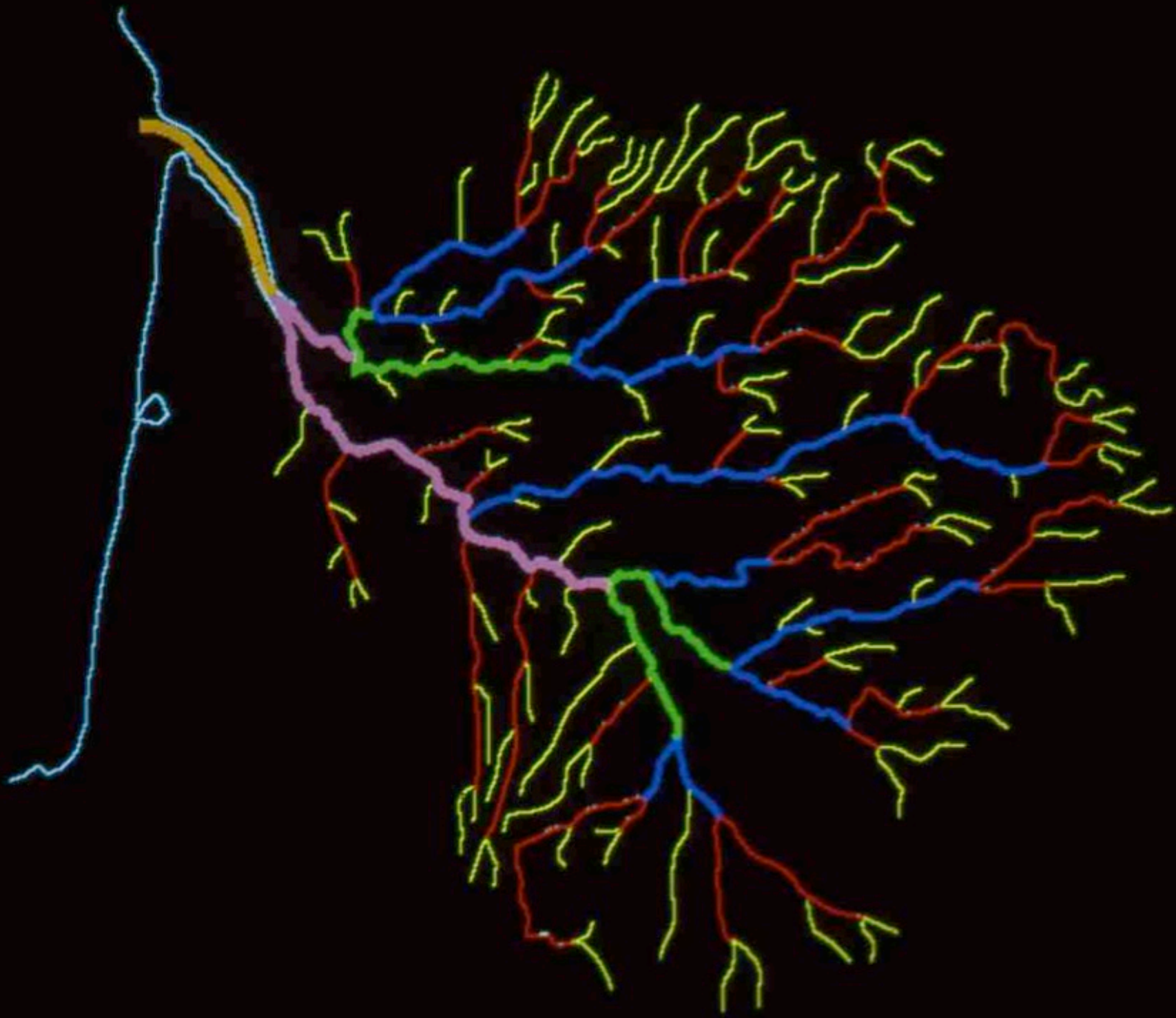


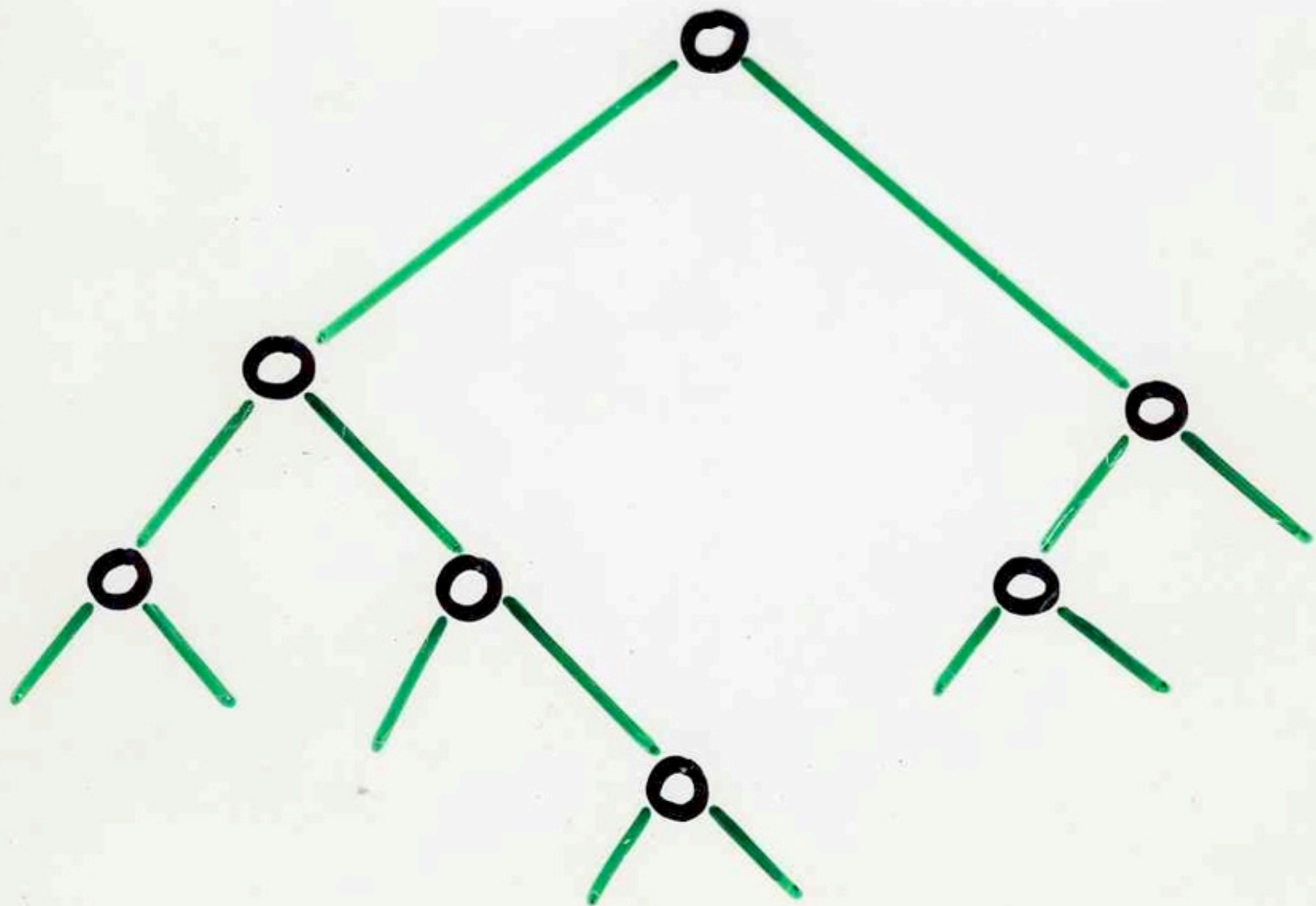




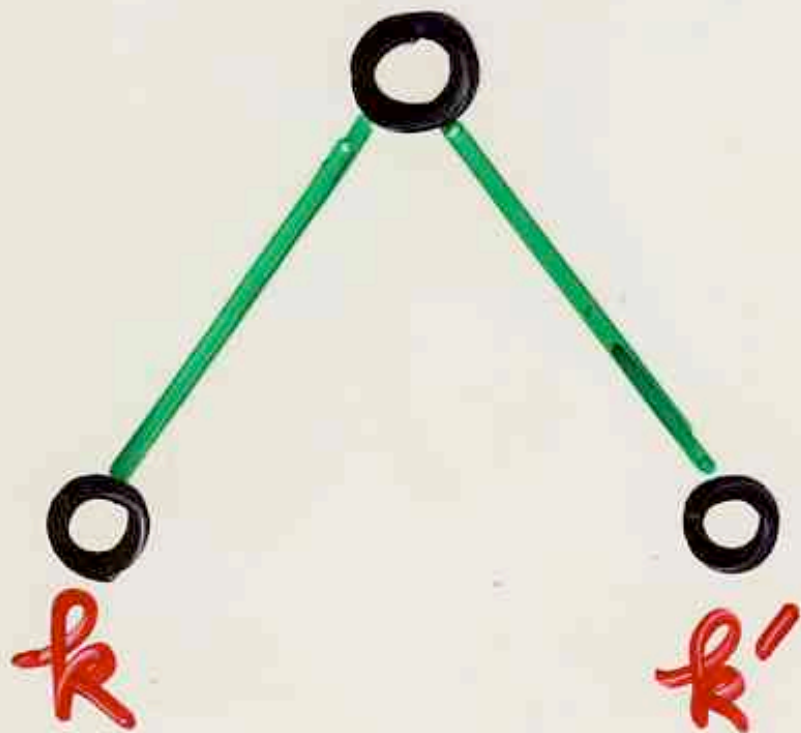




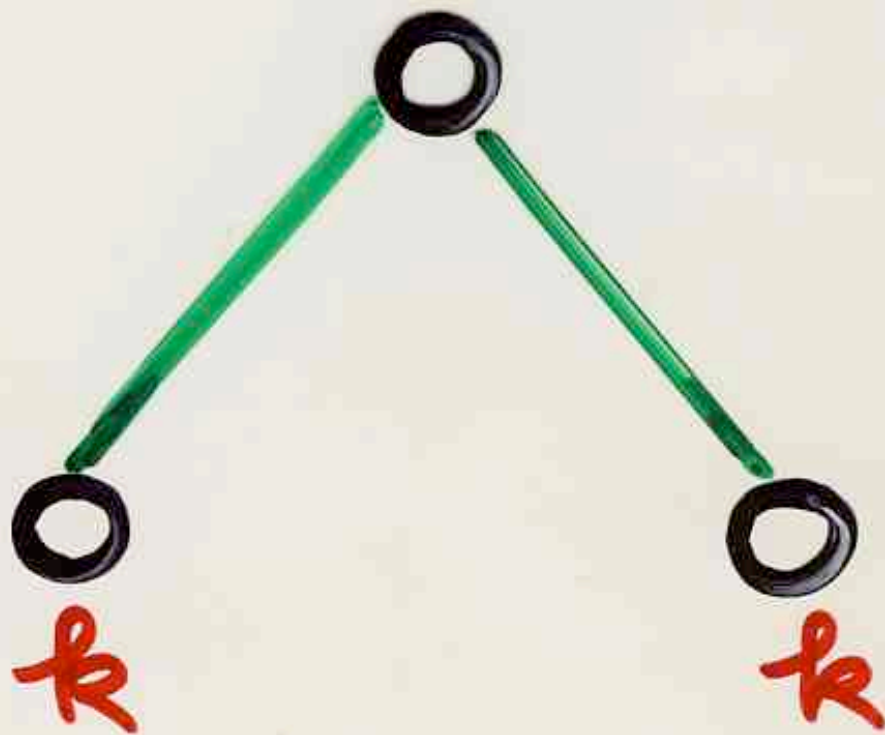




$\max(k, k')$



$k+1$



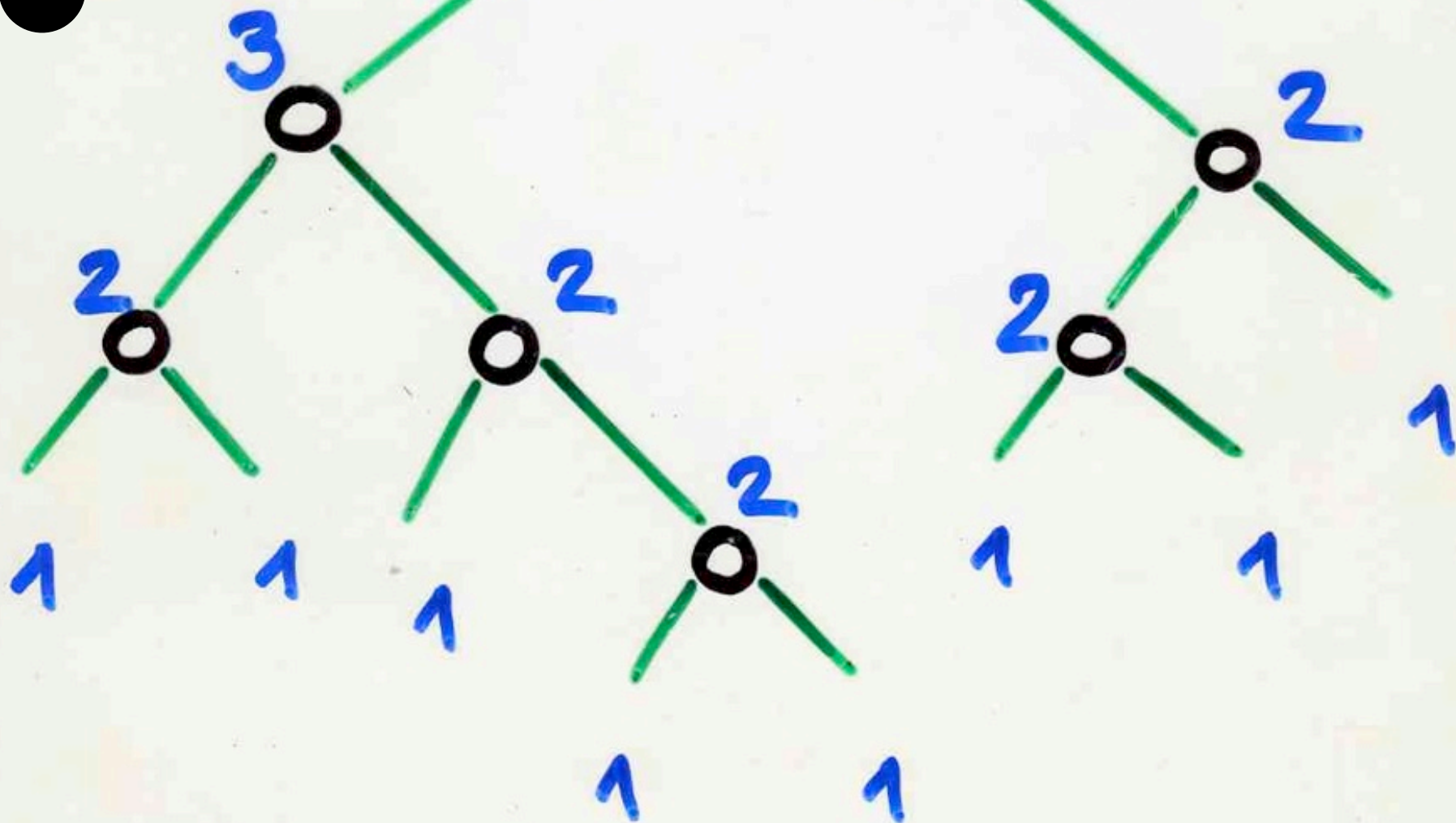


= nombre
minimum
de registres

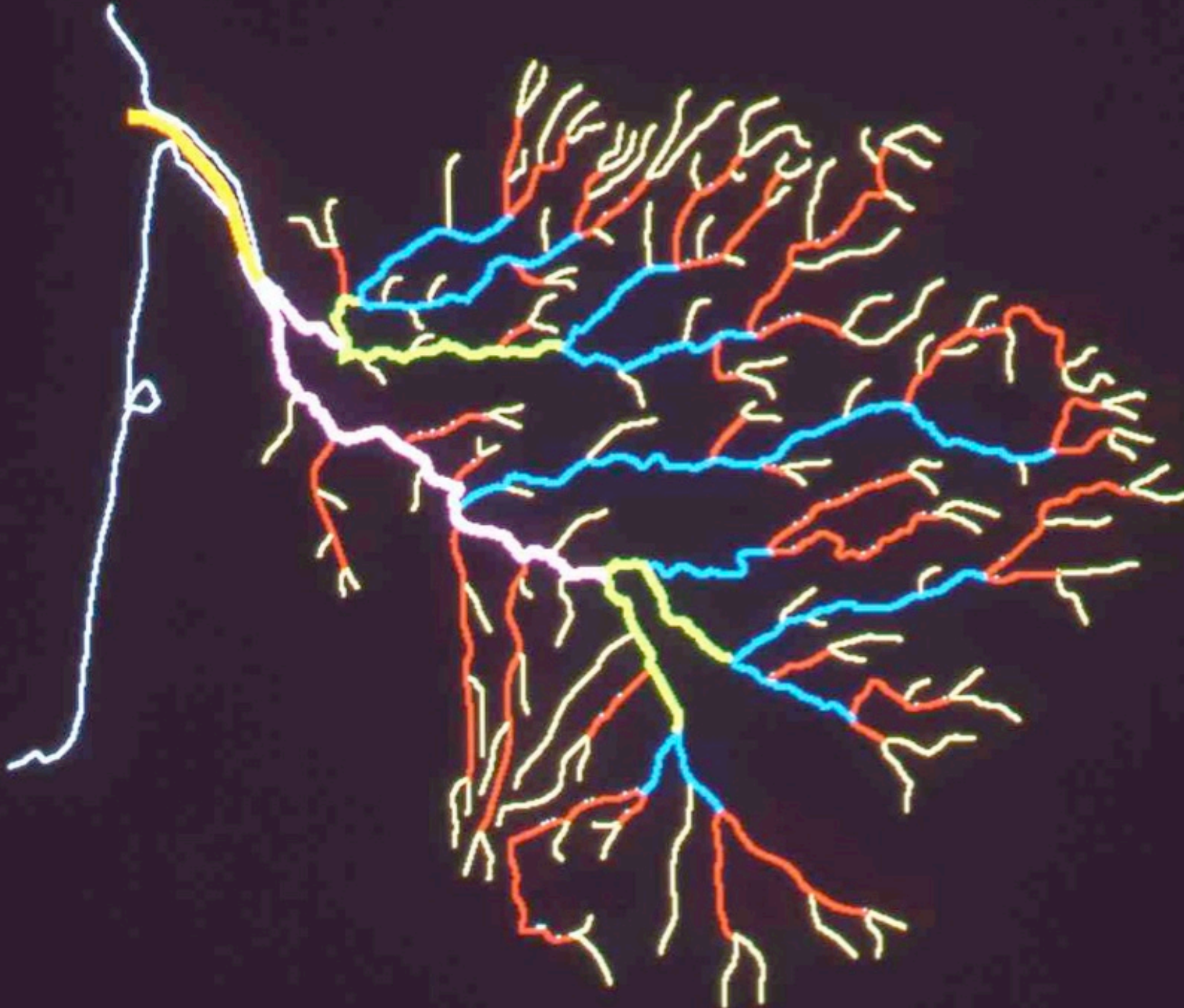
3

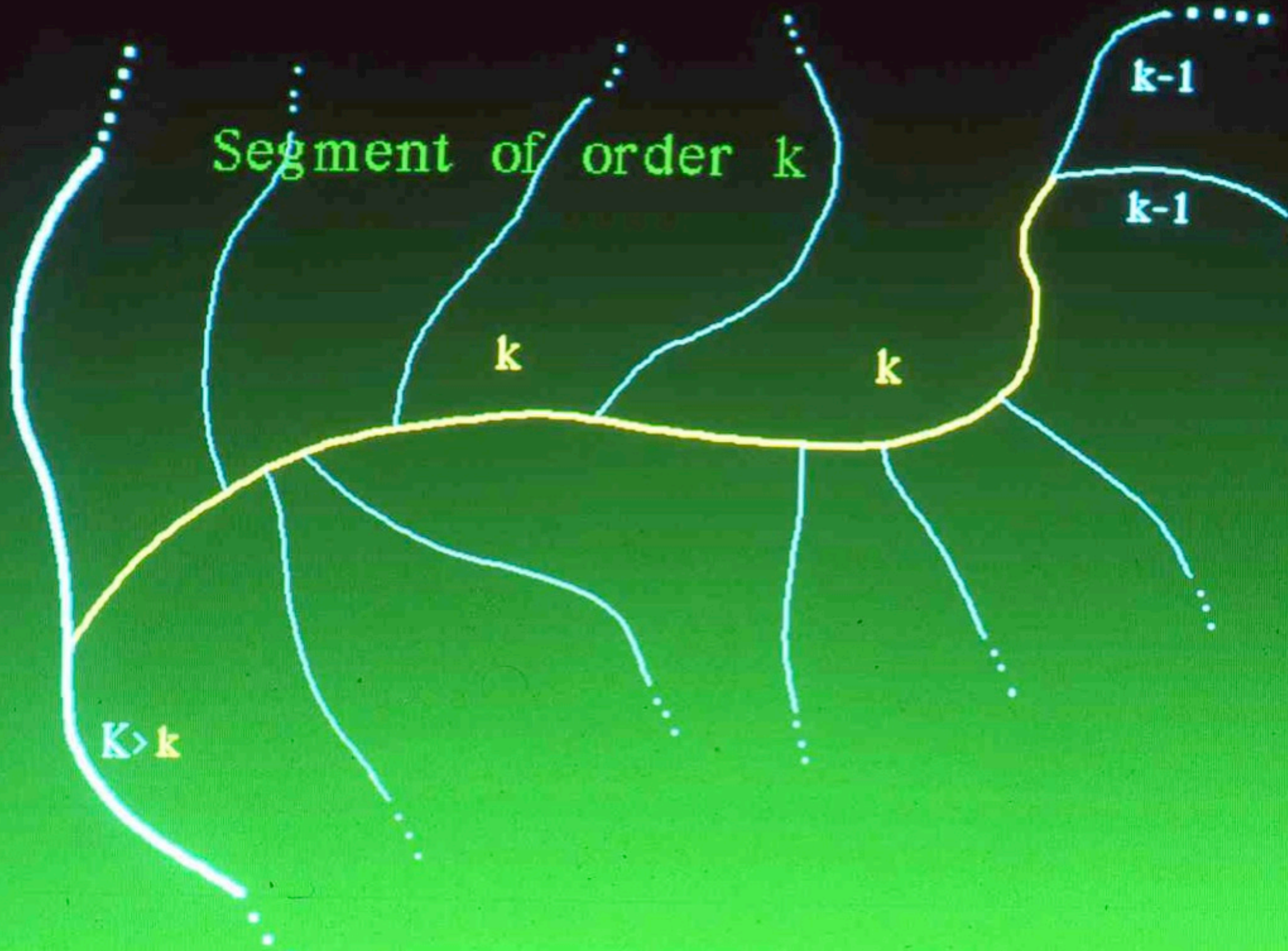
= $St(B)$

nombre de
Strahler



rivière ou segment d'ordre k



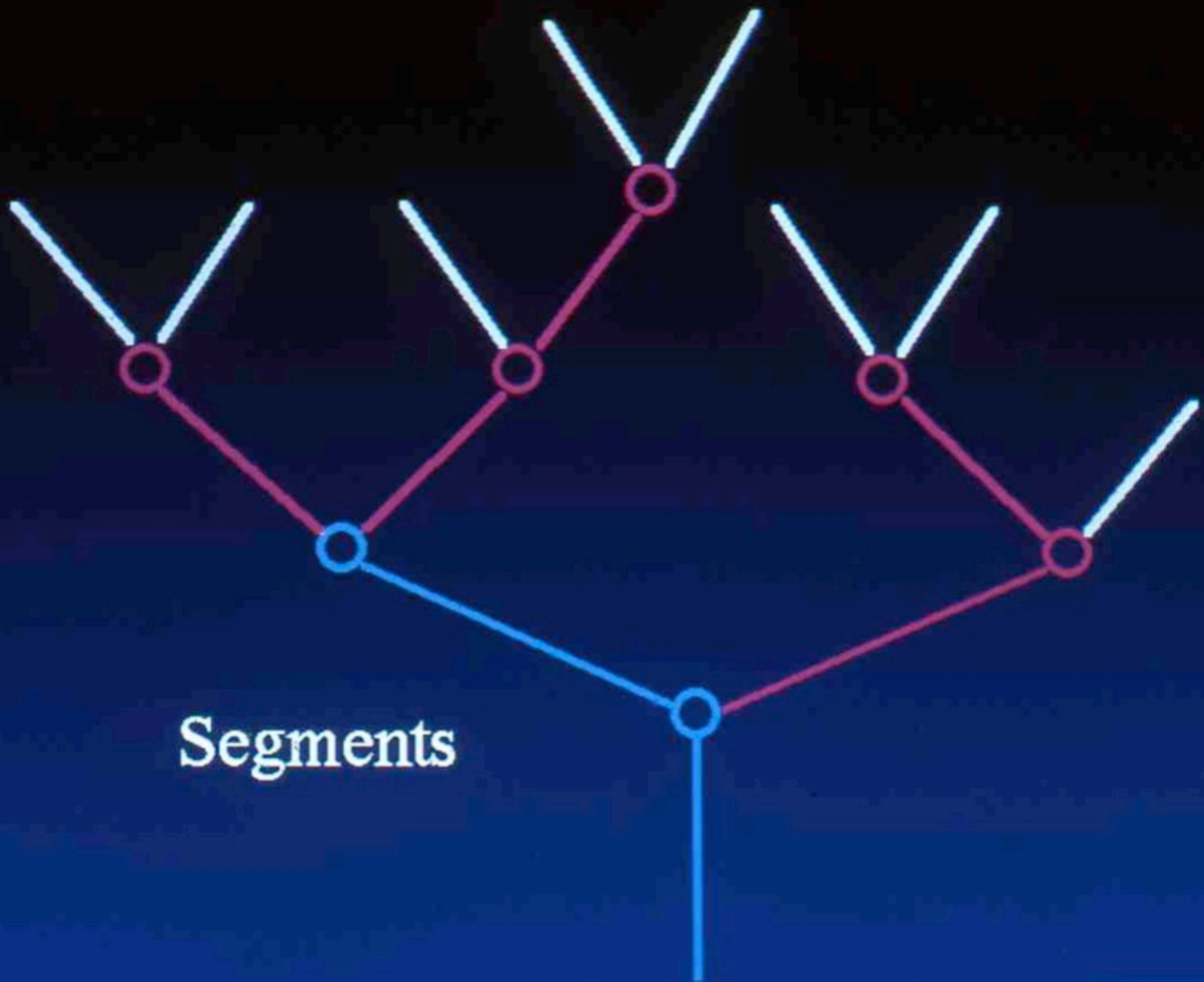


Rapport de branchement

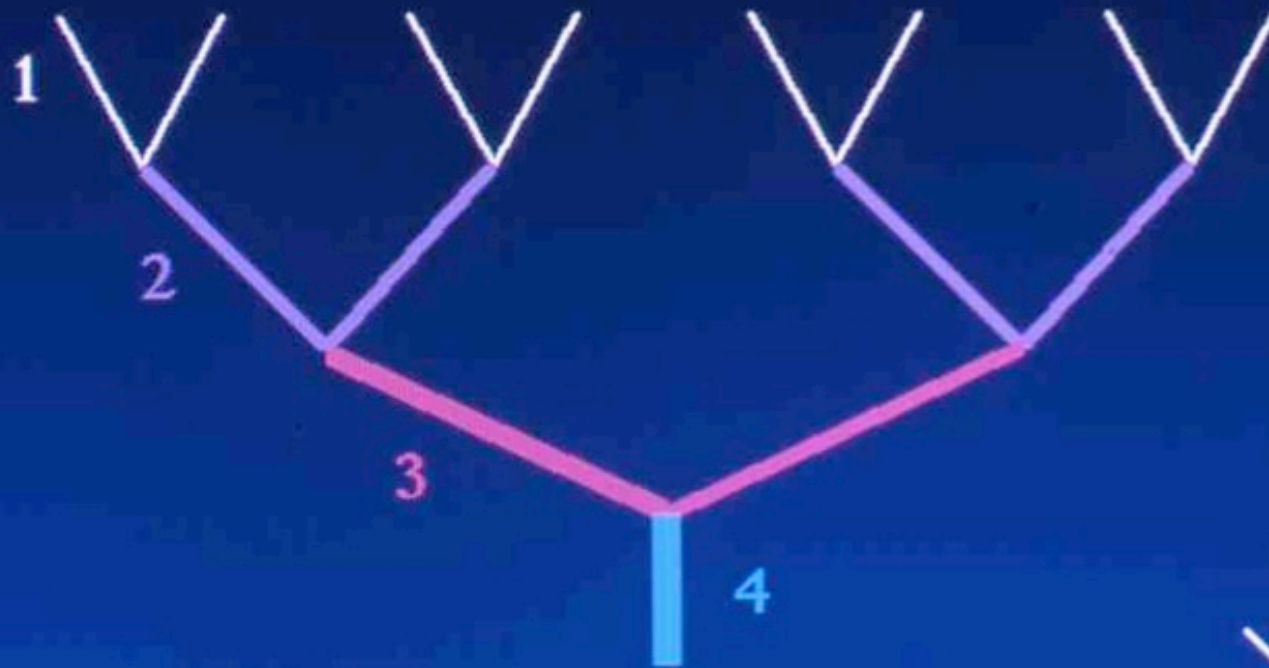
$$\beta_k = \frac{b_k}{b_{k+1}}$$

$$3 < \beta_k < 5$$

b_k = nombre de segments (rivières)
d'ordre k



Segments



arbre binaire parfait



arbre binaire «très fin»

corrélation entre la «forme» du réseau fluvial
et
la structure du sous-sol profond

Prud'homme, Nadeau, Vigneaux, 1970, 1980

informatique graphique

matrice de ramification
d'un arbre binaire

Arquès, Eyrolles, Janey, X.V.

SIGGRAPH'89, IMAGINA' 90



LE CORBUSIER

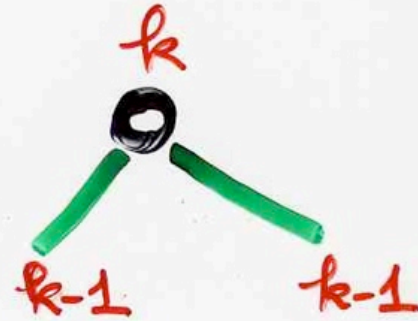
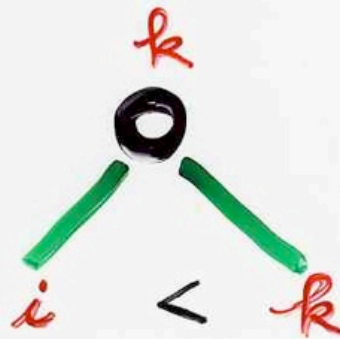
Images de synthèse
d'arbres, feuilles, paysages, ...

Anquetin, Eyrolles, Jamey, X.G.V.

SIGGRAPH '89

AIA

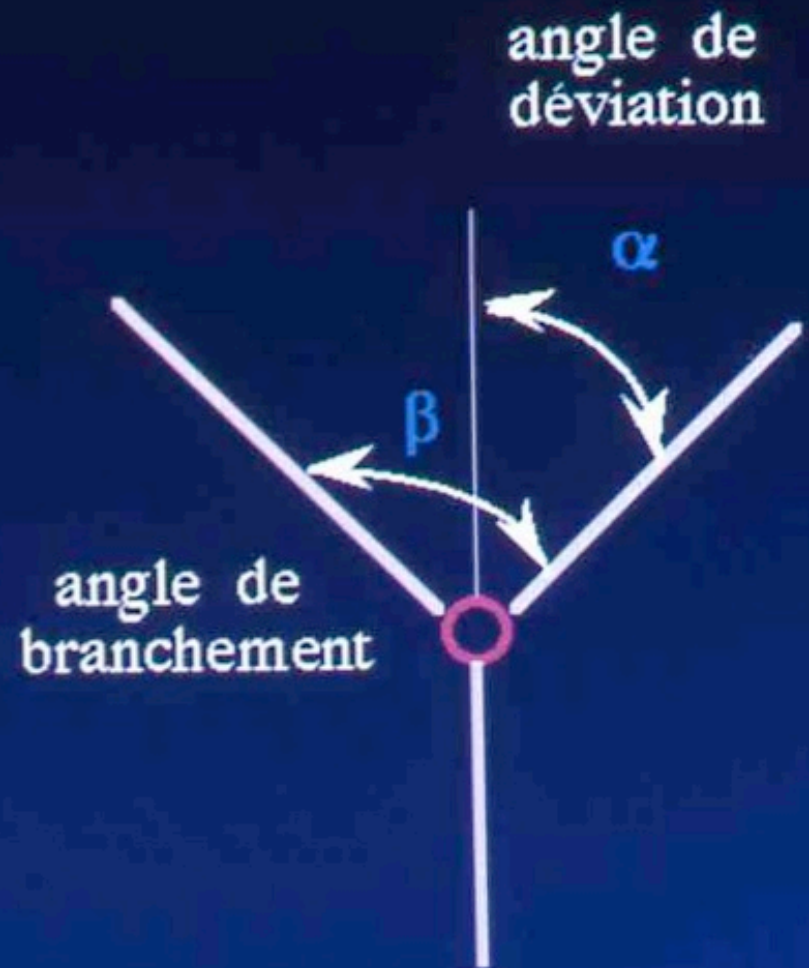
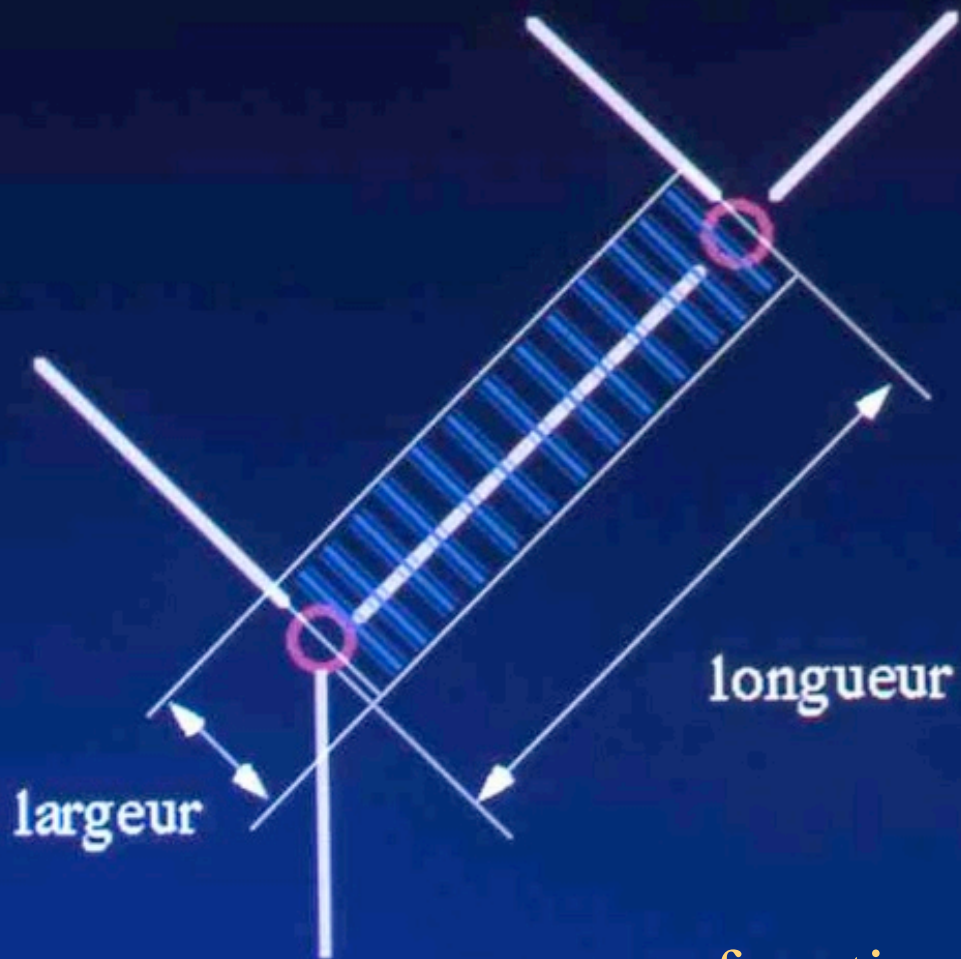
matrice
de
ramification



$$P_{k,i} = \frac{b_{k,i}}{a_k}$$

biordre (k, i)

tableau
de
probabilités



fonctions de l'ordre k
et du biordre (k,i)

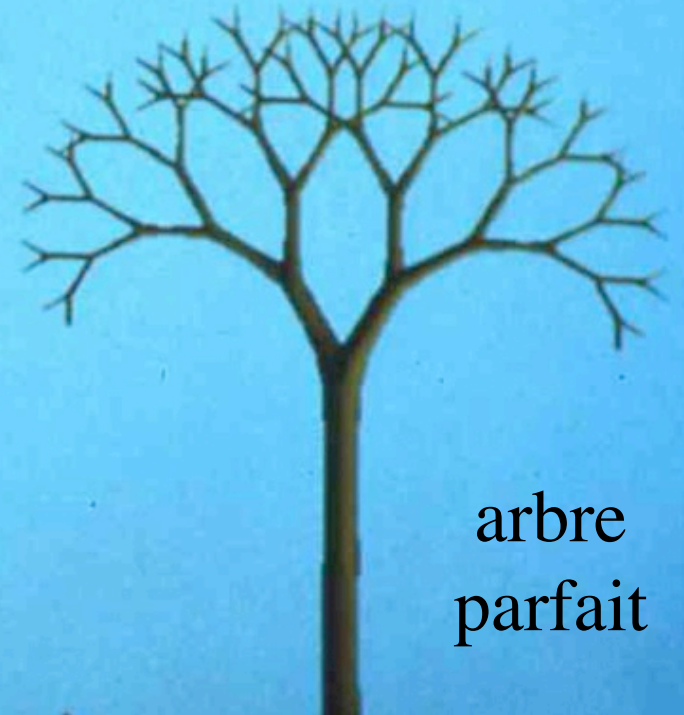
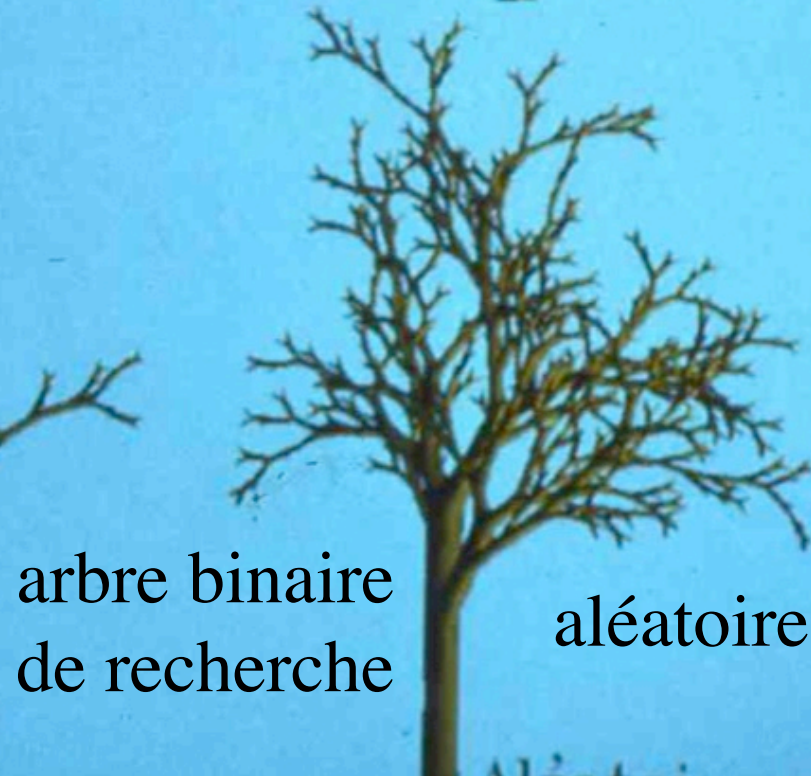
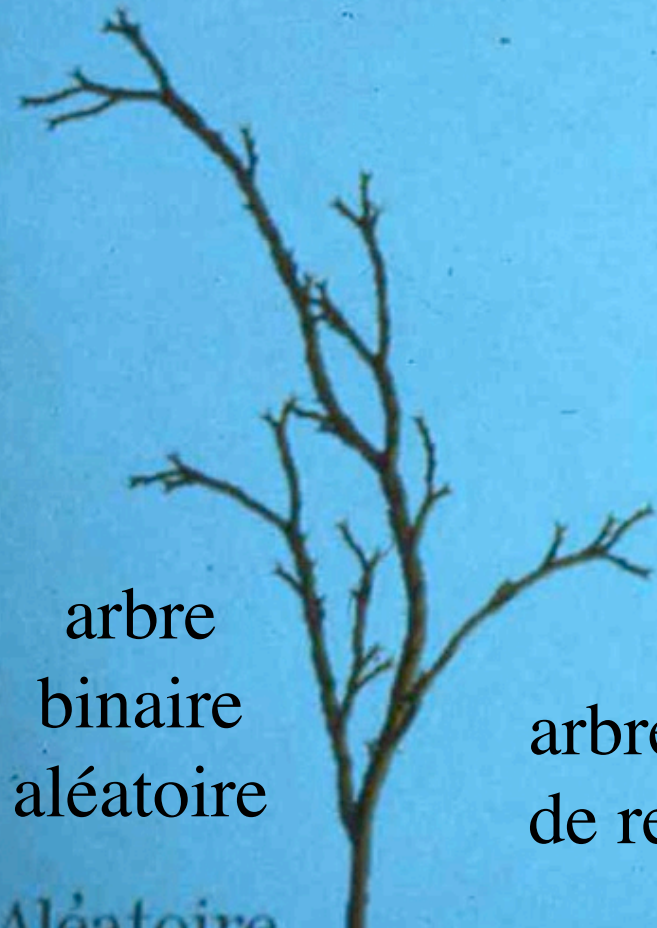






ASA

mélange de 3 matrices de ramification



mélange
de 3 matrices
de ramification

3 «formes»



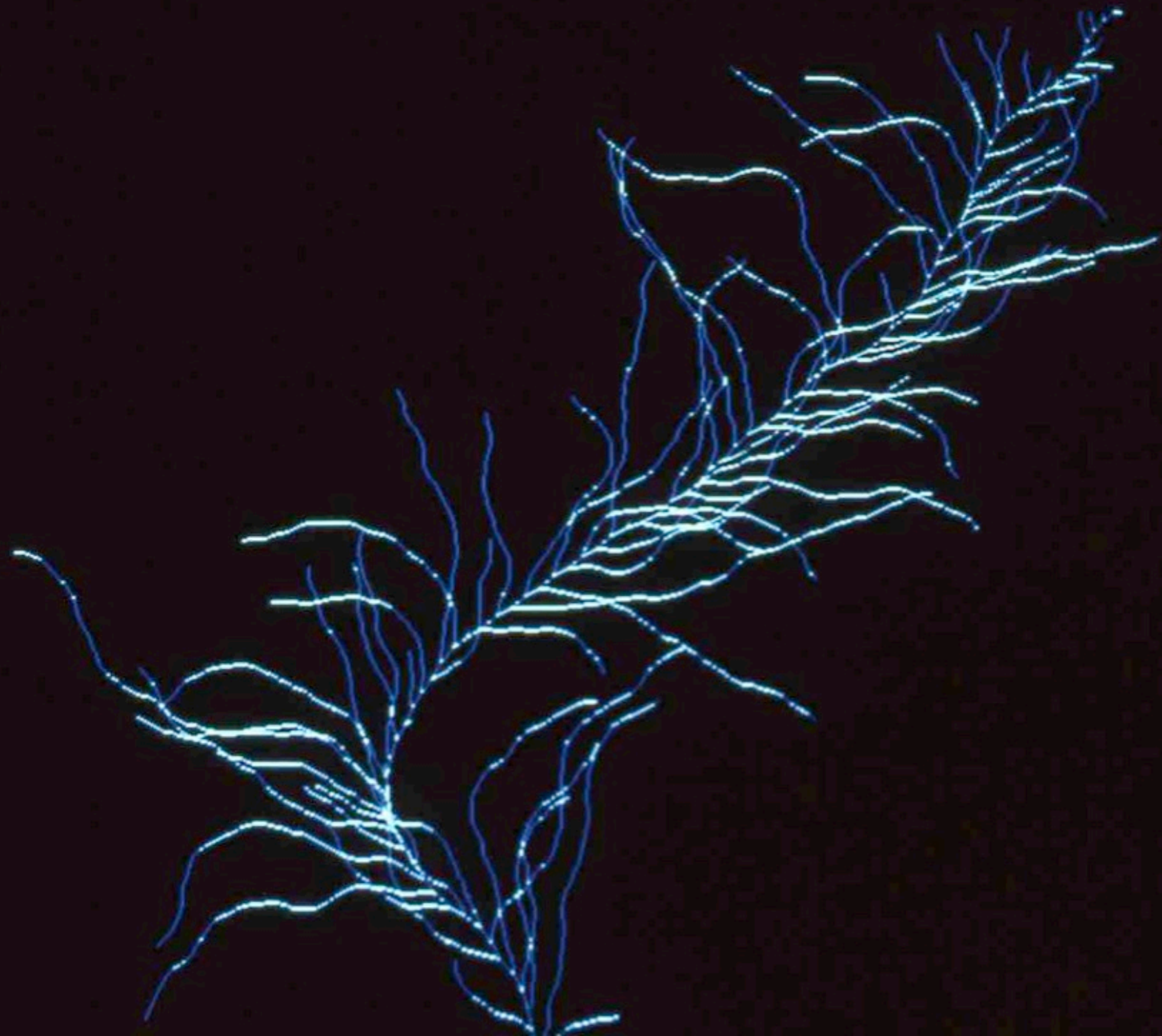
2 :	0	10000										
3 :	0	0	10000									
4 :	0	0	0	10000								
5 :	5000	2500	1250	625	625							
6 :	5000	2500	1250	625	313	312						
7 :	125	250	500	1000	2000	3000	3125					
8 :	63	125	250	500	1000	2000	3000	3062				
9 :	31	63	125	250	500	1000	2000	3000	3031			
10 :	15	31	63	125	250	500	1000	2000	3000	3016		
11 :	7	15	31	63	250	125	500	1000	2000	3000	3009	





A\$A















S'il existe une quelconque beauté dans ces images de synthèse d'arbres, elle n'est que le pâle reflet de l'extraordinaire beauté des mathématiques se cachant derrière les algorithmes générant ces images.

Nombre de Strahler

moyen

parmi tous les arbres binaires
ayant n sommets

$$St_n = \log_4 n + f(\log_4 n) + o(1)$$

Flejole, Raoult, Vuillemin (1979)
Kemp (1979)

f fonction
périodique

Théorie des nombres

$T(n)$ = nombre total de 1 dans
les écritures binaires
des nombres $1, 2, \dots, (n-1)$.

série génératrice

$S_{n,k}$ = nombre d'arbres binaires B
ayant n sommets (internes)
et nombre de Strahler
 $st(B) = k$

$$S_k(t) = \sum_{n \geq 0} S_{n,k} t^n$$

série formelle

$$S_1 = 1$$

$$S_2 = \frac{t}{1 - 2t}$$

$$S_3 = \frac{t^3}{1 - 6t + 10t^2 - 4t^3}$$

$$S_4 = \frac{t^7}{1 - 14t + 78t^2 - 220t^3 + 330t^4 - 252t^5 + 84t^6 - 8t^7}$$



Pafnuty Tchebychev
(1829-1894)

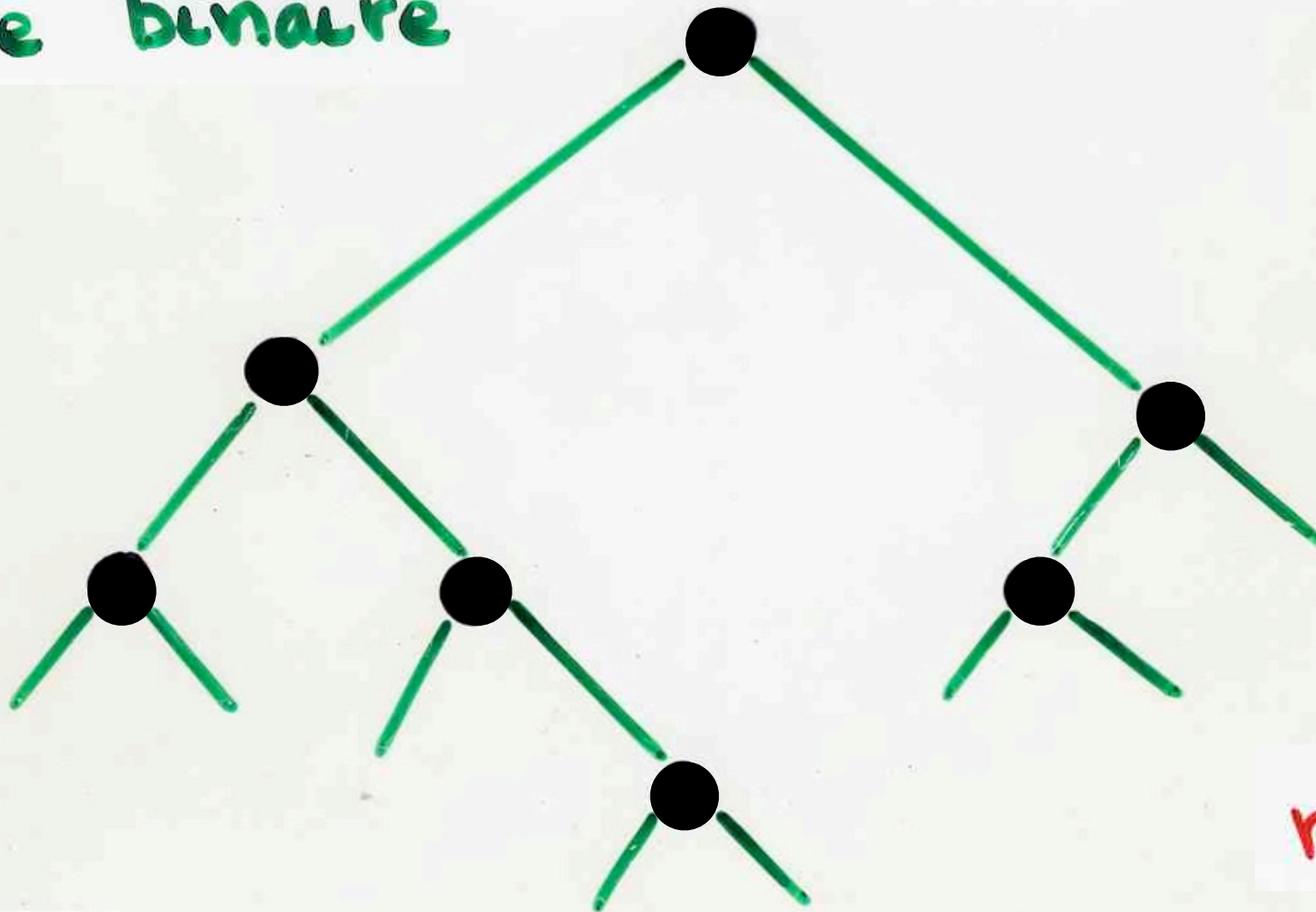
polynômes de
Tchebychev

Trigonométrie

$$\sin(n+1)\theta = (\sin\theta) U_n(\cos\theta)$$

Compter les arbres ...

arbre binaire



$$n = 7$$

$C_n =$

nombre d'arbres binaires
ayant n sommets (internes)

[ou $n+1$ sommets externes]
feuilles

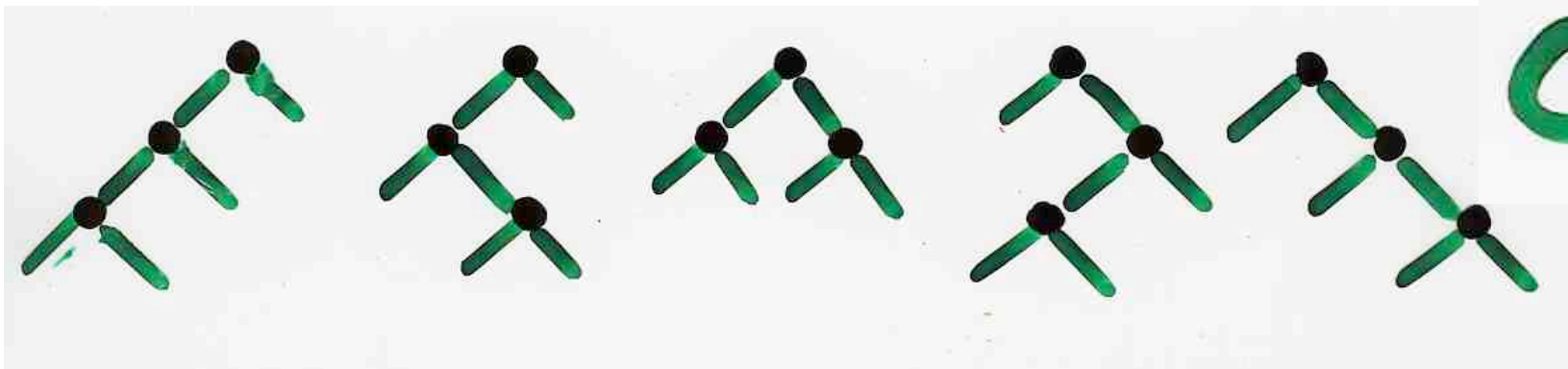




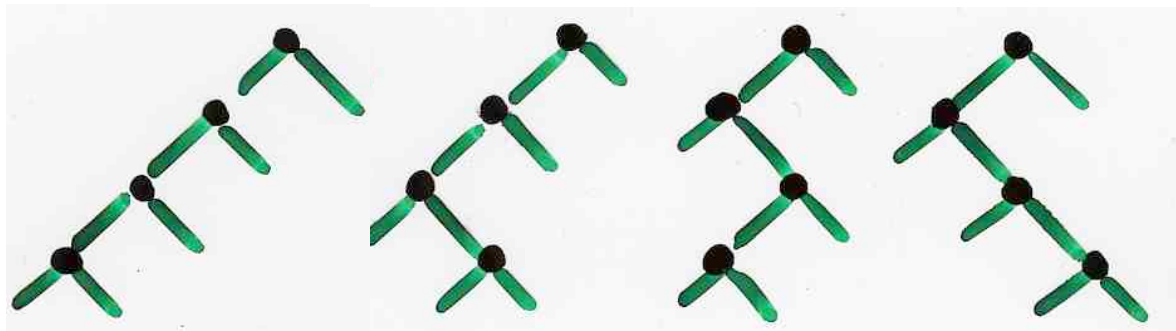
$$C_1 = 1$$



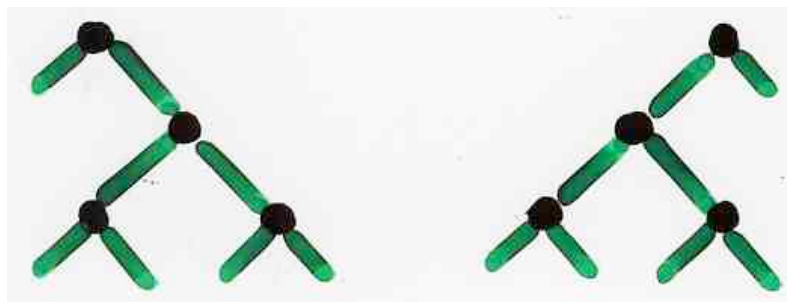
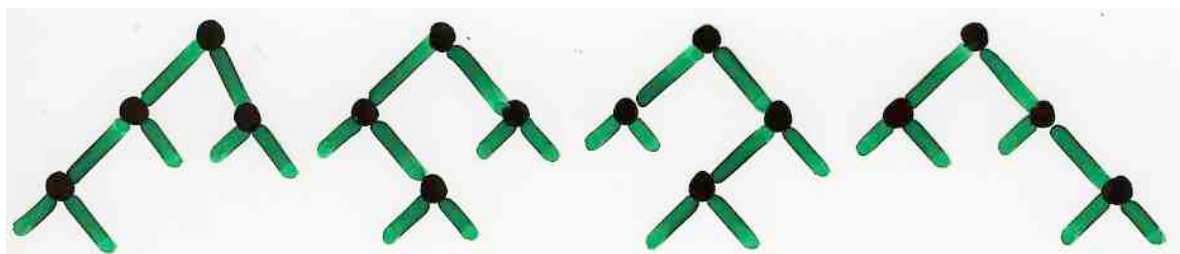
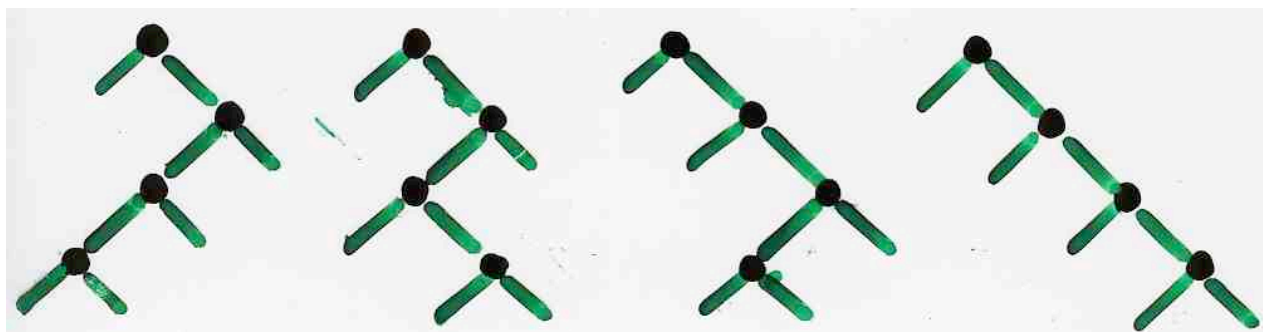
$$C_2 = 2$$



$$C_3 = 5$$



$$C_4 = 14$$



nombre
de
Catalan

$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

$$= \frac{(2n)!}{(n+1)! n!}$$

$$n! = 1 \times 2 \times \dots \times n$$

nombres de Catalan

1 1 2 5 14 42



E. Catalan
(1814 - 1894)

Vierfeld, und steht hier auf 8 nicht bestimmbare Stellen geschrieben
 fünf der Diagonale I. a^2 ; II. b^2 ; III. c^2 ; IV. d^2 ; V. e^2

Wenn hier ein Dreieck fünf 3 Diagonale in 4 Triangula
 zerlegt, und steht hier auf 14 bestimmbare Stellen geschrieben

Hier ist die Frage Generaliter. In ein Polygon von n Seiten
 fünf $n-3$ Diagonale in $n-2$ Triangula zerlegt, und auf

wie viele Stellen geschrieben haben, steht geschrieben worden.
 Auf wie viele Stellen diese Bestimmung stehen = x

wenn $n = 1, 2, 5, 14, 42, 132, 429, 1430, \dots$

ist $x = 1, 2, 5, 14, 42, 132, 429, 1430, \dots$
 Hieraus sieht man den Zusammenhang. In generaliter

$$x = \frac{2 \cdot 6 \cdot 10 \cdot 14 \cdot 18 \cdot 22 \cdot \dots \cdot (2n-10)}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot \dots \cdot (n-1)} = \frac{(2n)!}{(n+1)! \cdot n!}$$

$6 = 2 \cdot \frac{3}{1}, 14 = 5 \cdot \frac{3}{2}, 42 = 14 \cdot \frac{3}{2}, 132 = 42 \cdot \frac{3}{2}$

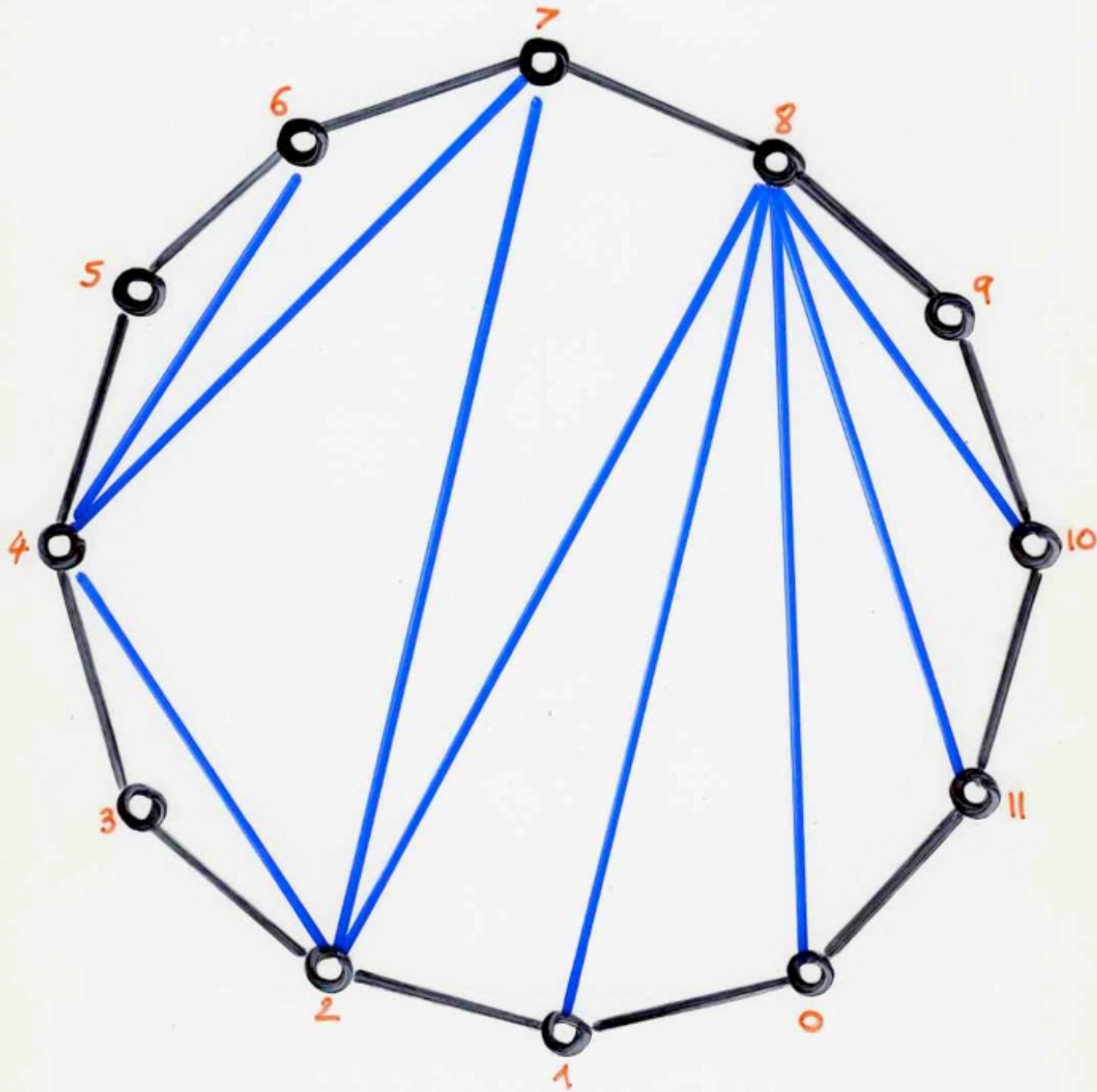
$$C_n = \frac{1}{n+1} \binom{2n}{n} \quad n! = 1 \times 2 \times 3 \times \dots \times n$$

Une lettre de Leonhard Euler
à Christian Goldbach

Berlin, 4 Septembre 1751

Leonhard
Euler
1707 - 1783





... die Mittel ist ungleich. ...

$$\frac{1 - 2a - \sqrt{1 - 4a^2}}{2a^2}$$

$$1 + 2a + 5a^2 + 14a^3 + 42a^4 + 132a^5 + \text{etc}$$

gemeinlich. ...

$$1 + 2a + 5a^2 + 14a^3 + 42a^4 + 132a^5 + \text{etc} = \frac{1 - 2a - \sqrt{1 - 4a^2}}{2a^2}$$

alle wenn $a = \frac{1}{4}$ ist $1 + \frac{2}{4} + \frac{5}{16} + \frac{14}{64} + \frac{42}{256} + \text{etc} = 4$.

Die ... die ... die ...
 ... die ... die ...
 ... die ... die ...
 ... die ... die ...

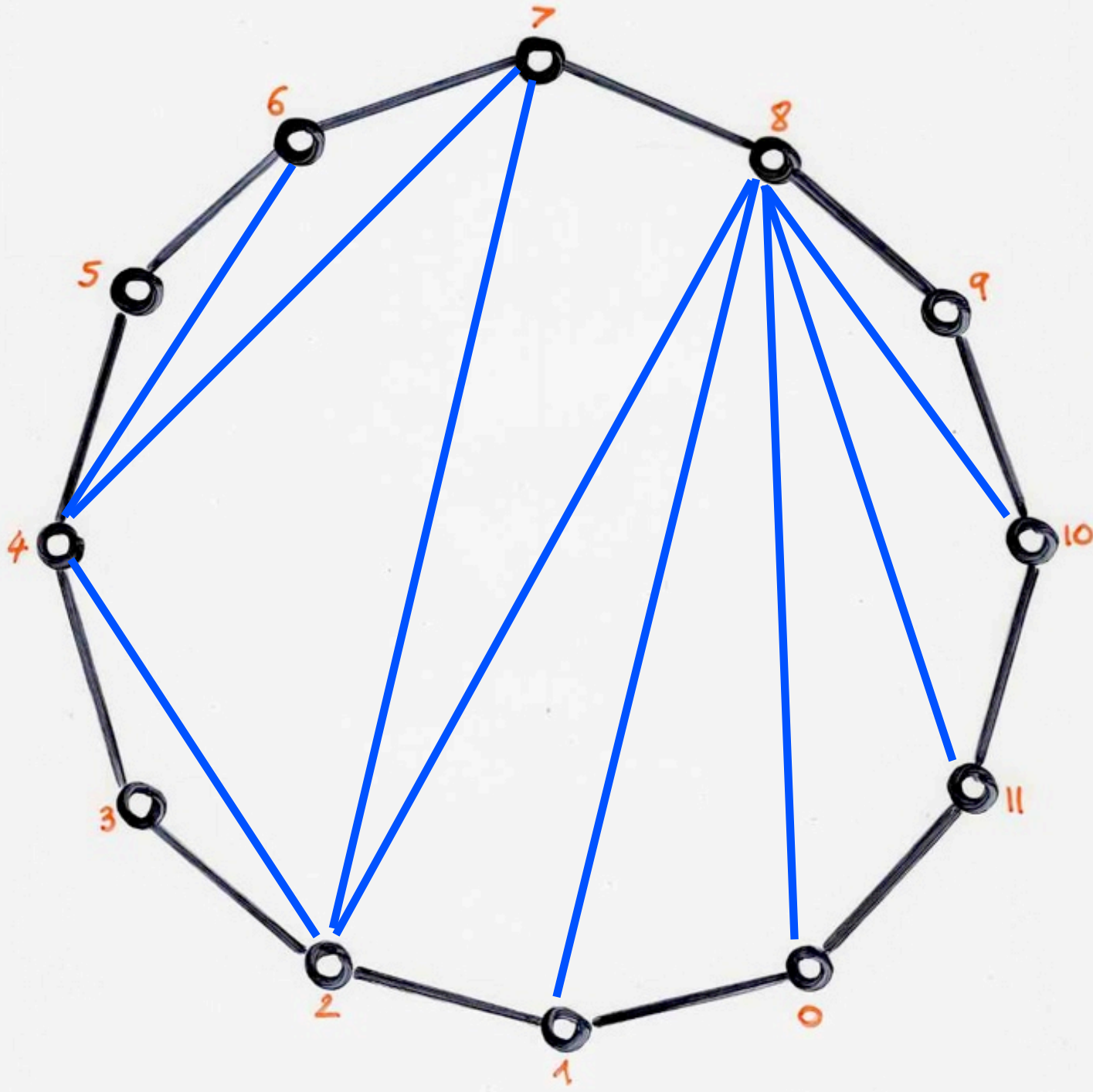
... die ...

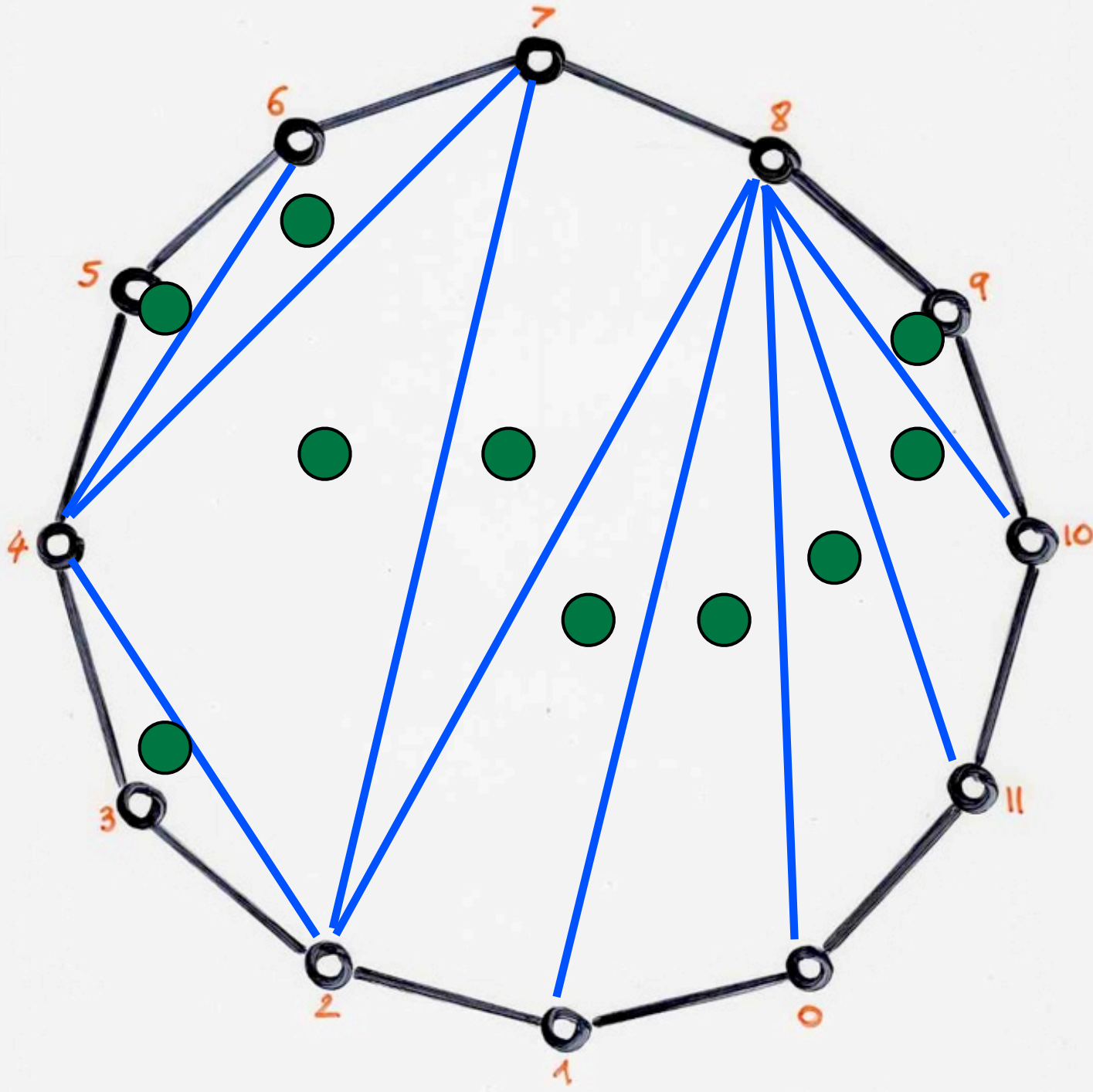
... 4^{te} Sept
 1751.

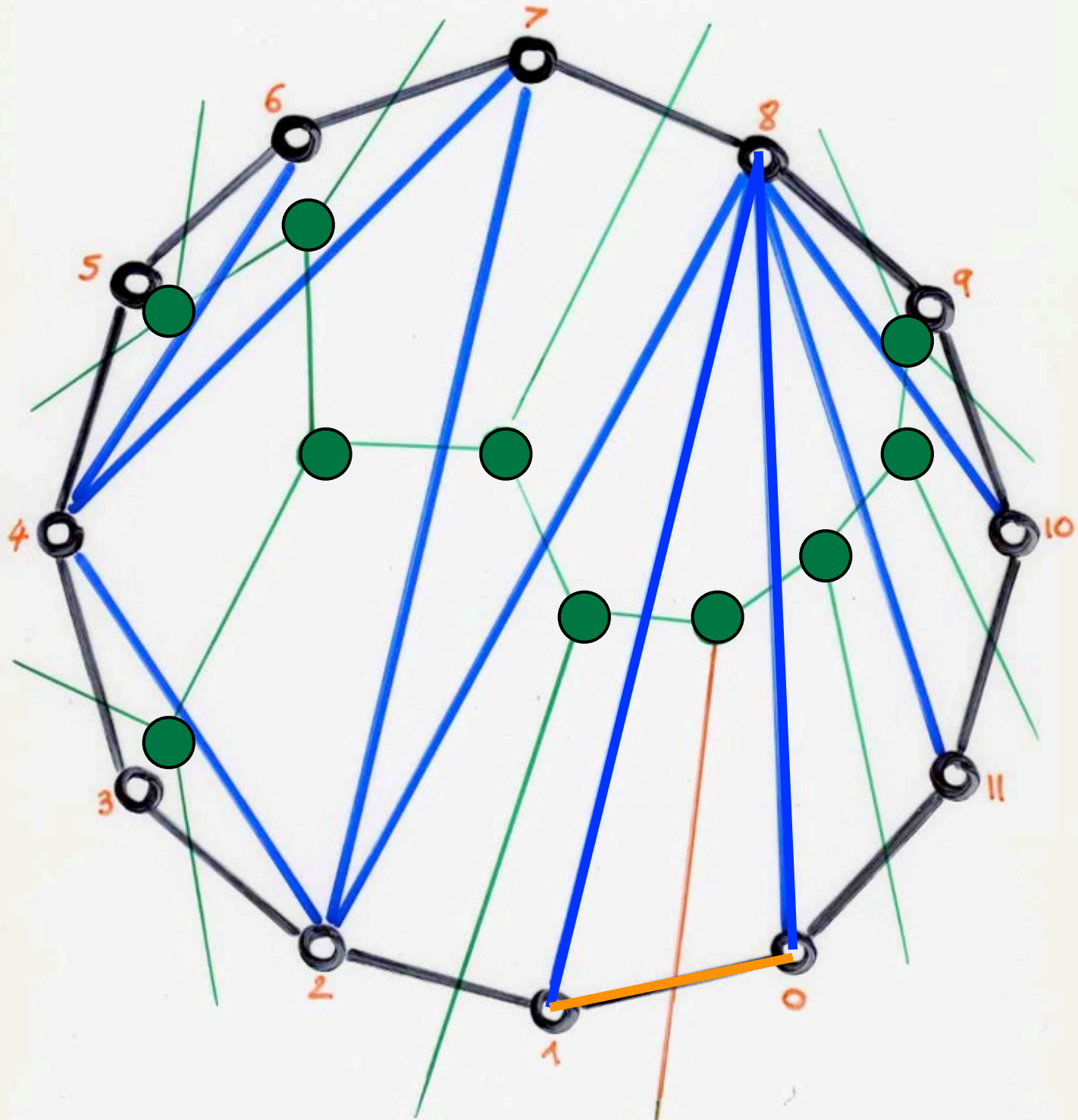
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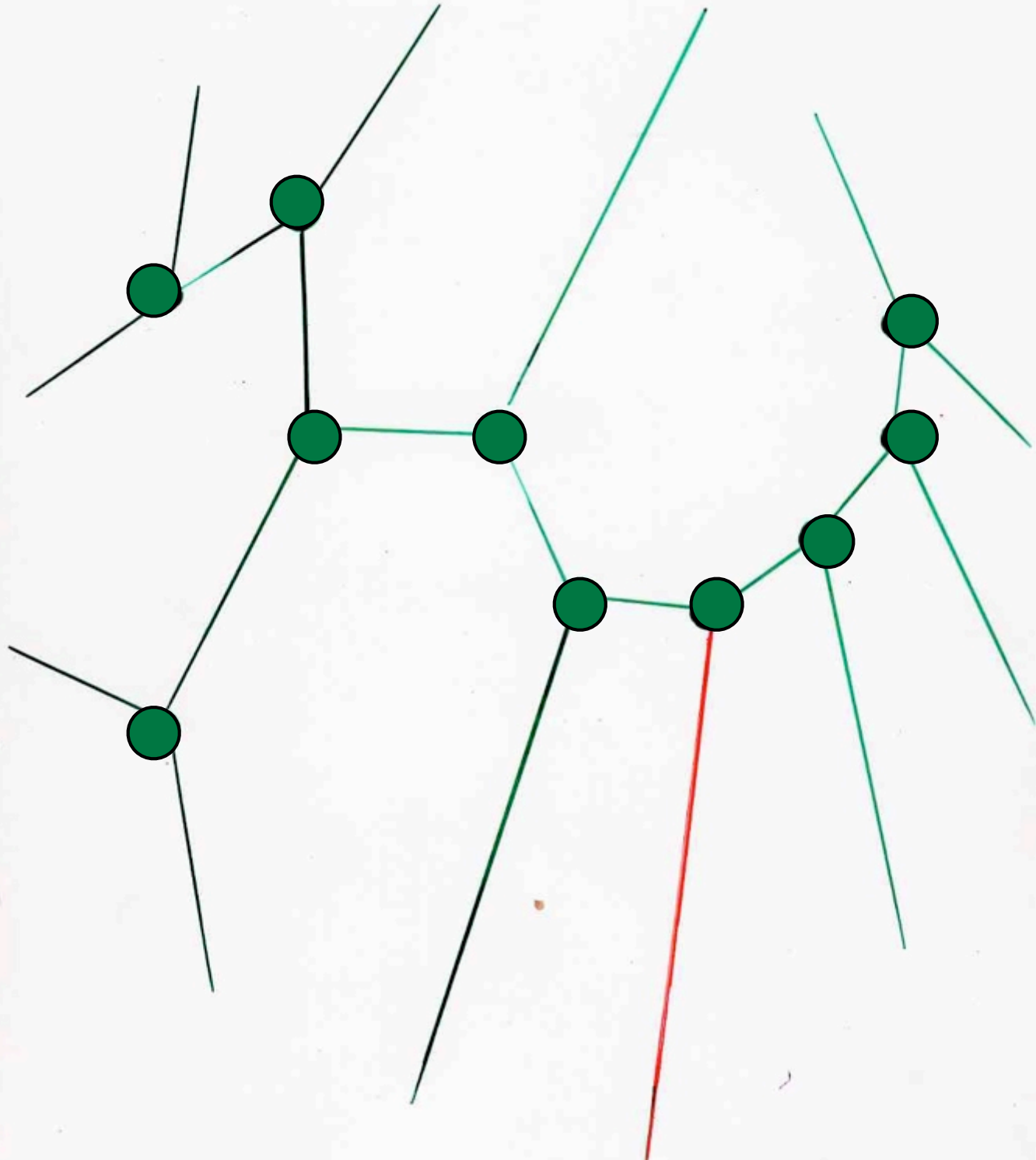
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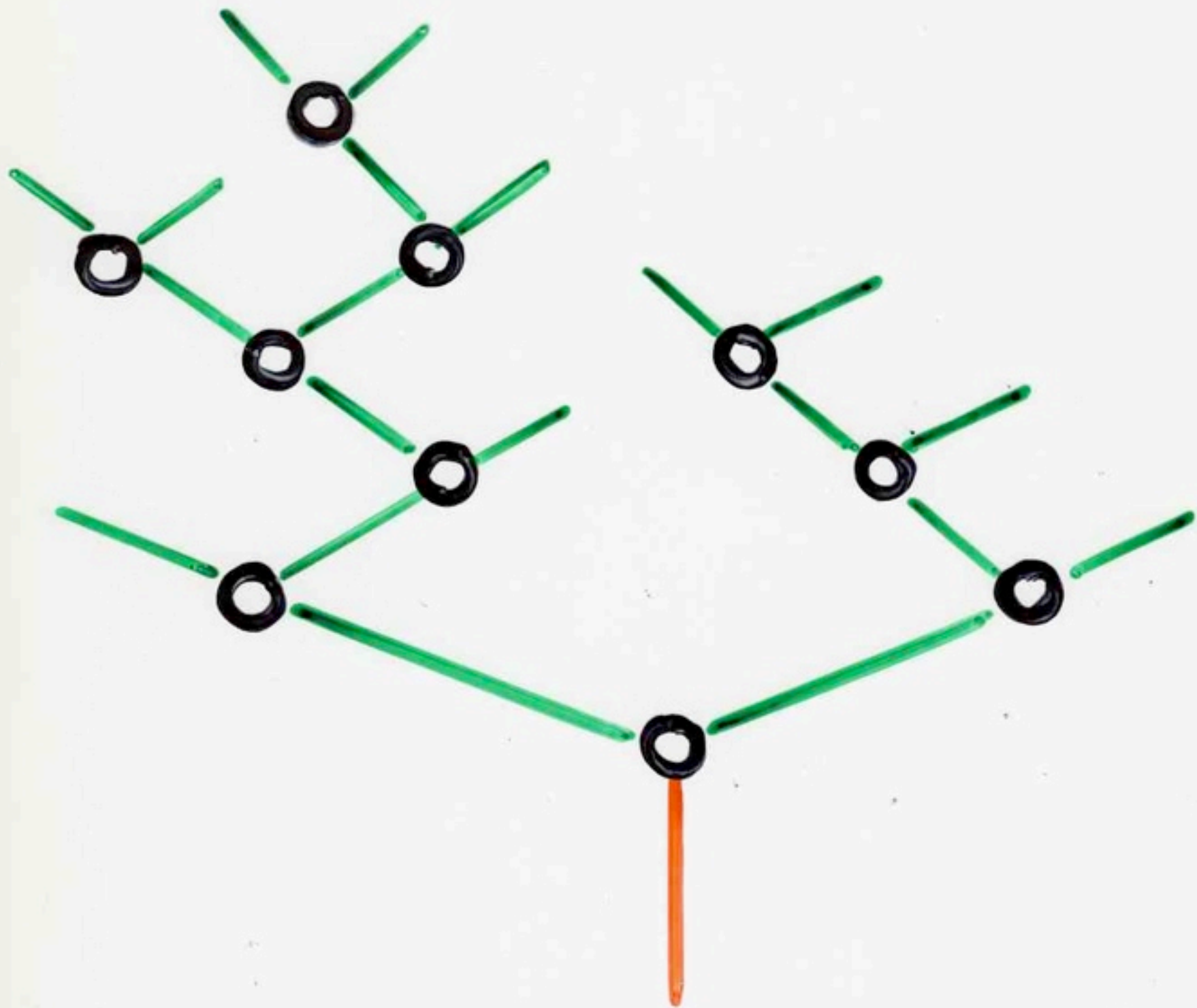
des triangulations
aux arbres binaires

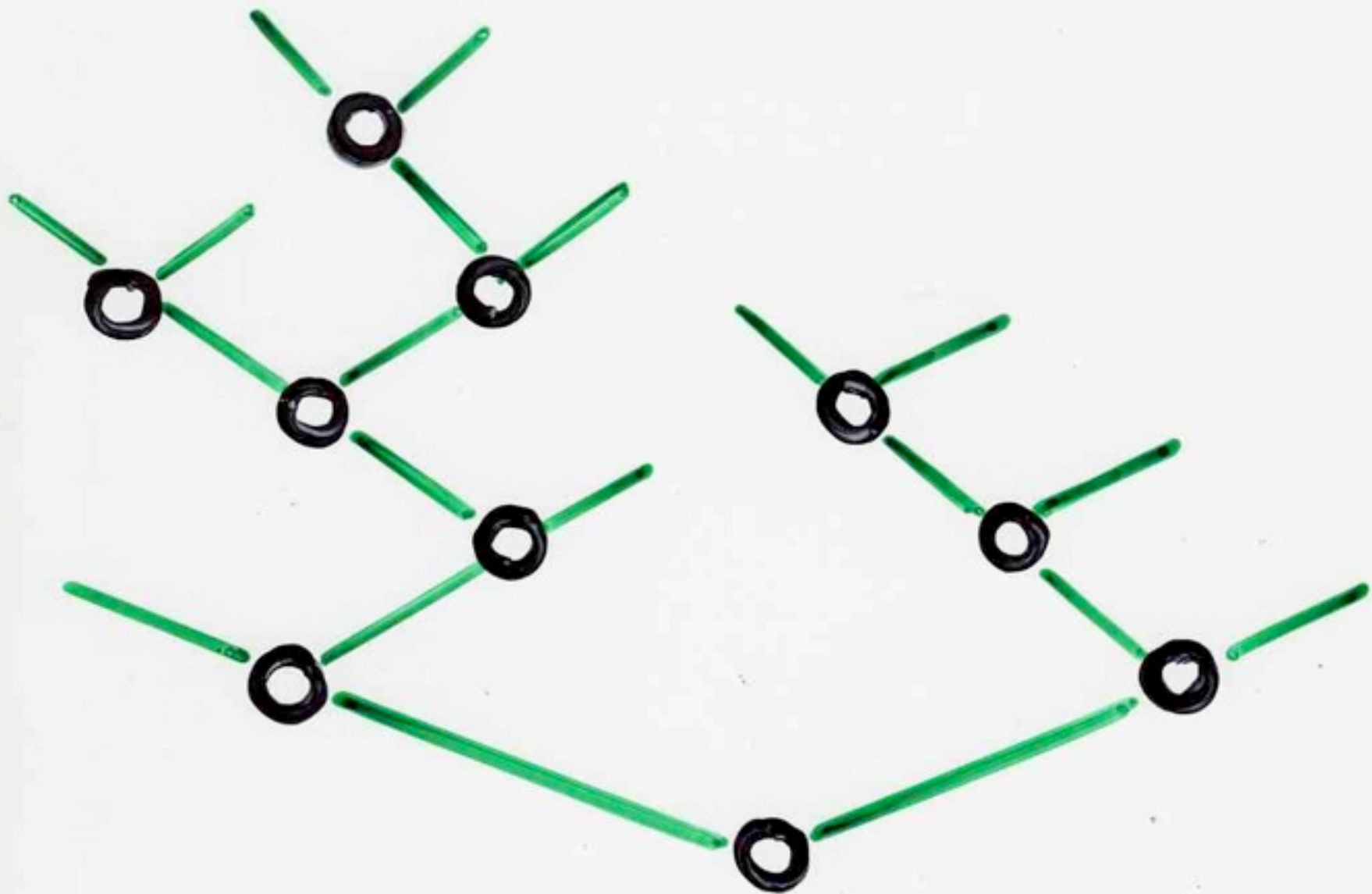








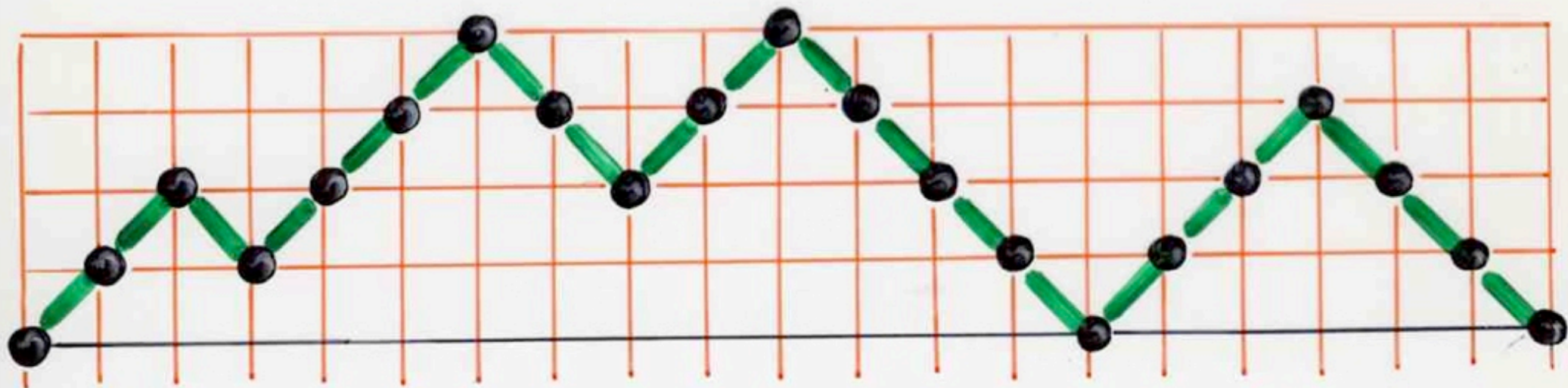




Comment démontrer la relation existant entre la distribution des nombres de Strahler et les polynômes de Tchebychev ?

$$S_k(t) = \sum_{k \geq 0} S_{n,k} t^n$$

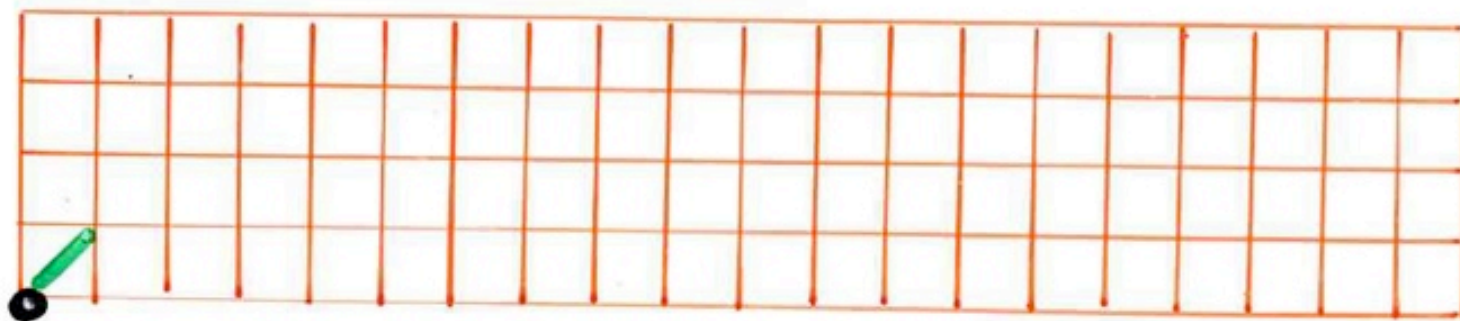
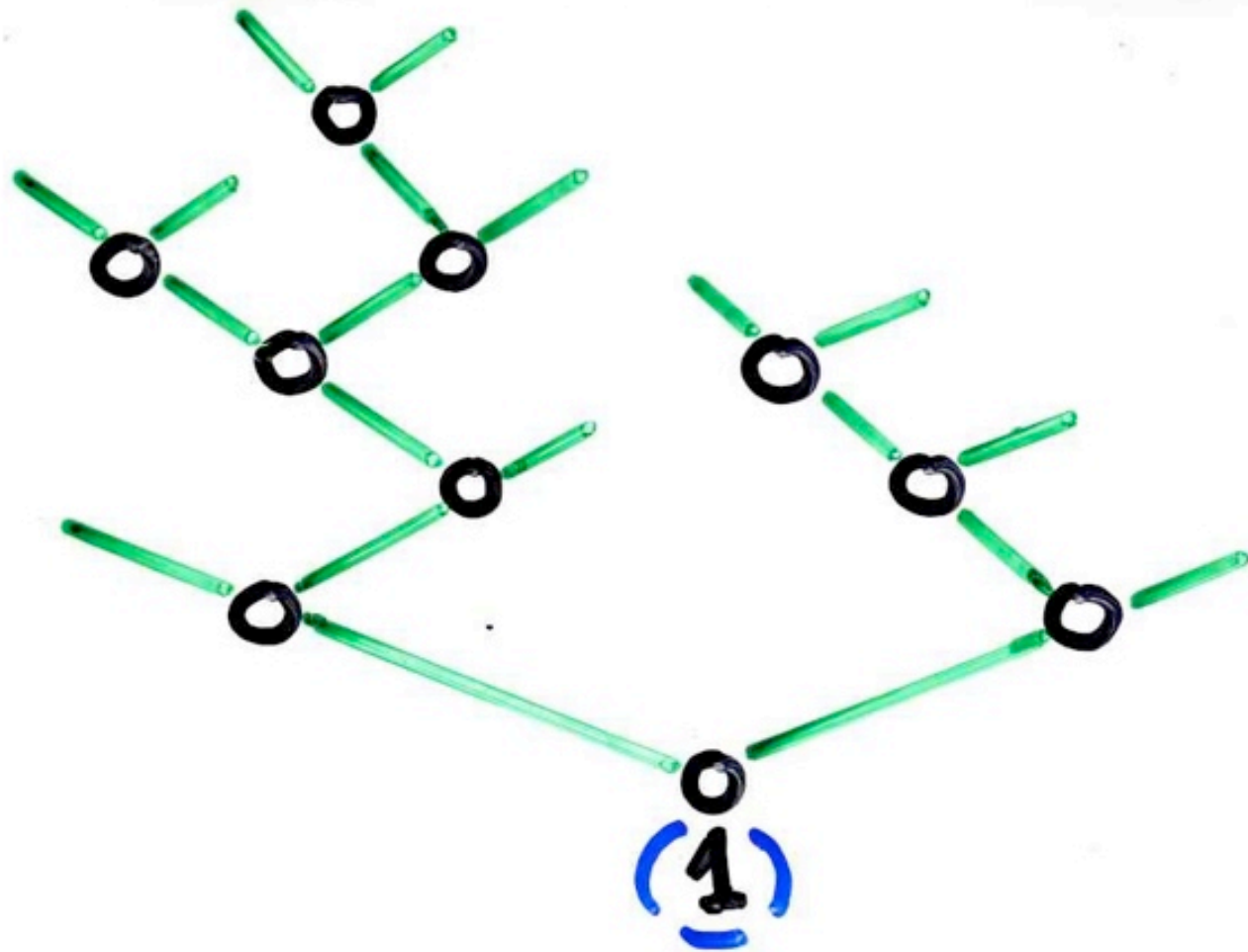
$$\sin(n+1)\theta = (\sin\theta) U_n(\cos\theta)$$

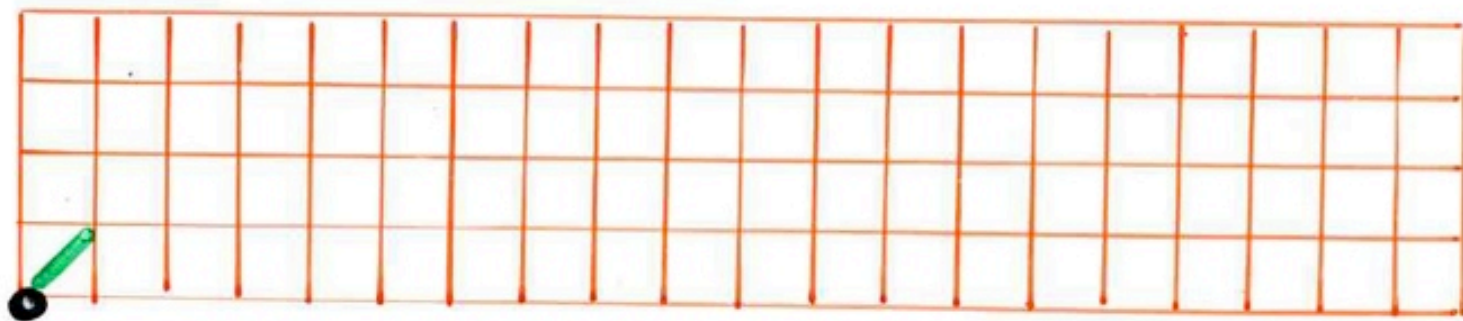
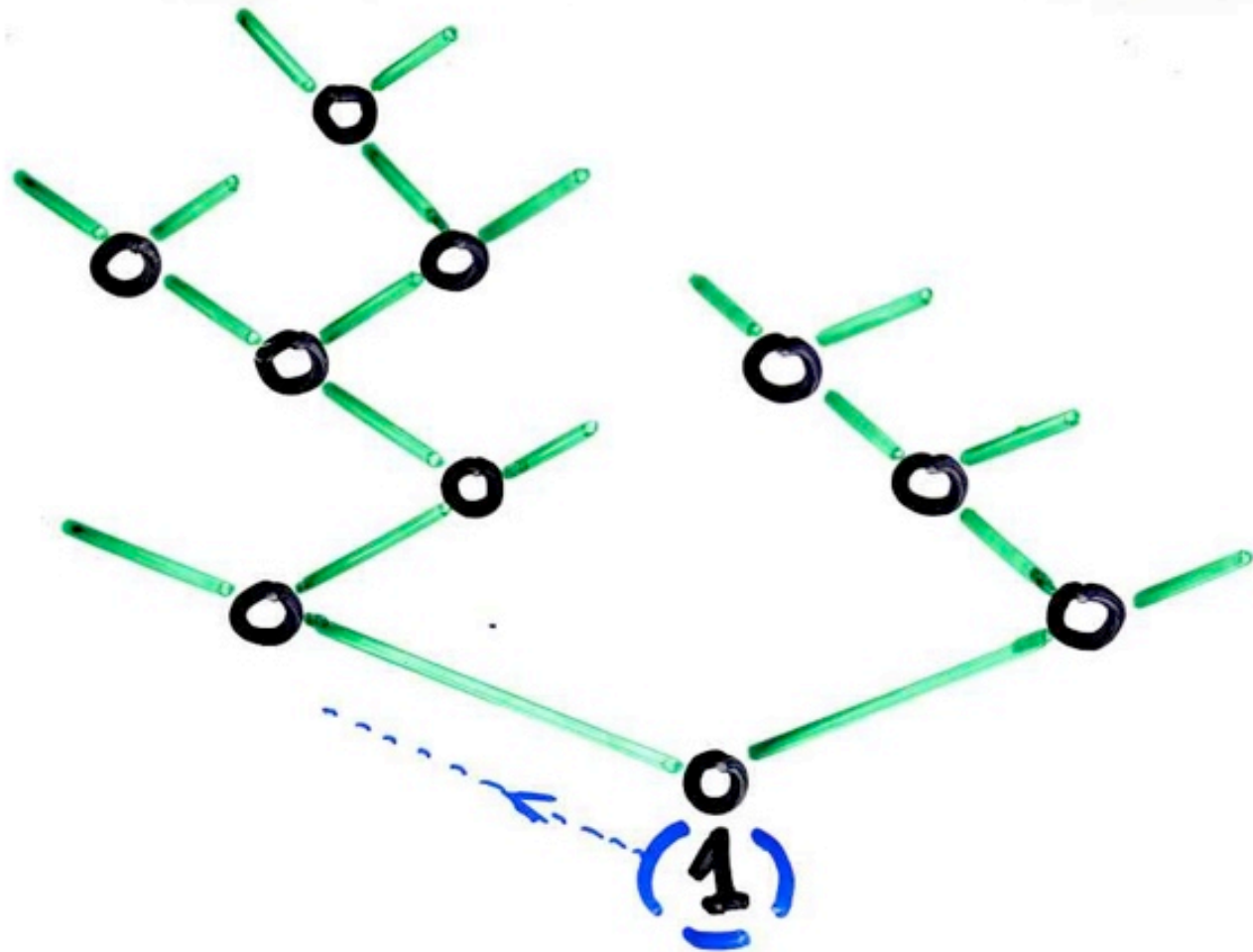


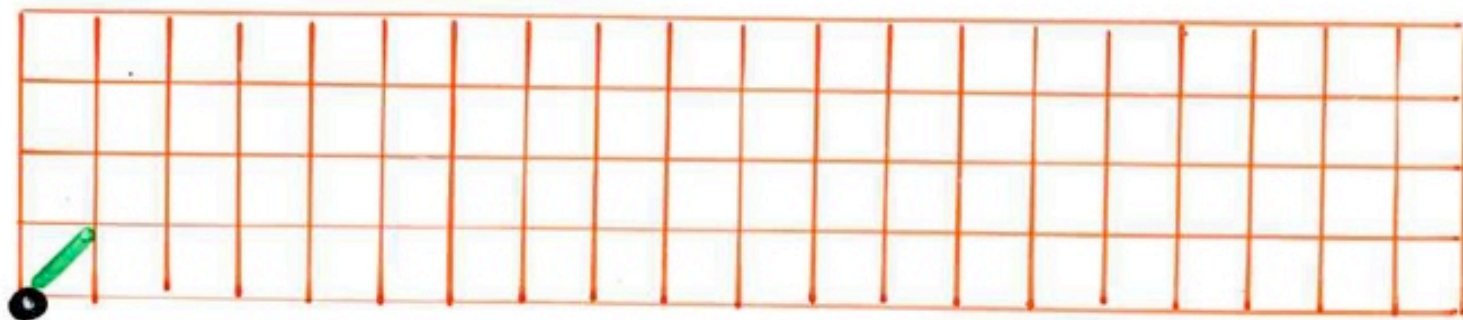
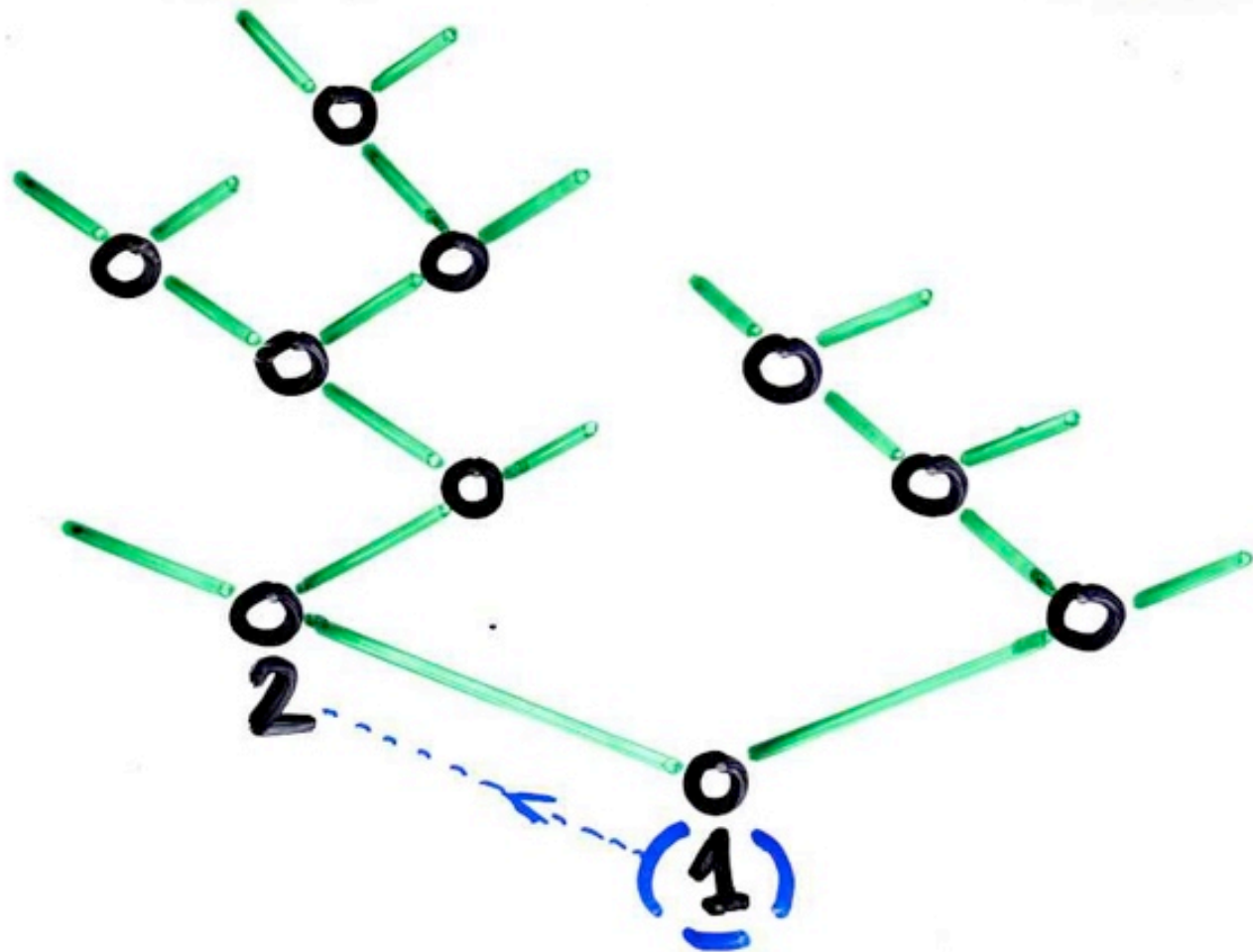
chemin de Dyck

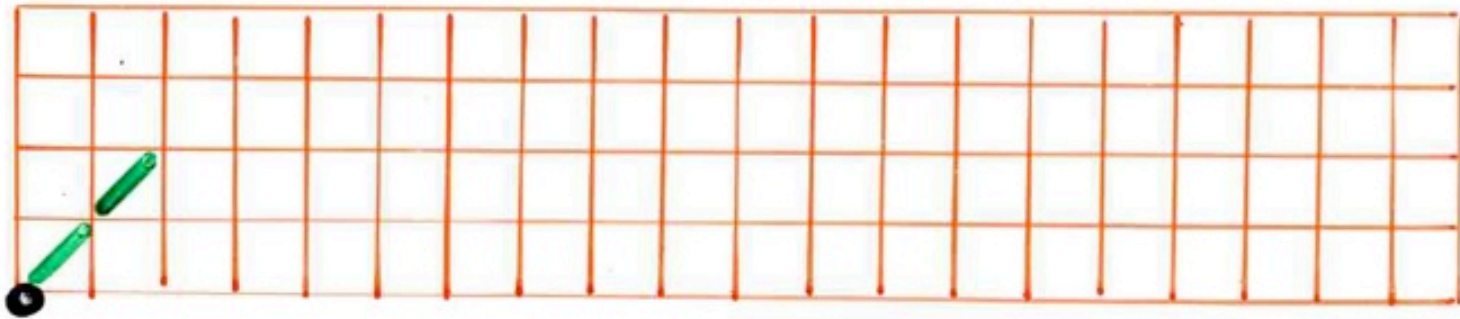
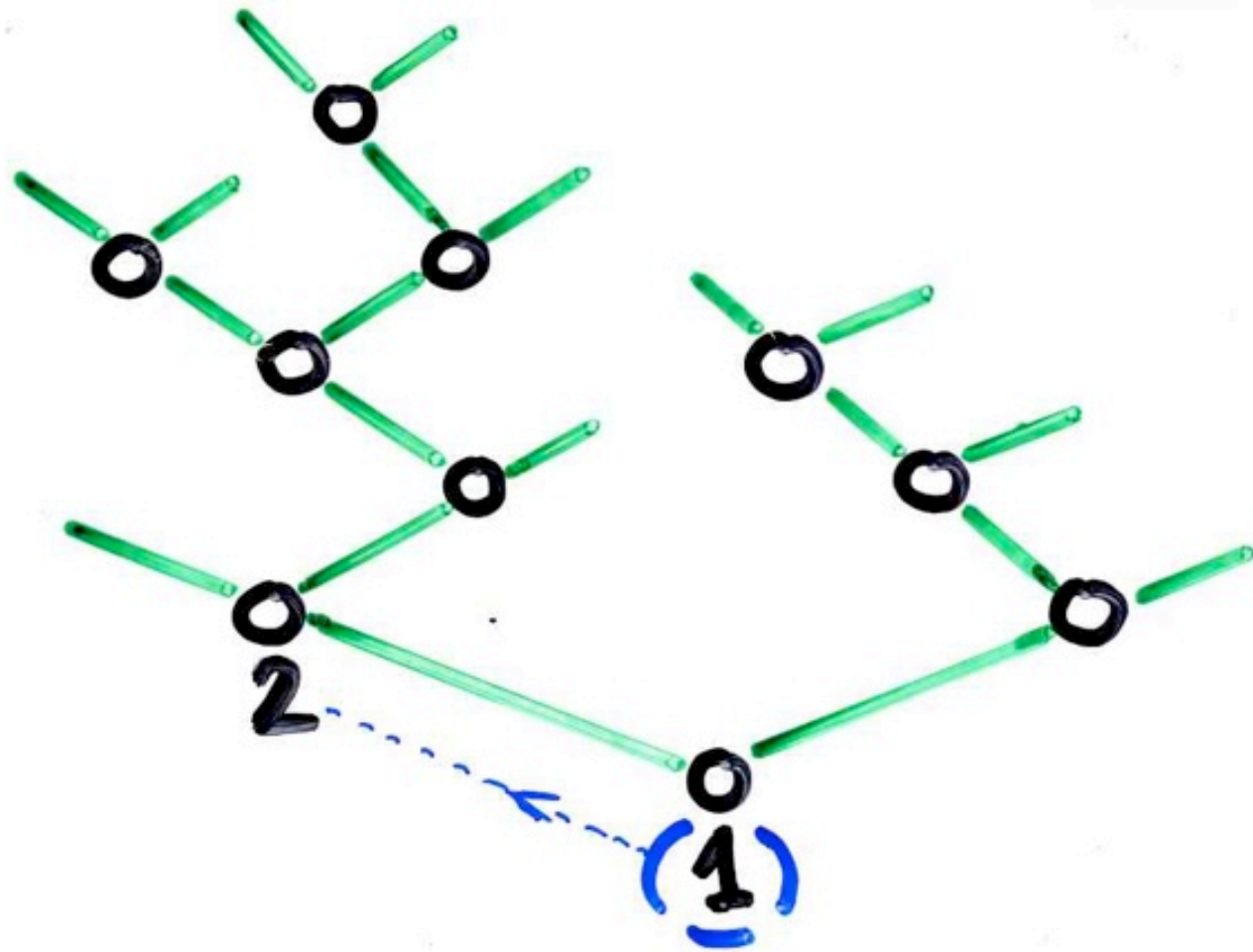
des arbres binaires ...

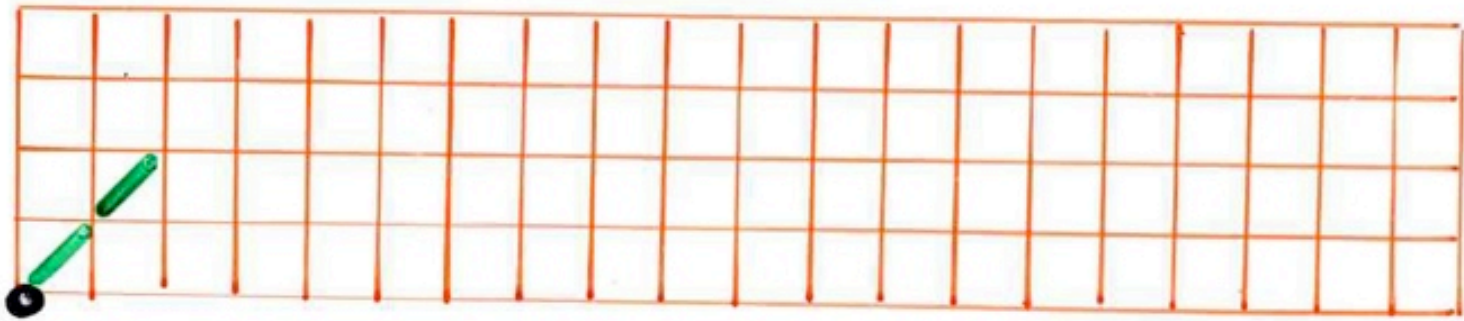
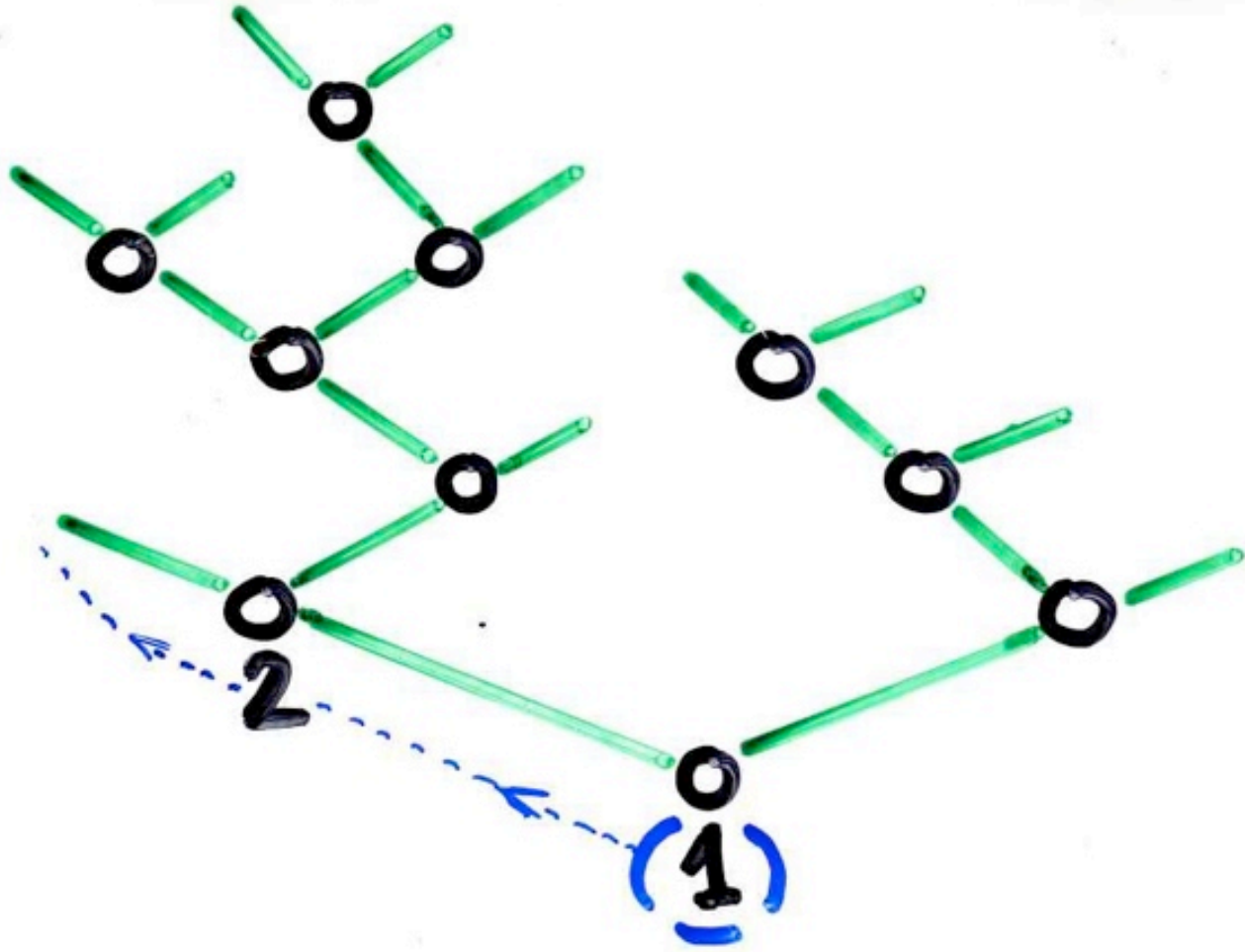
aux chemins de Dyck

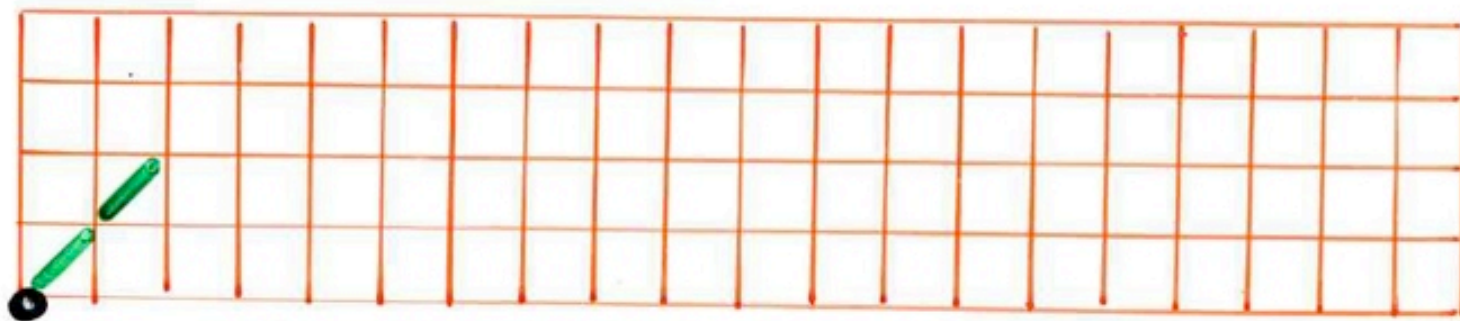
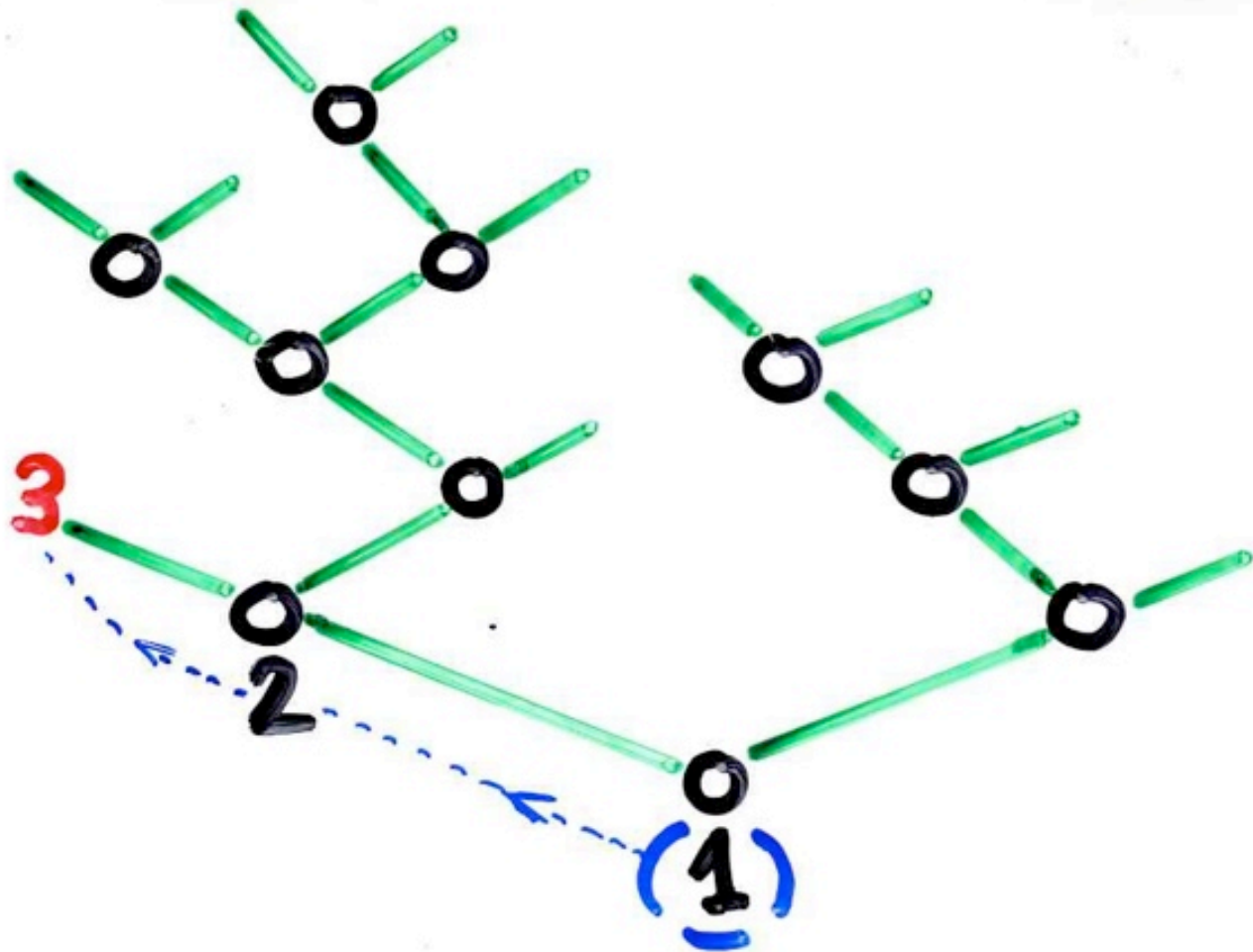


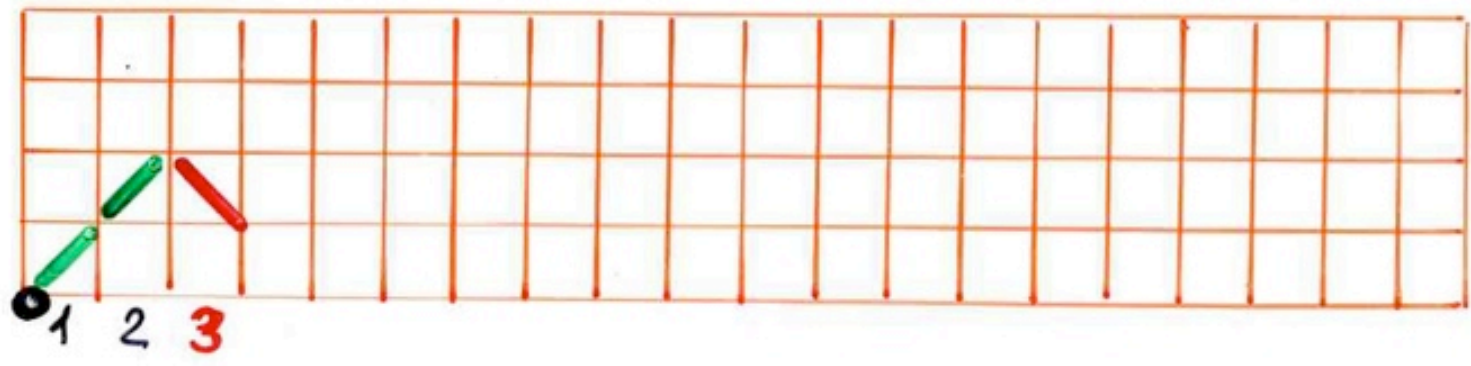
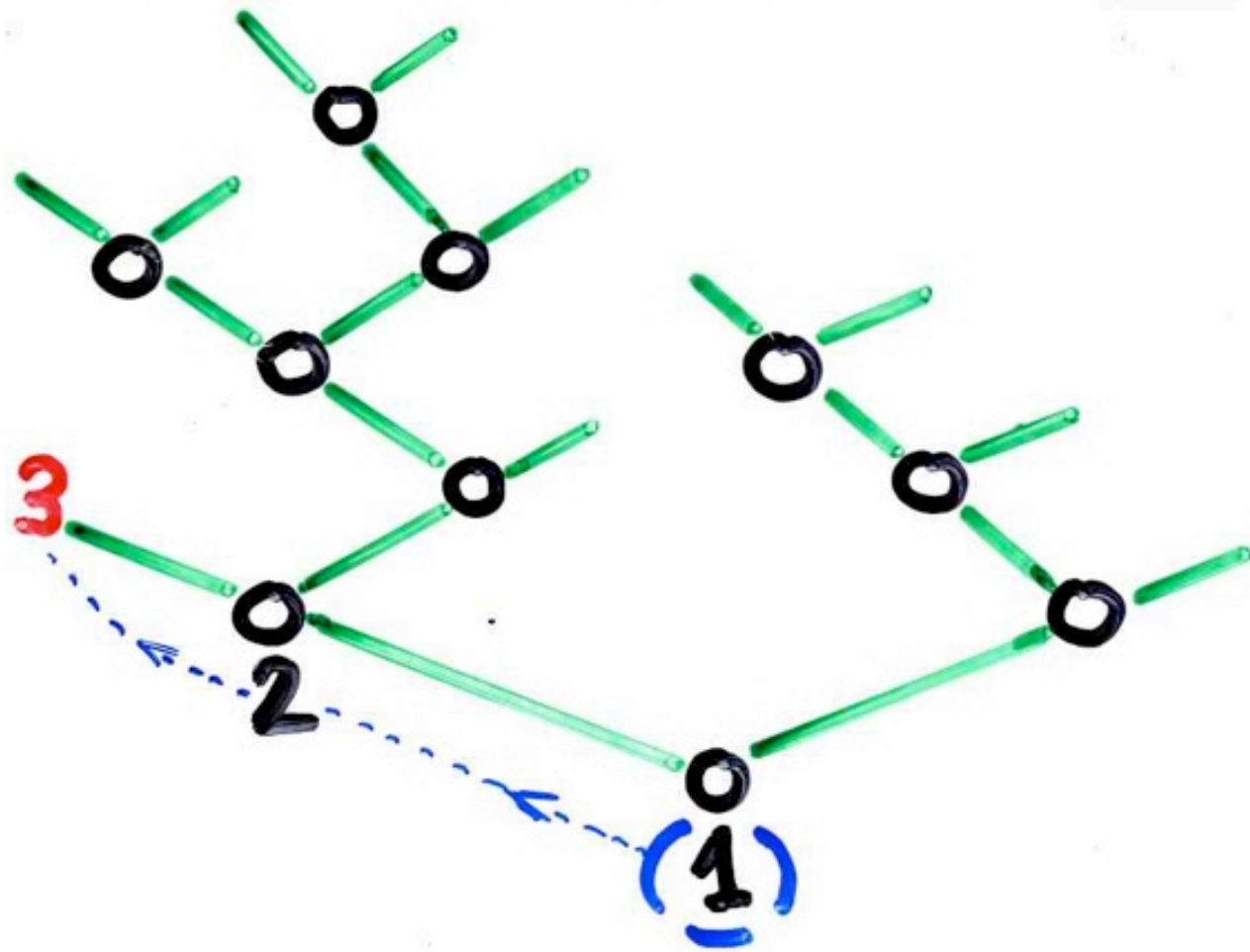


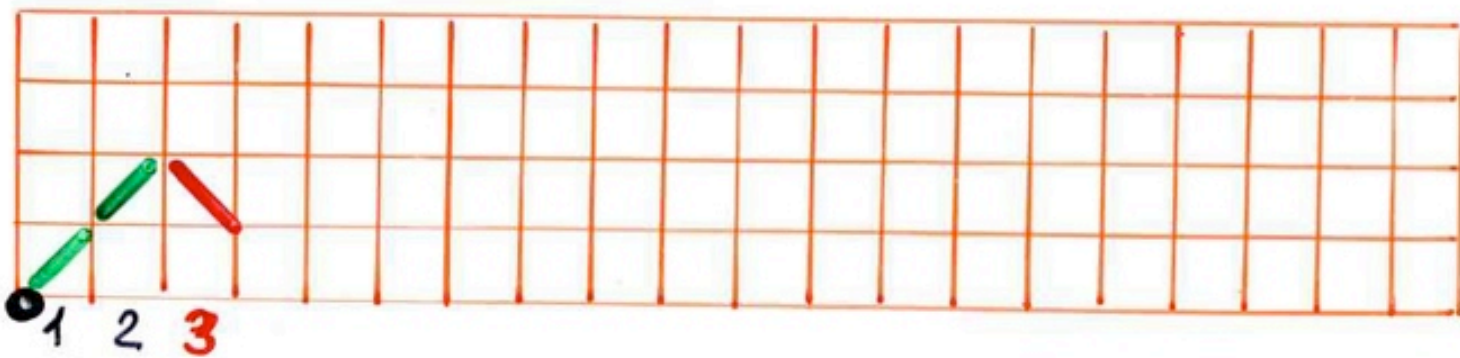
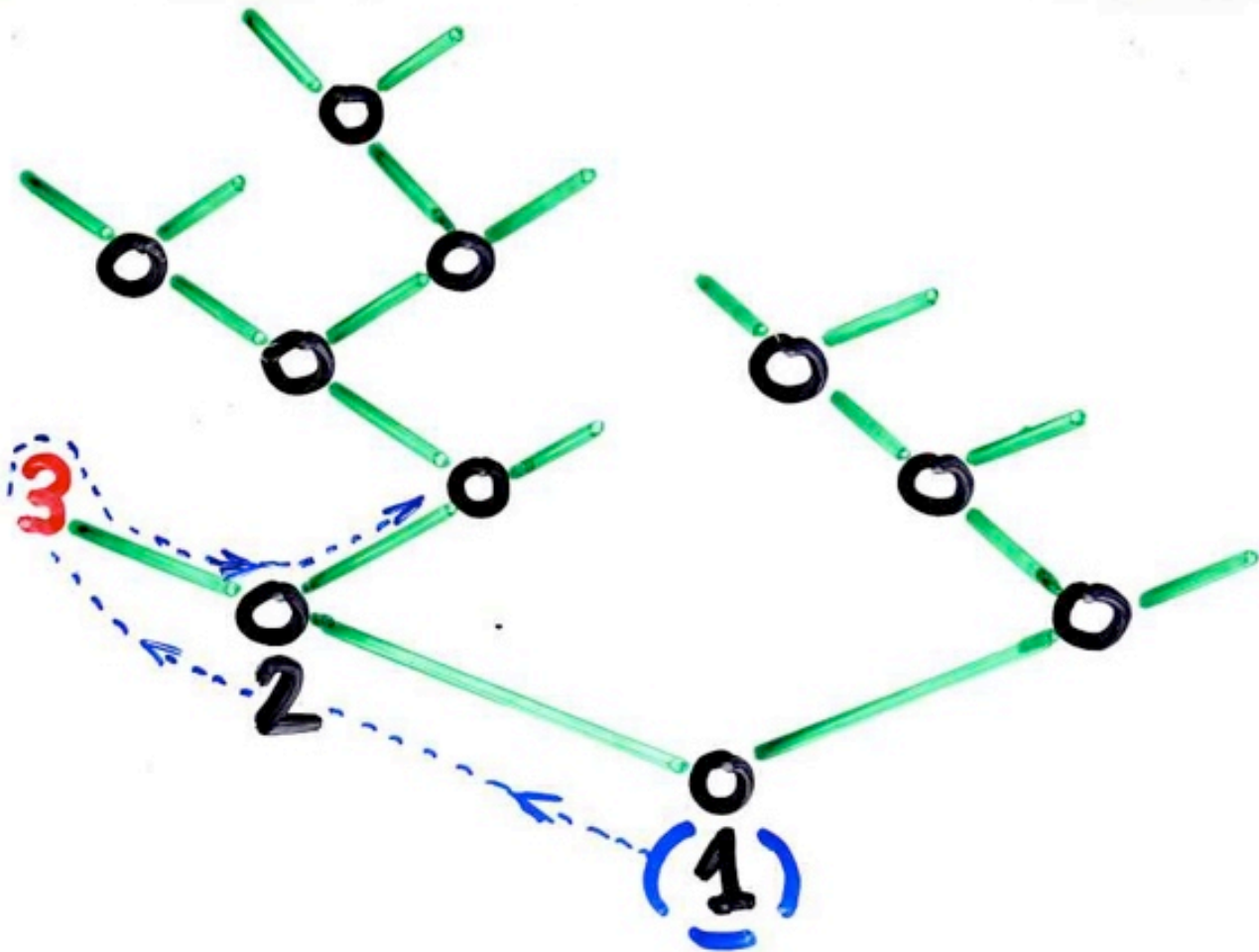


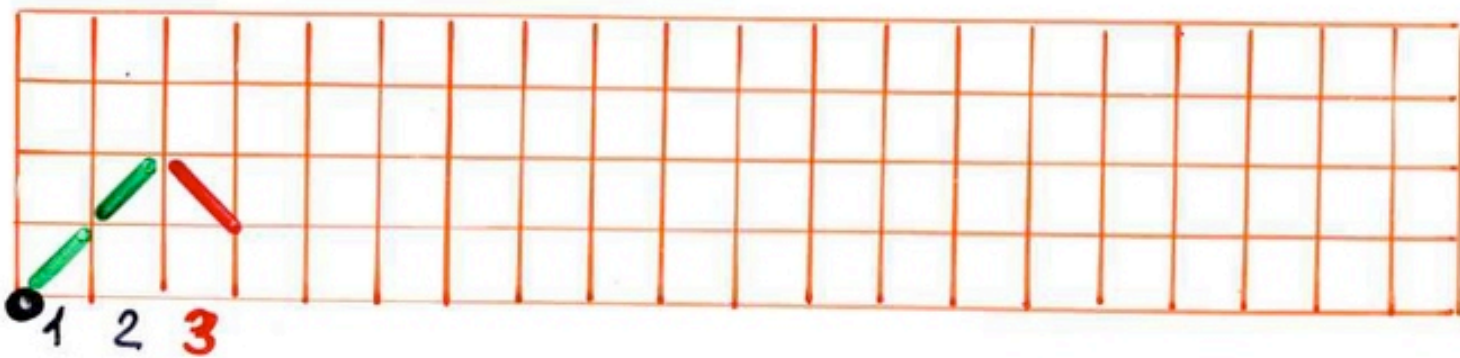
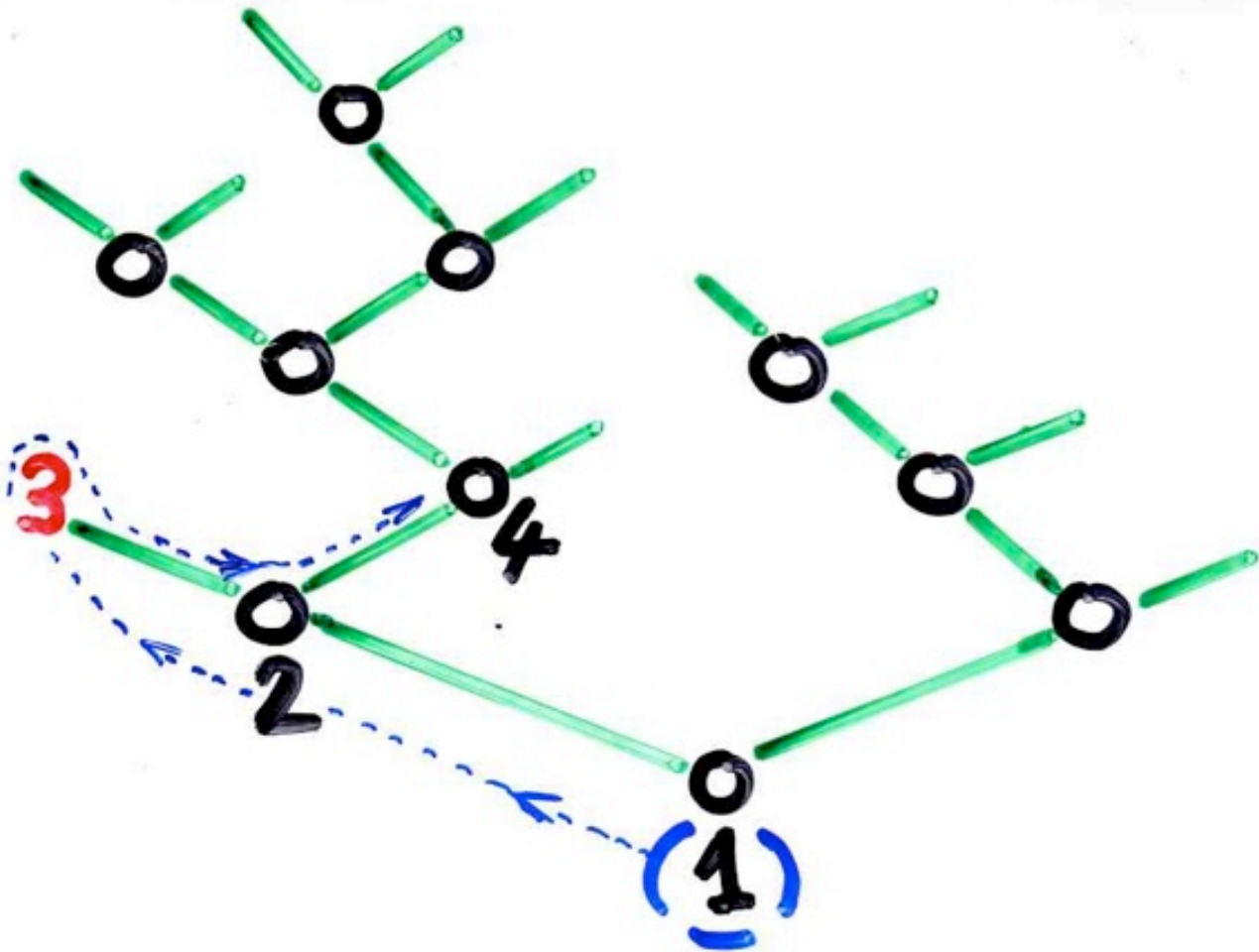


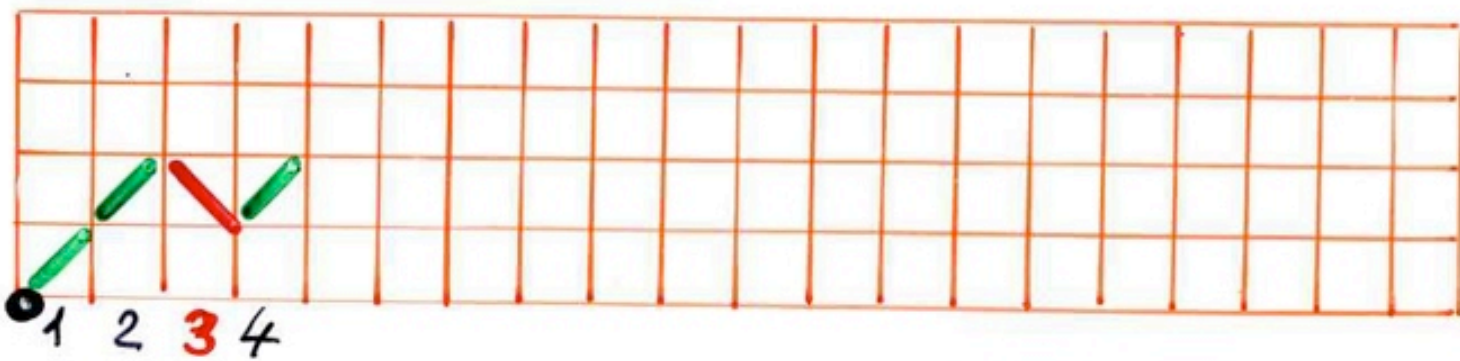
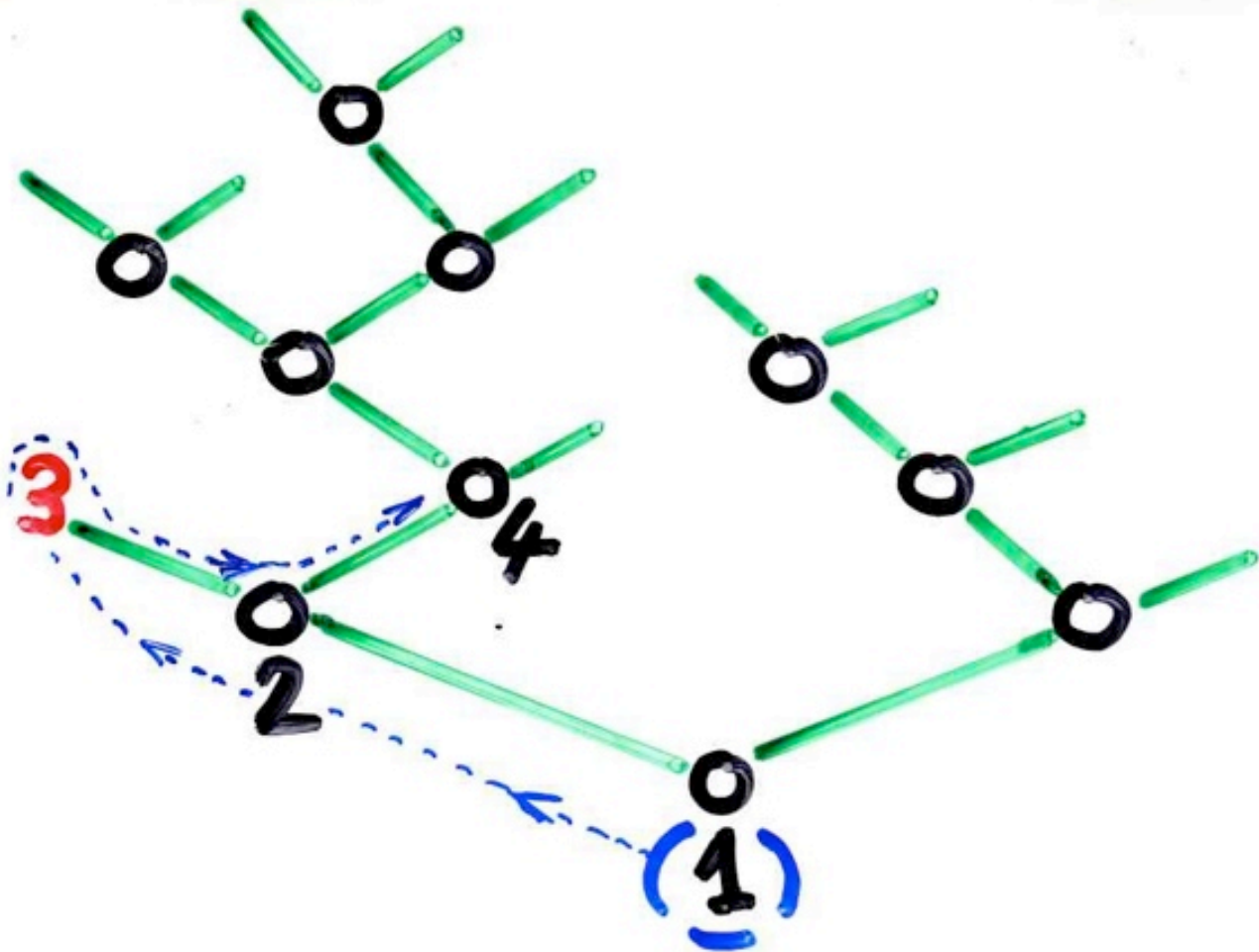


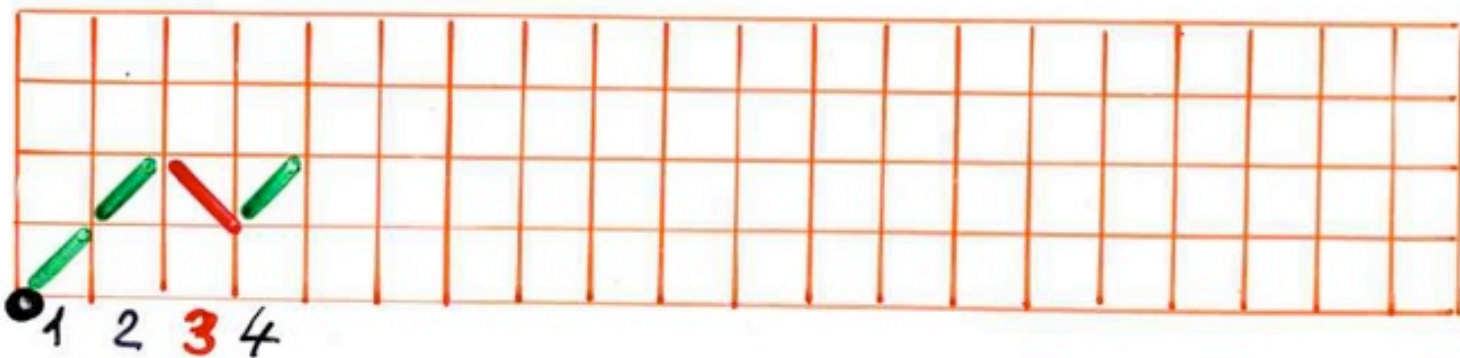
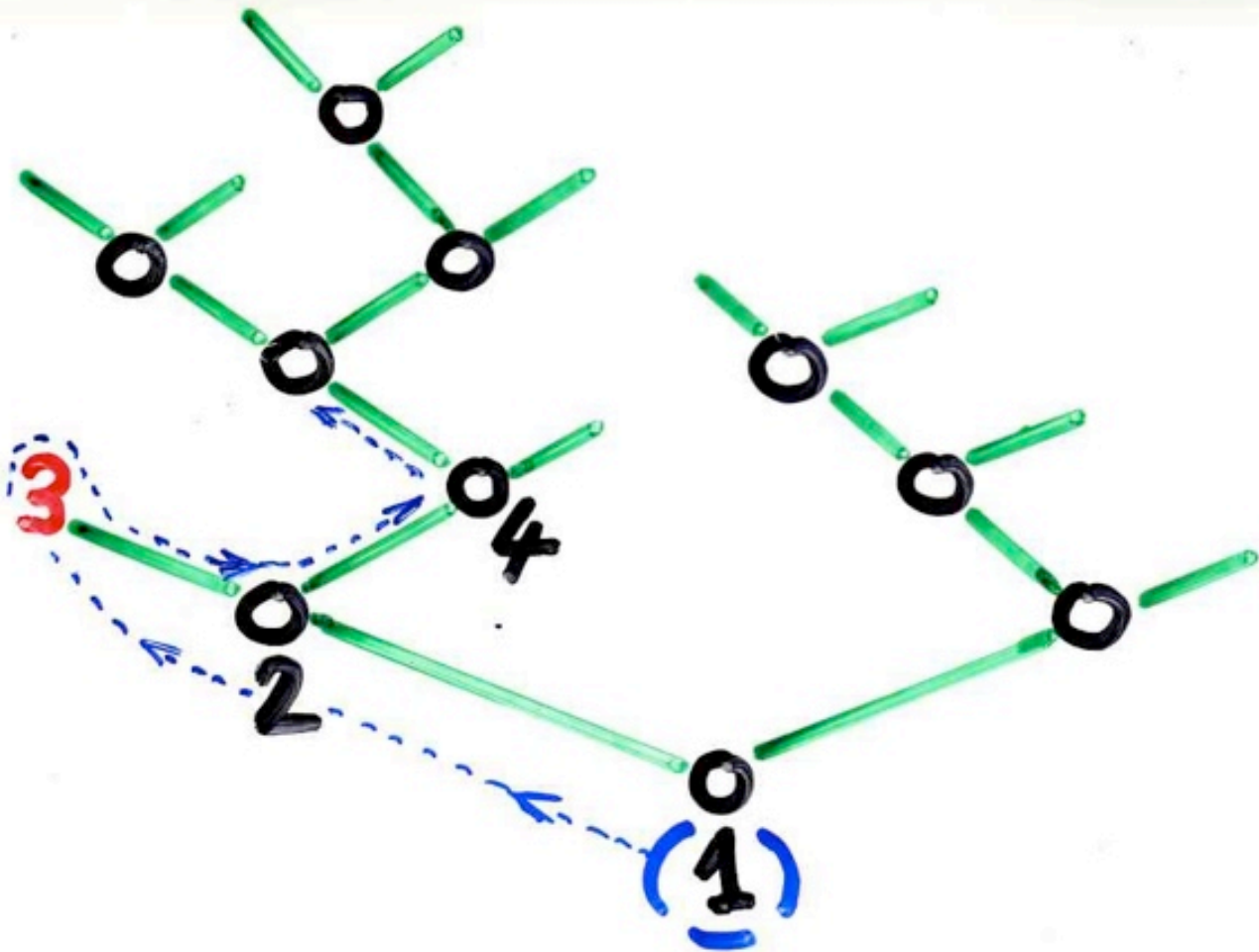


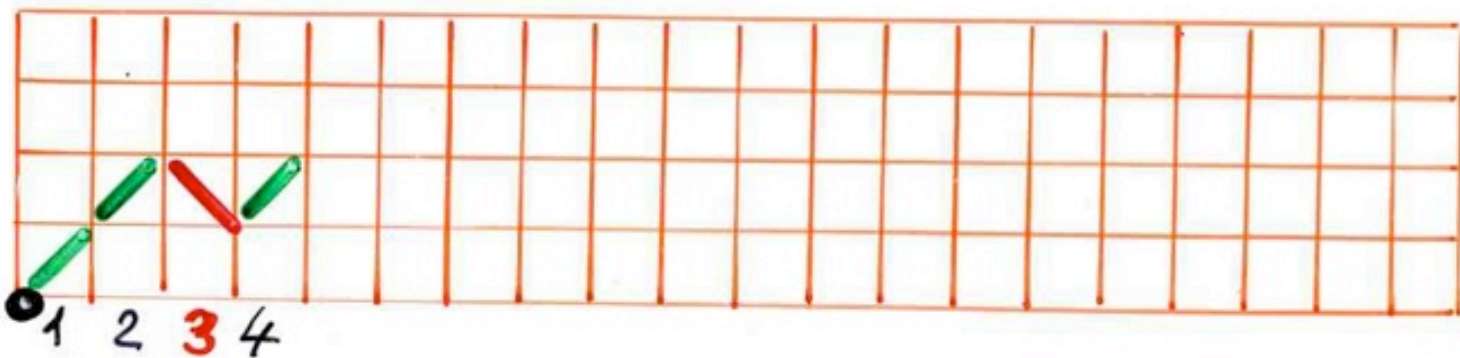
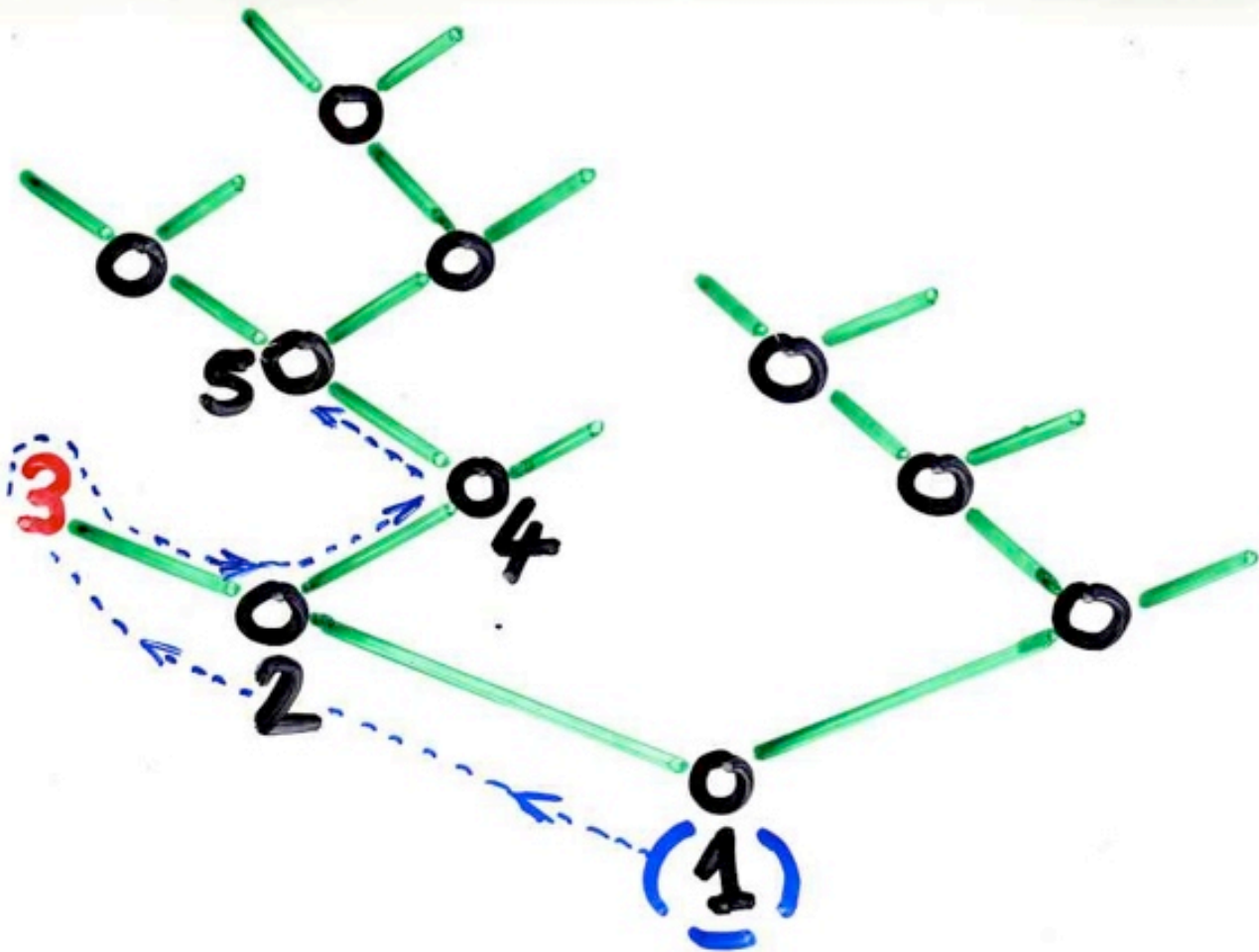


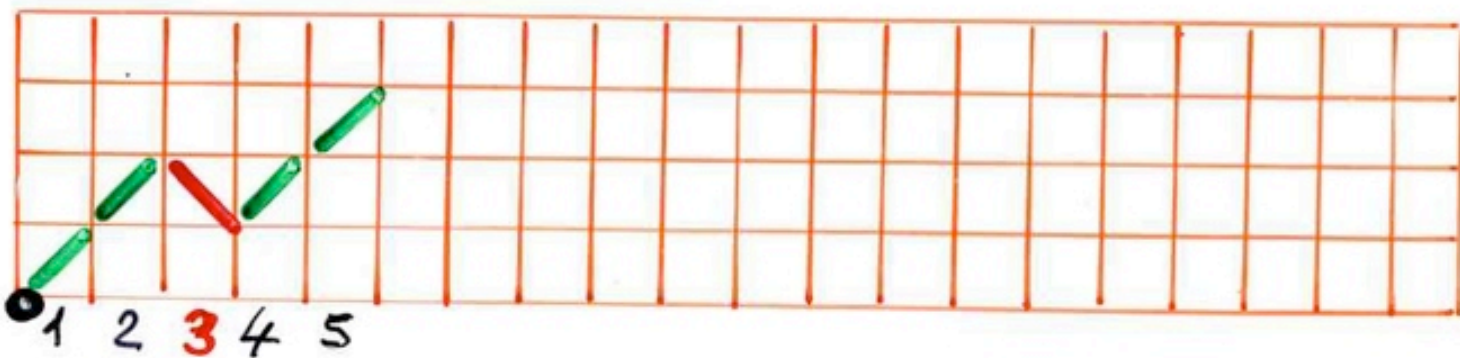
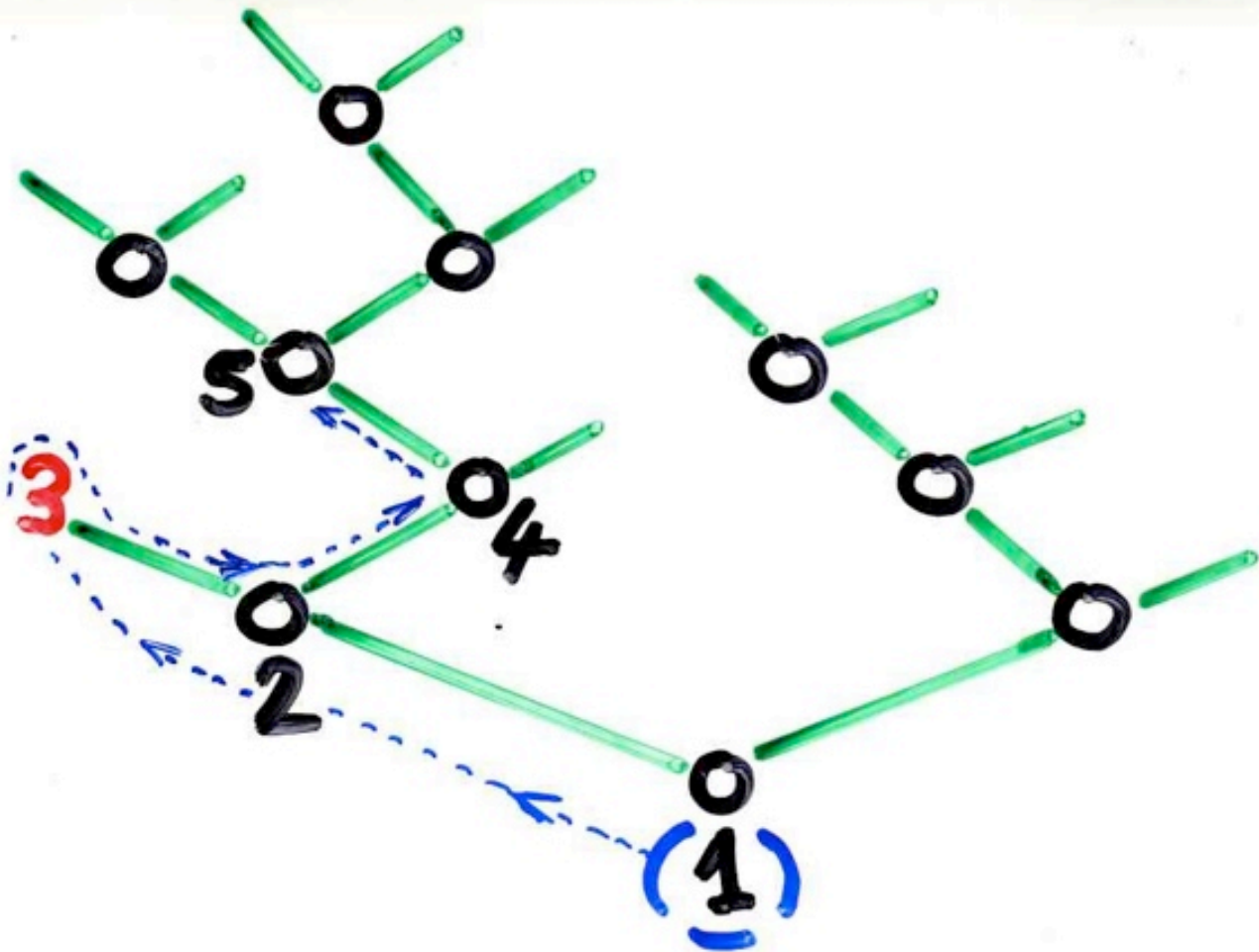


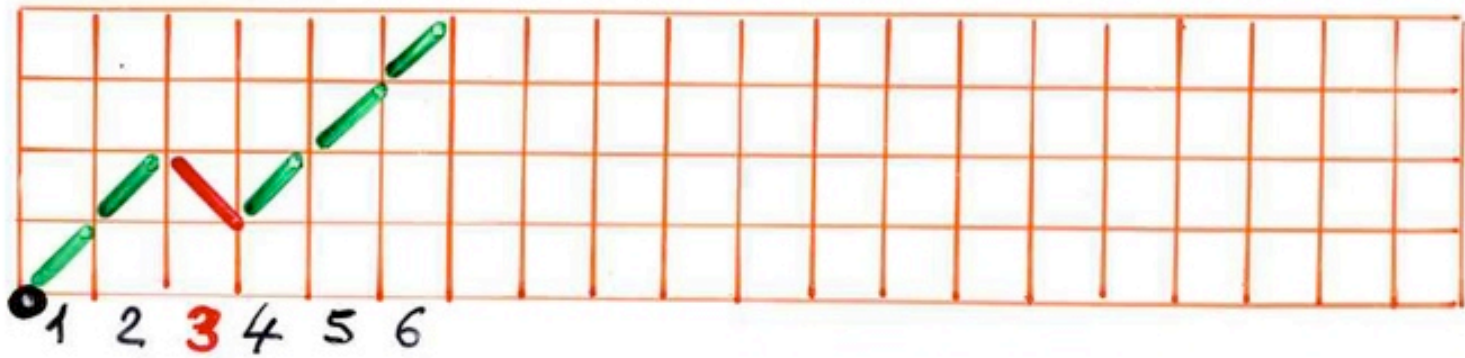
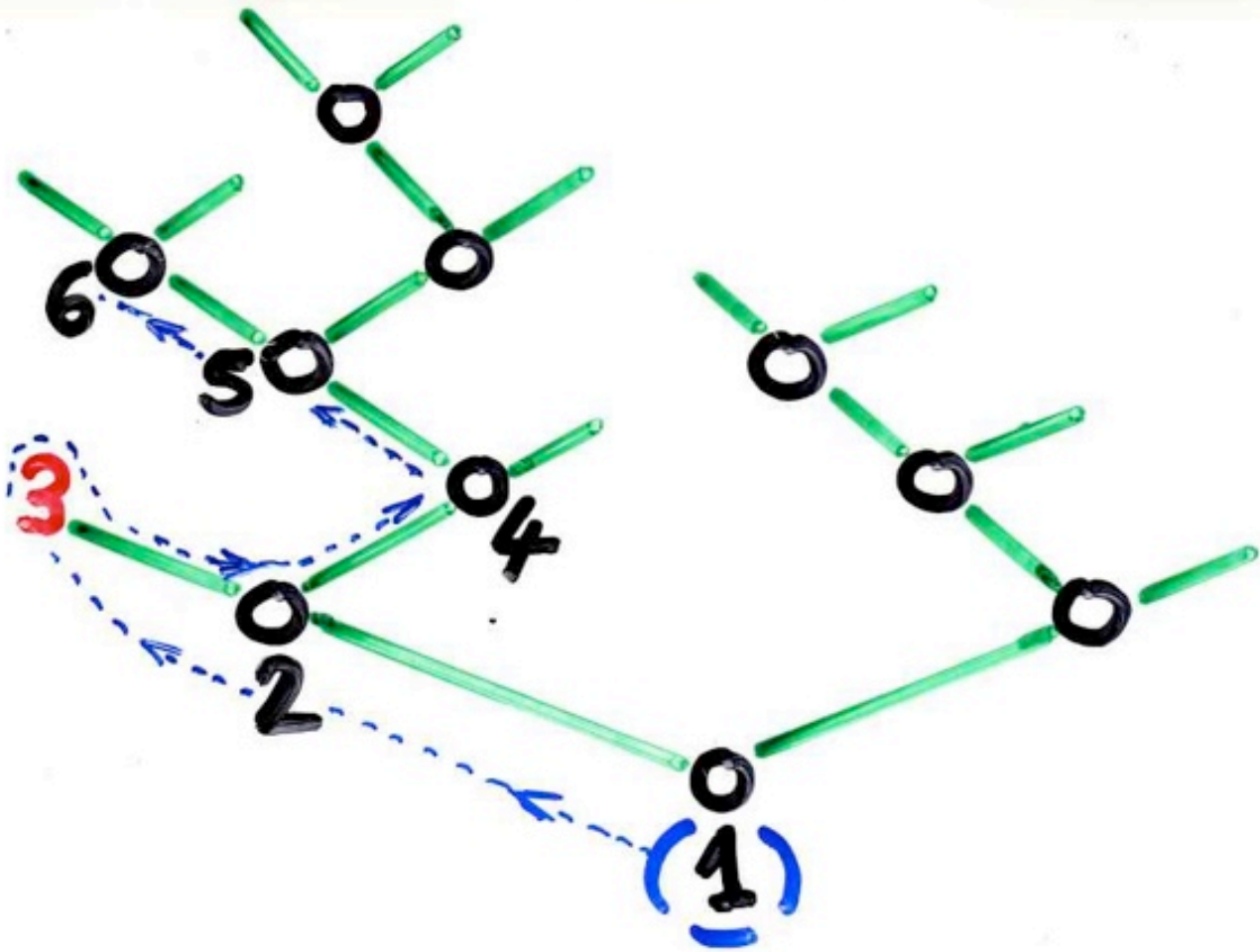


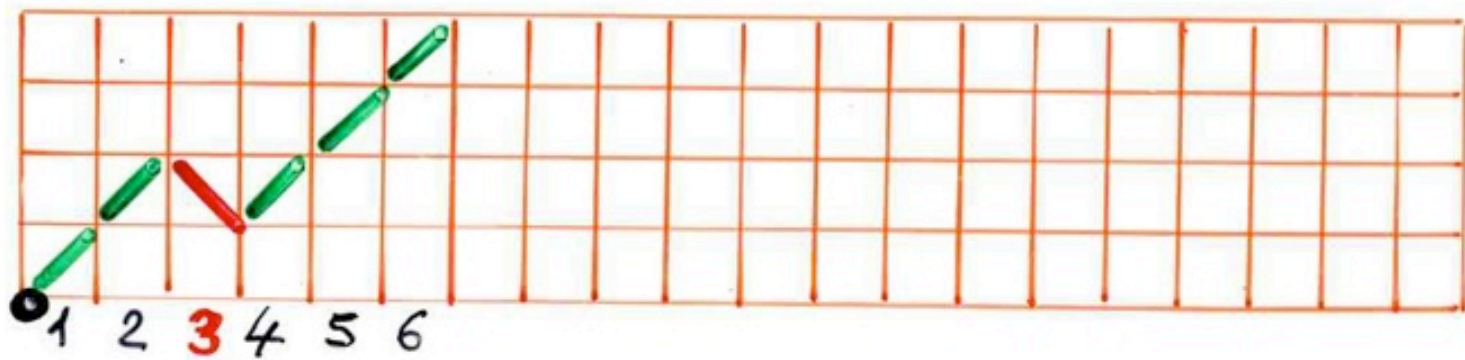
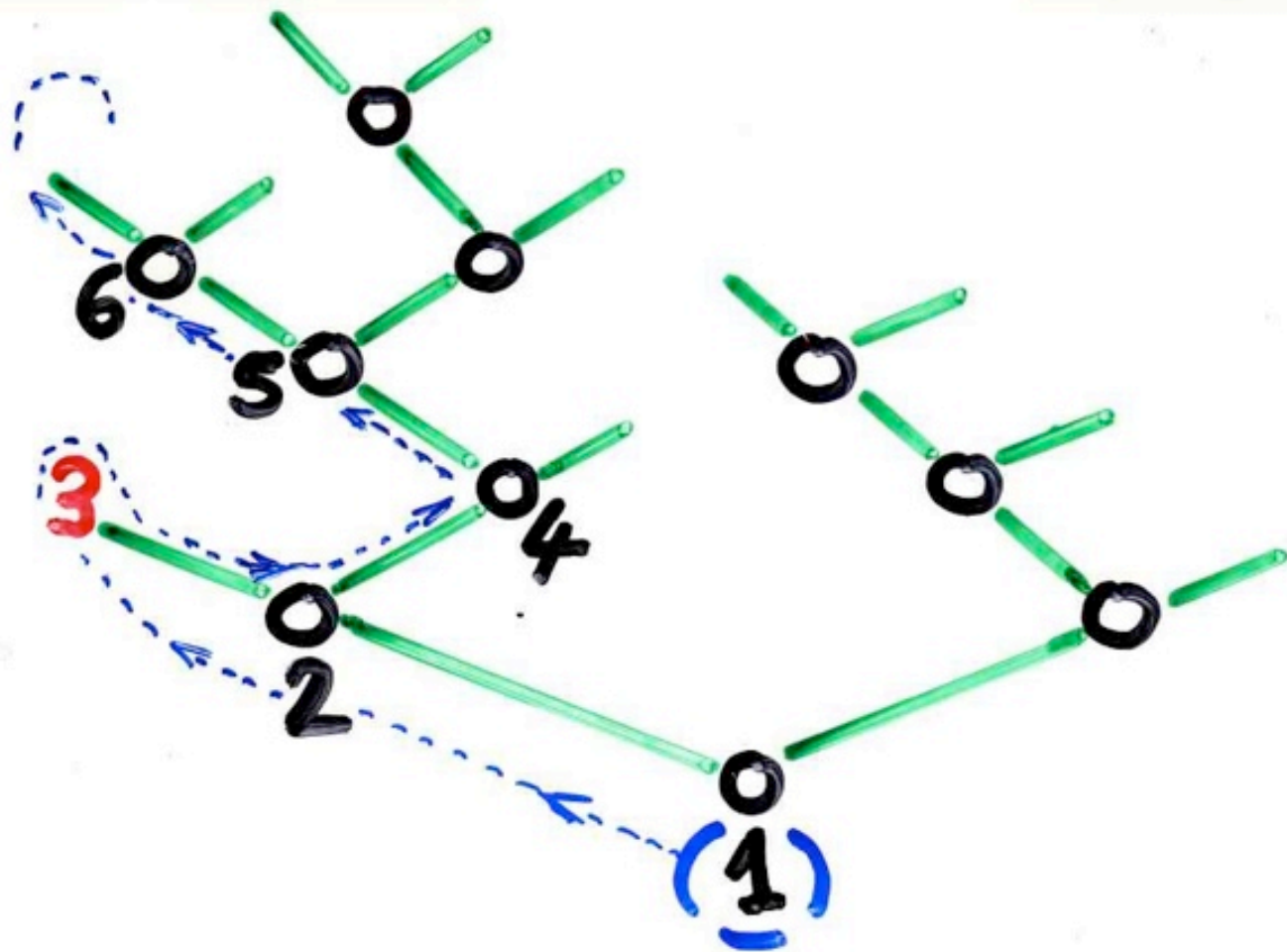


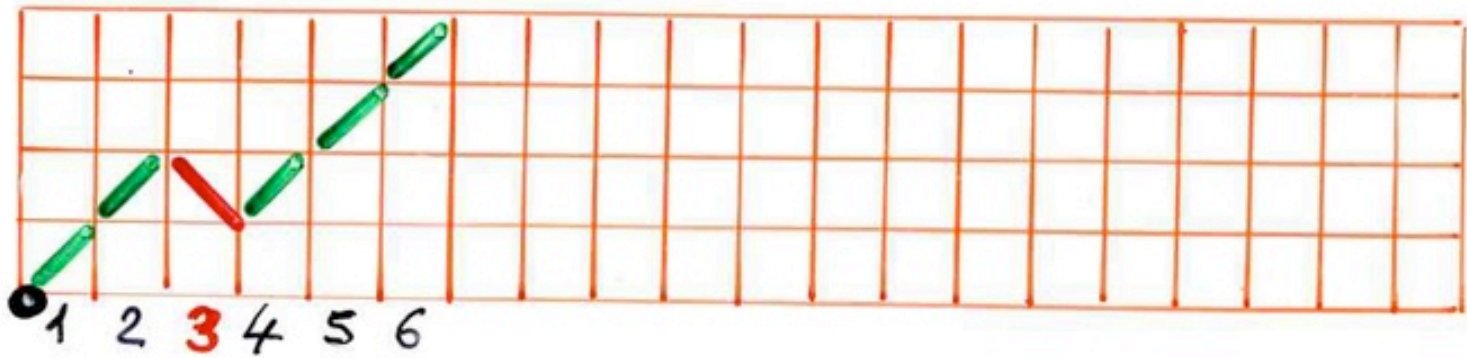
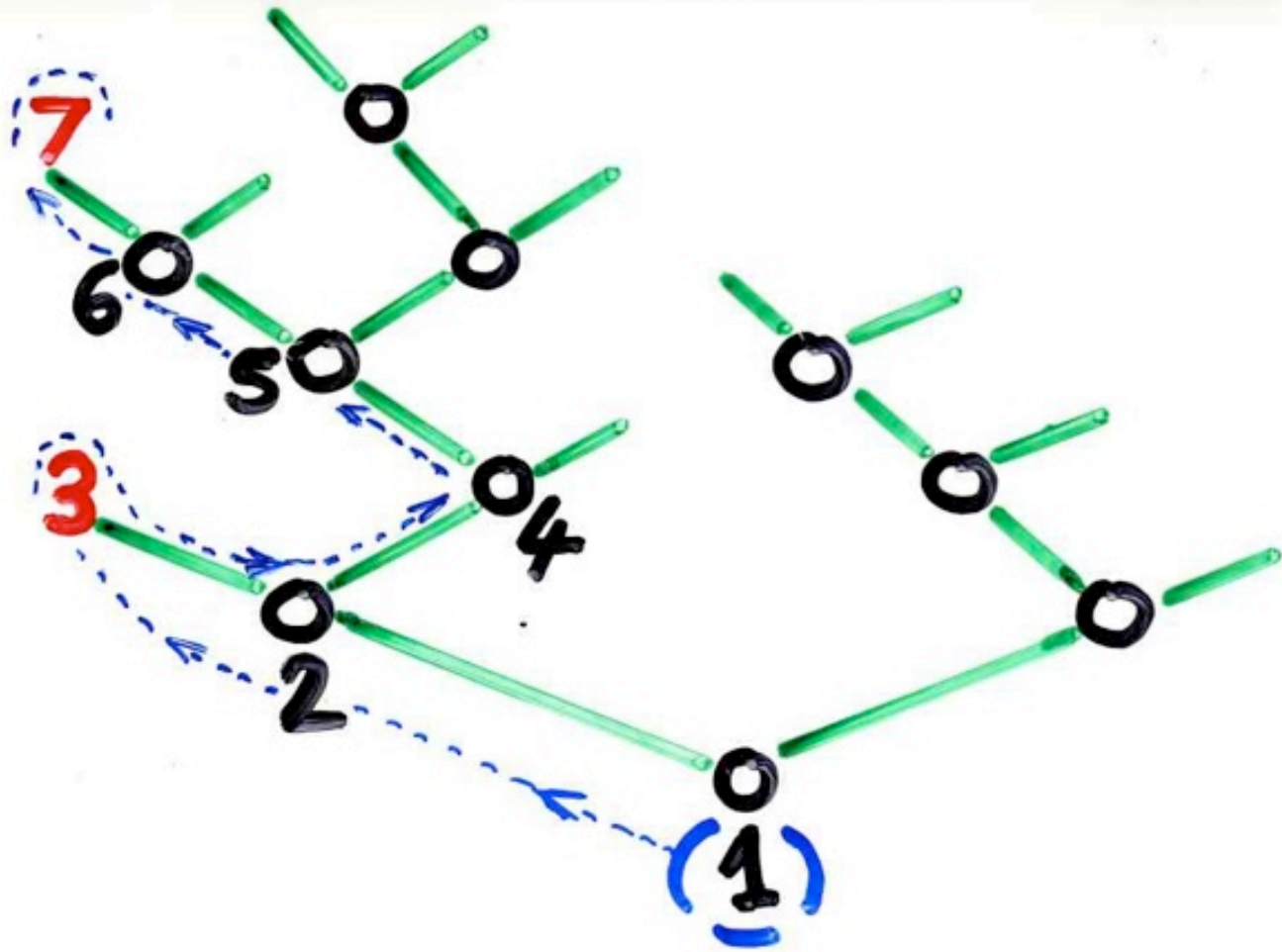


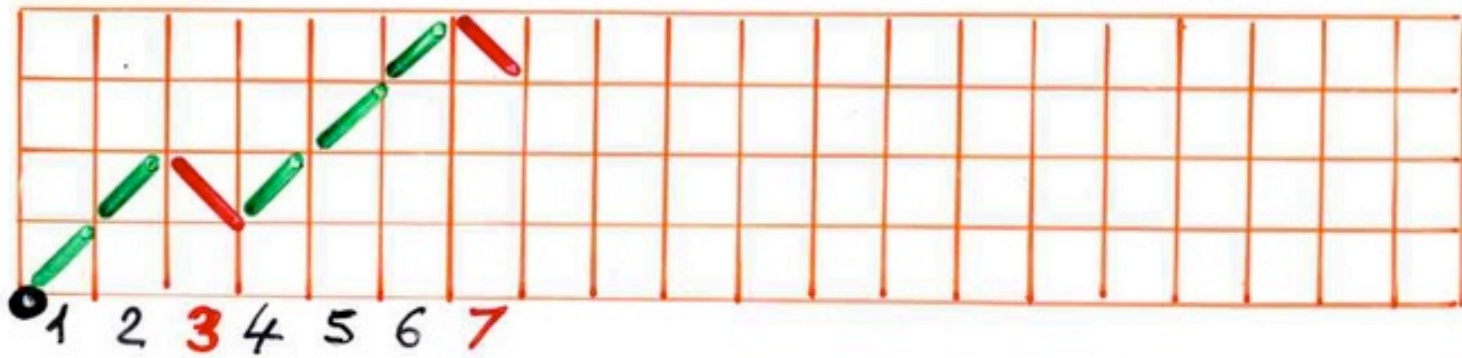
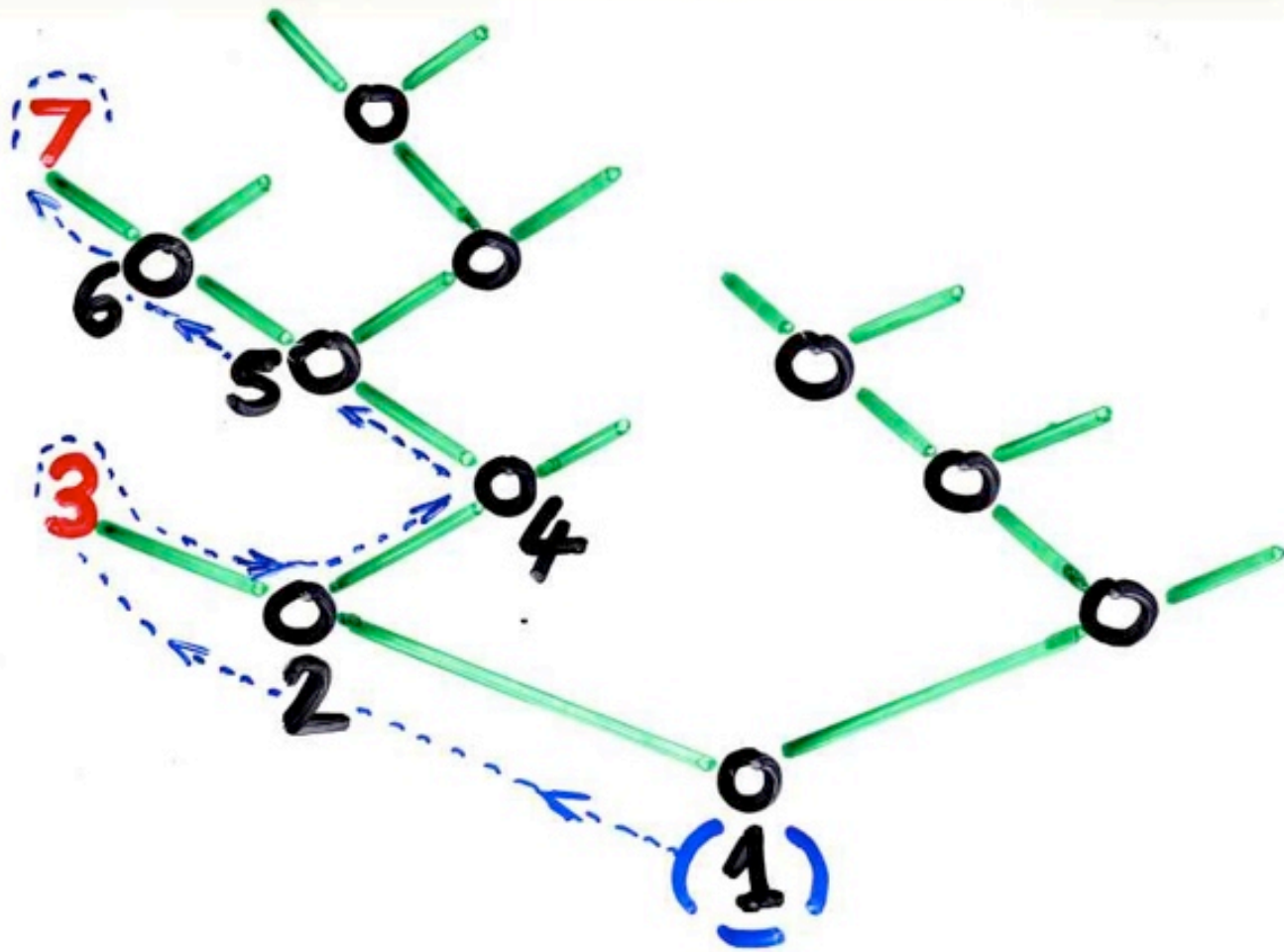


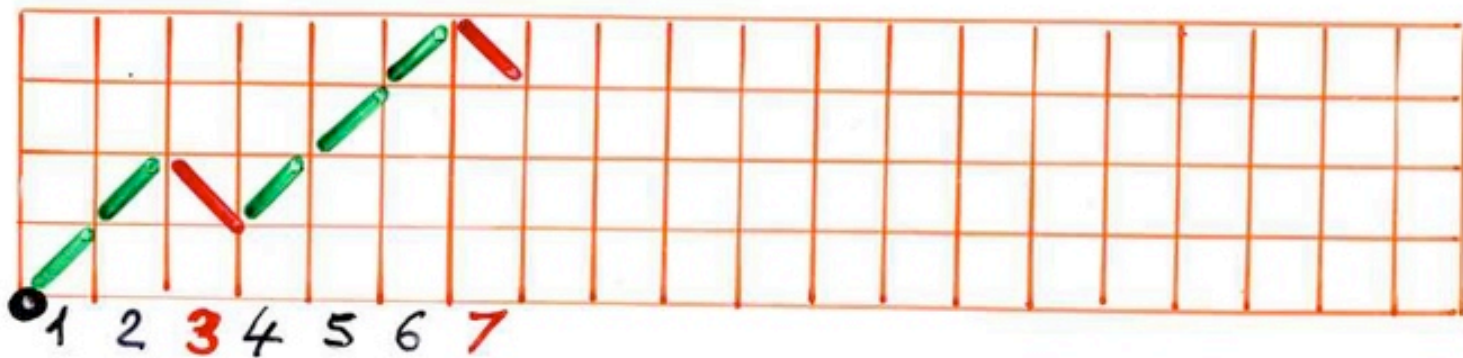
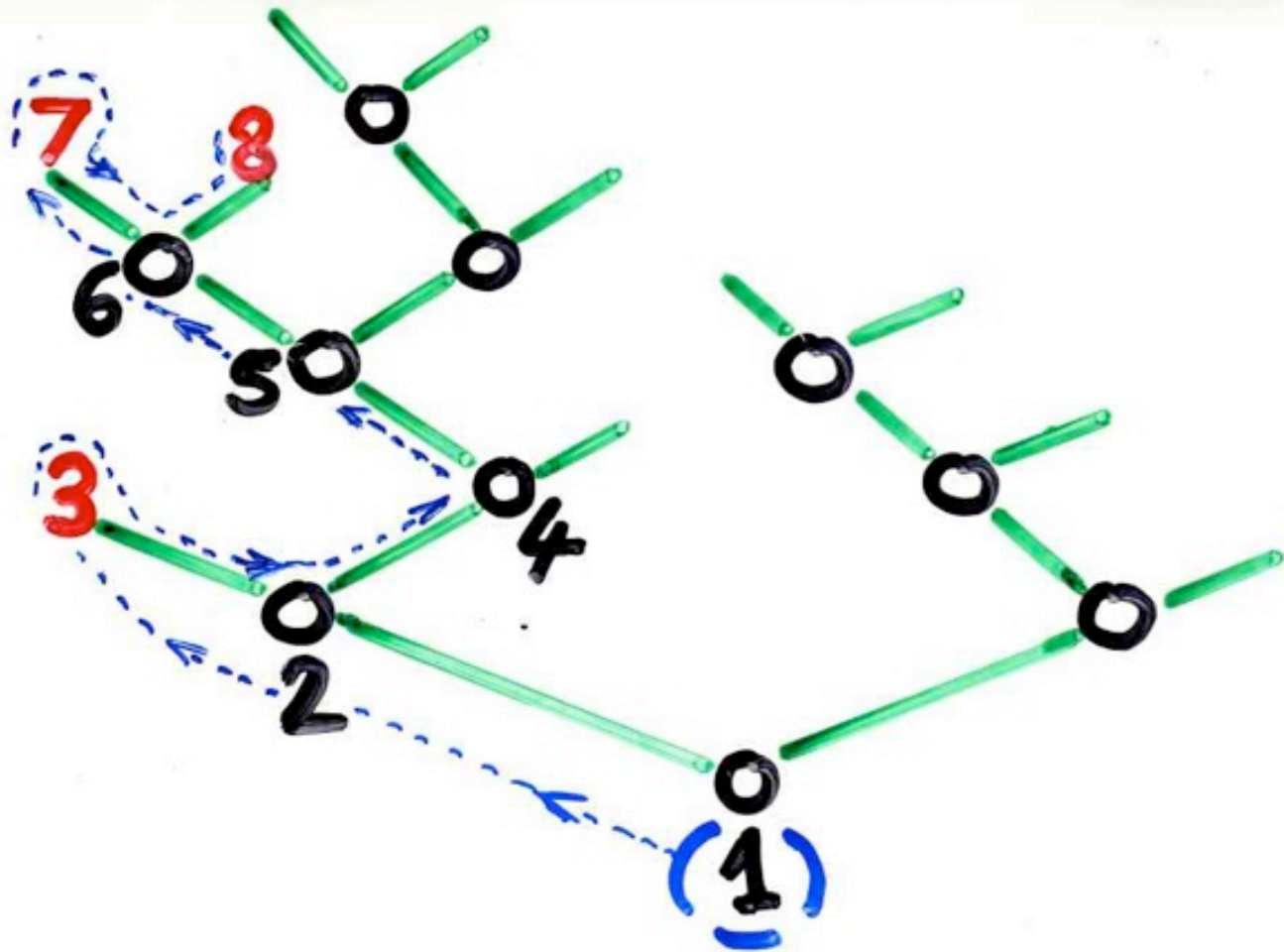


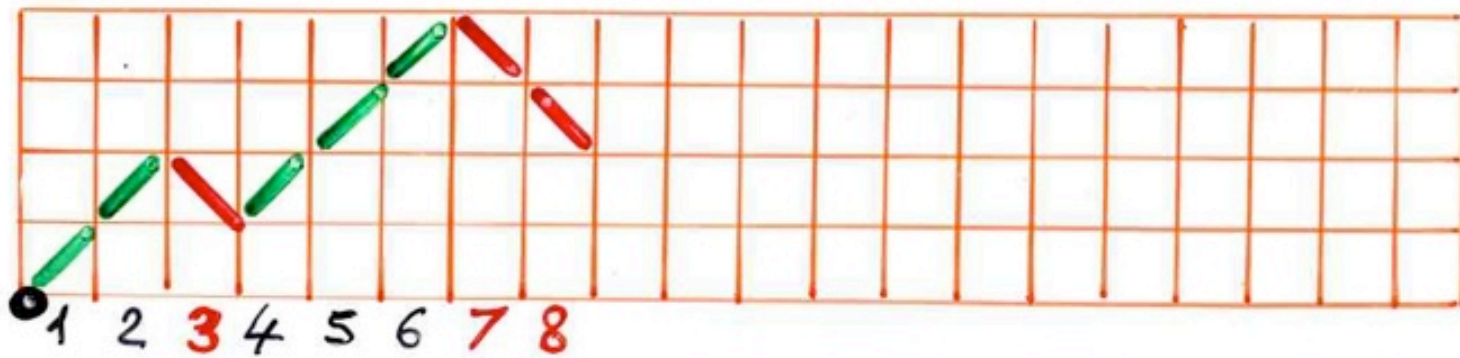
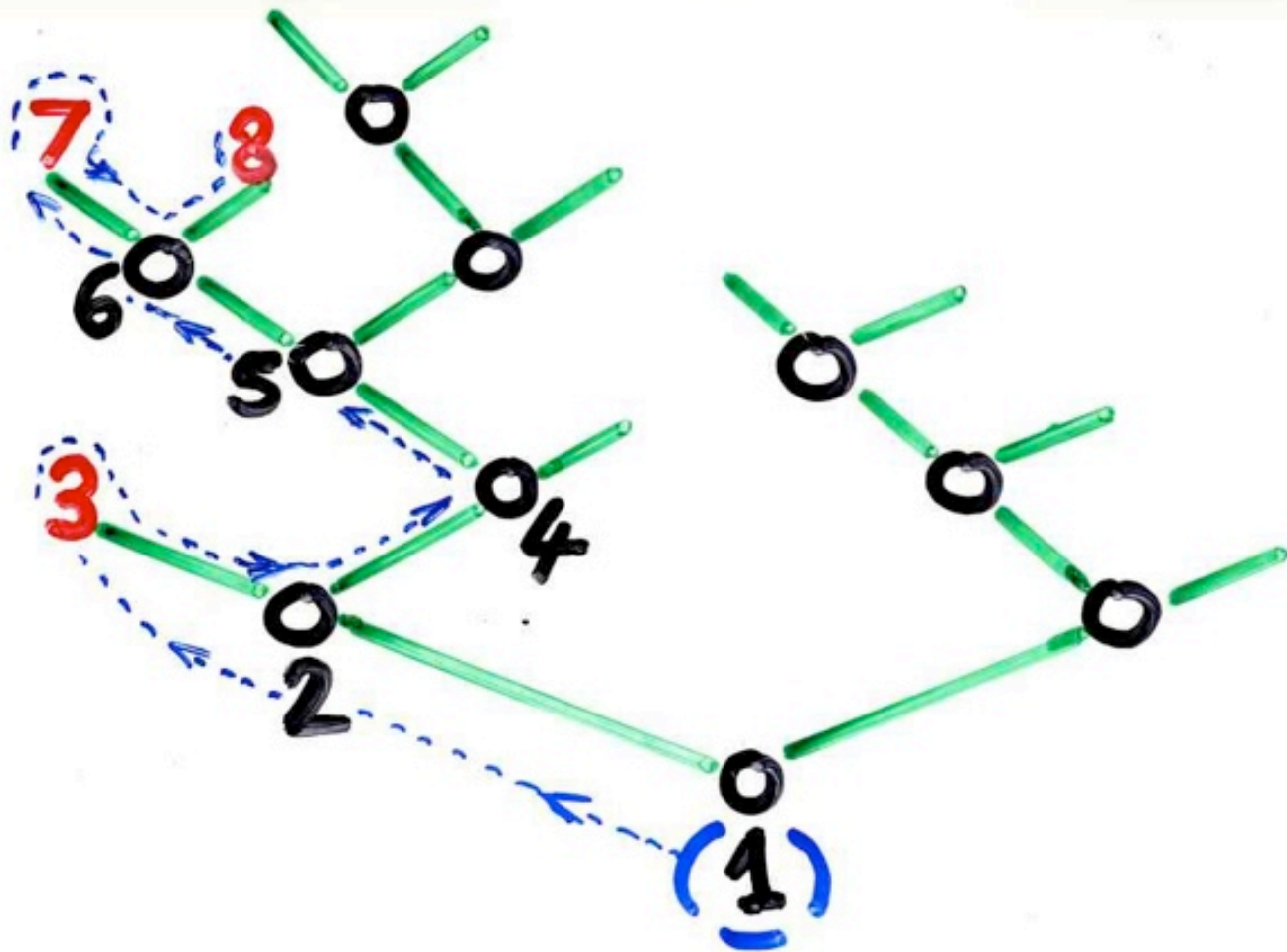


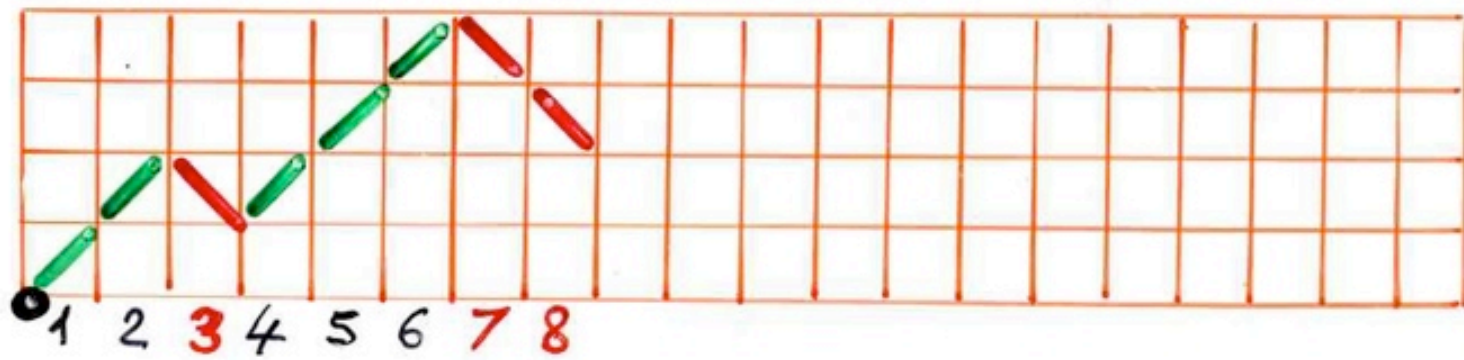
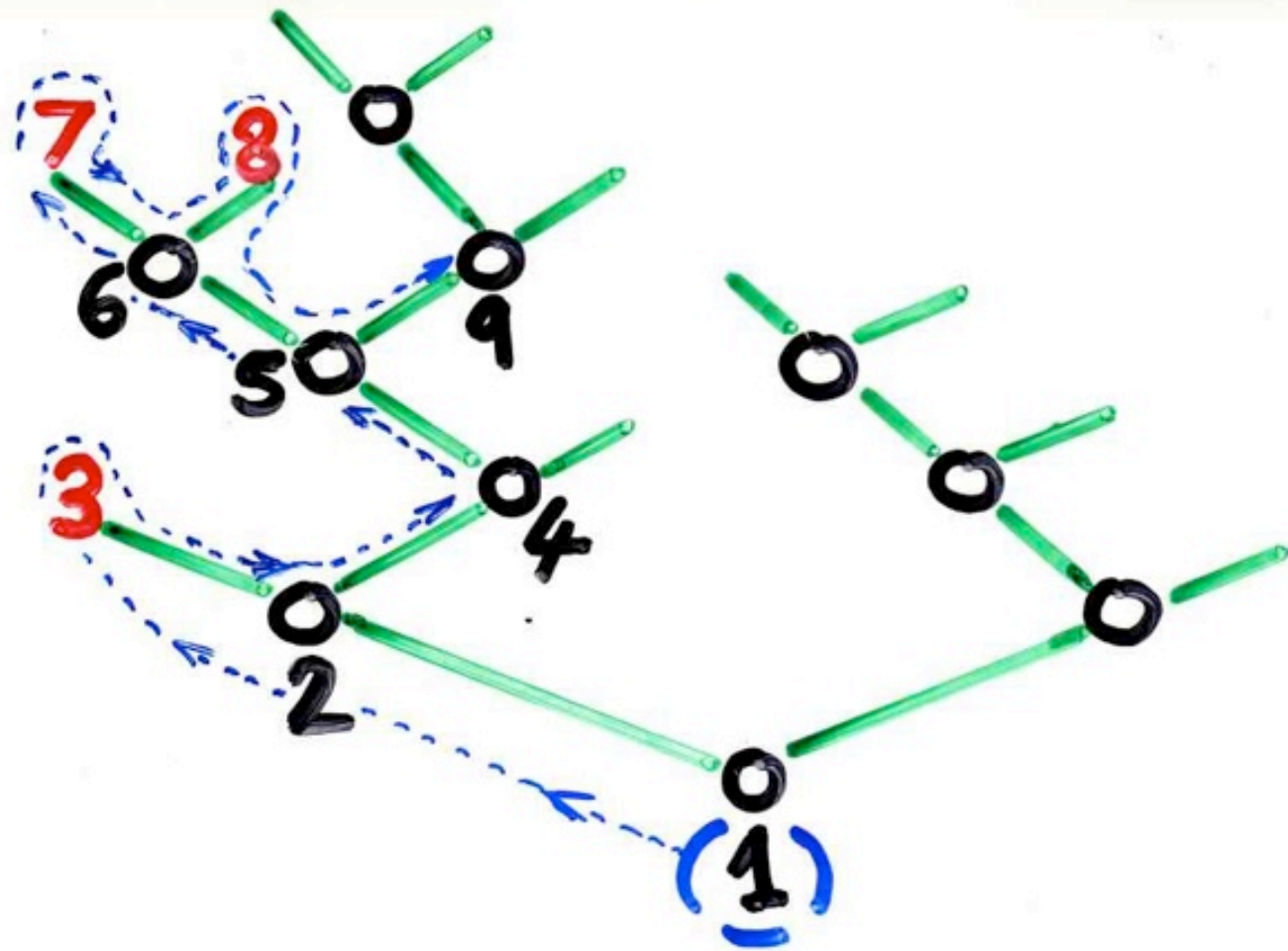


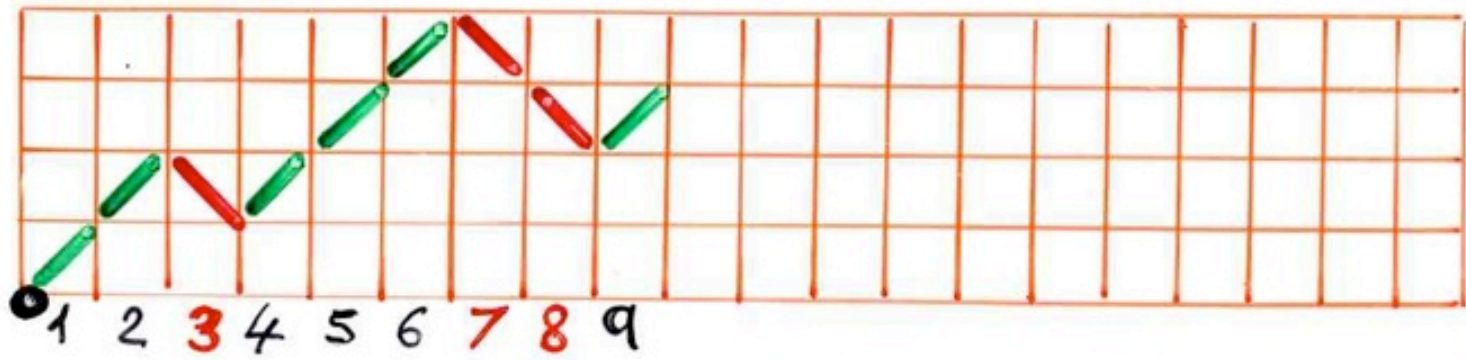
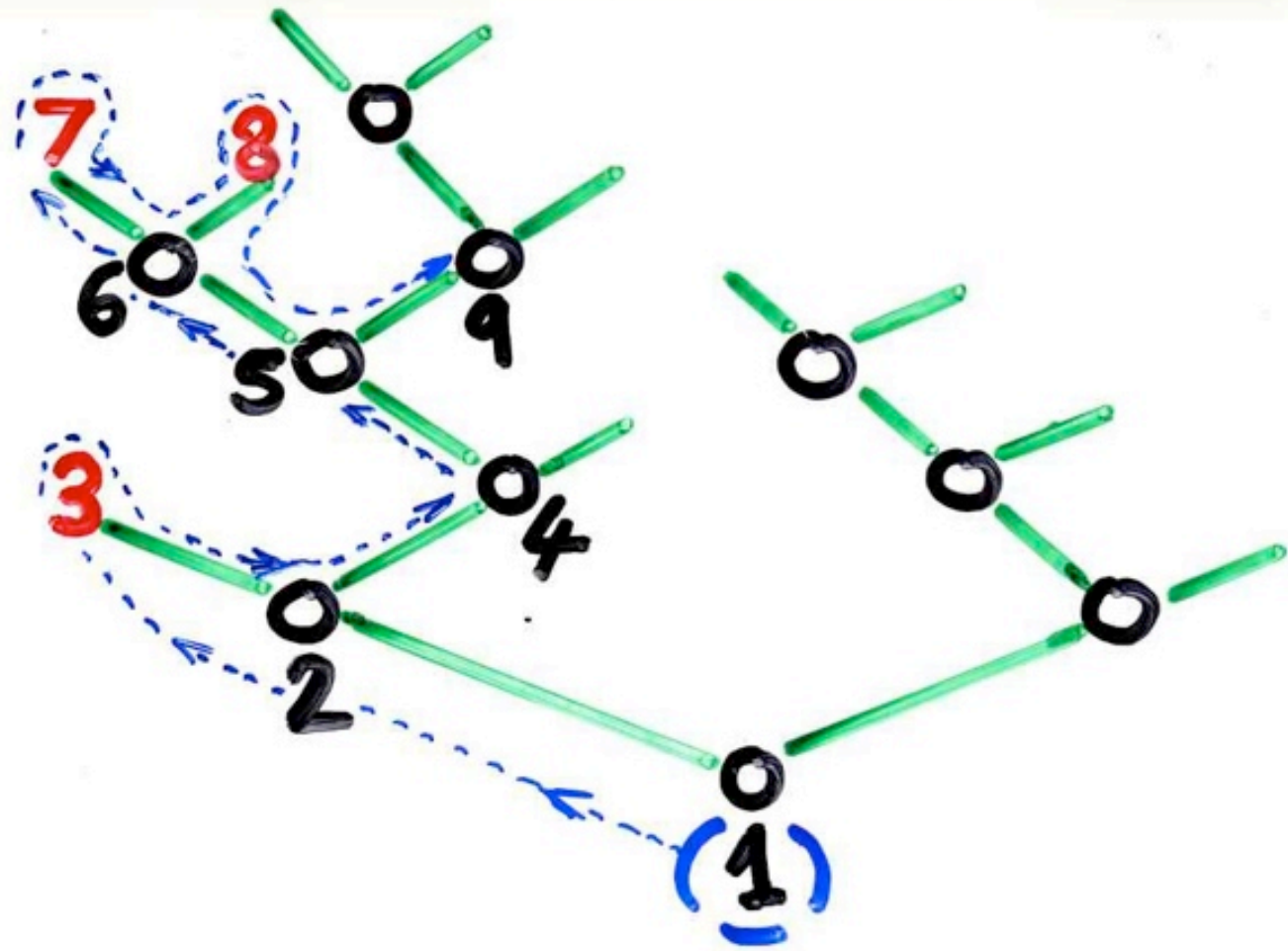


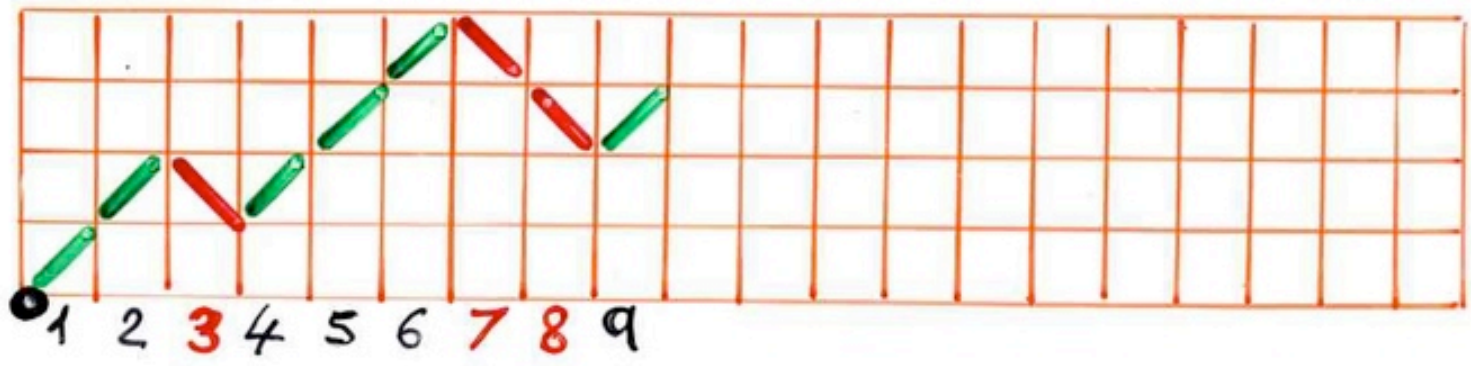
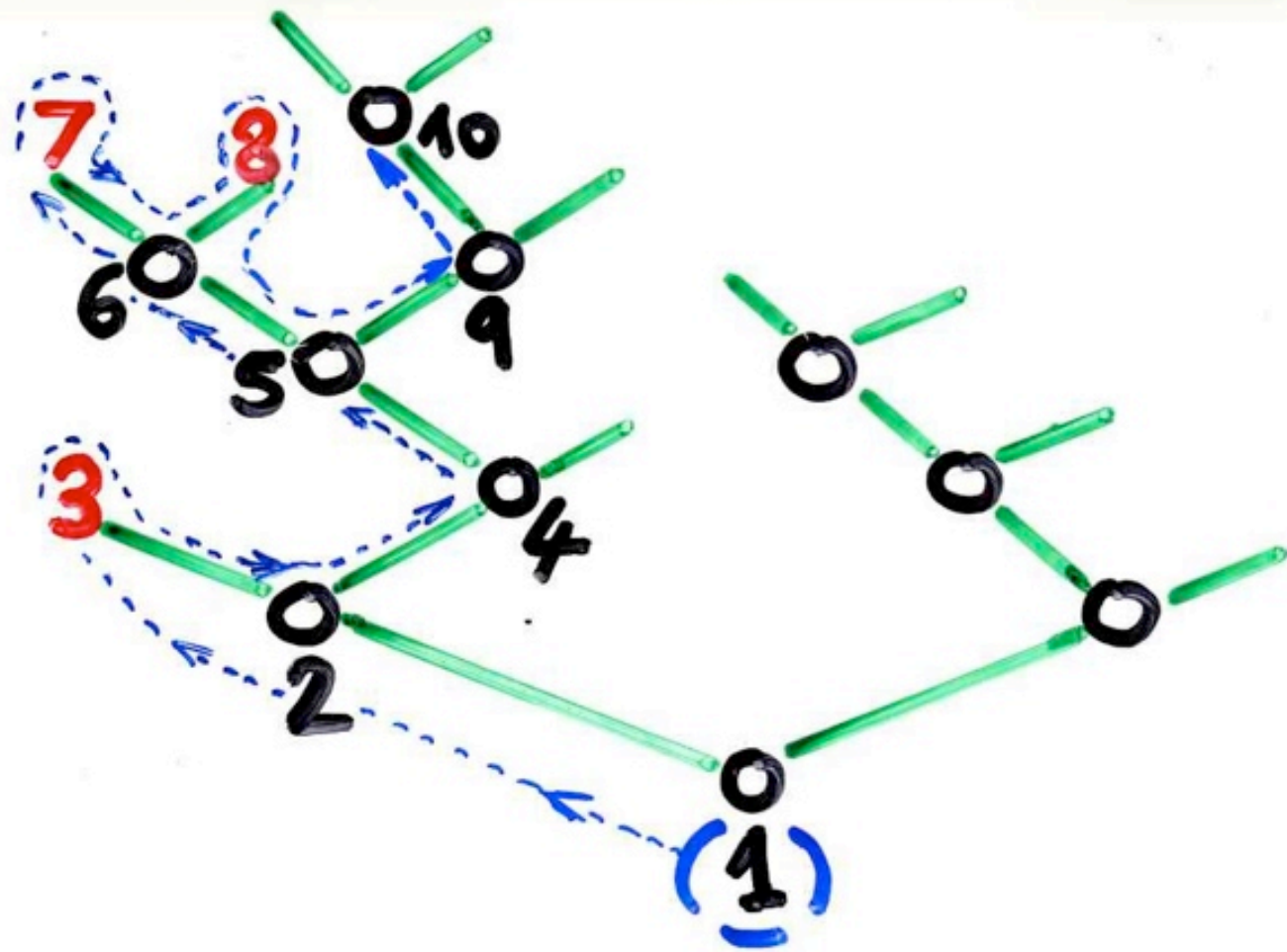


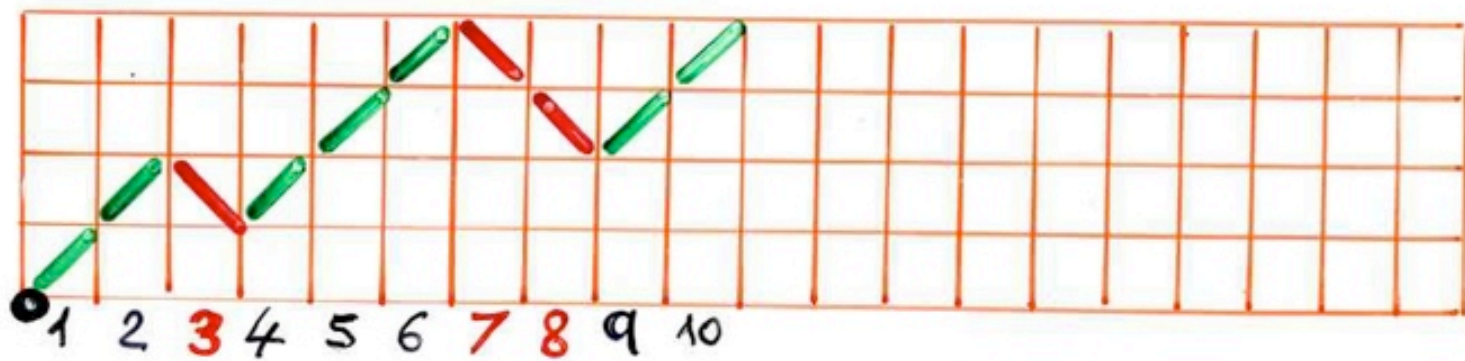
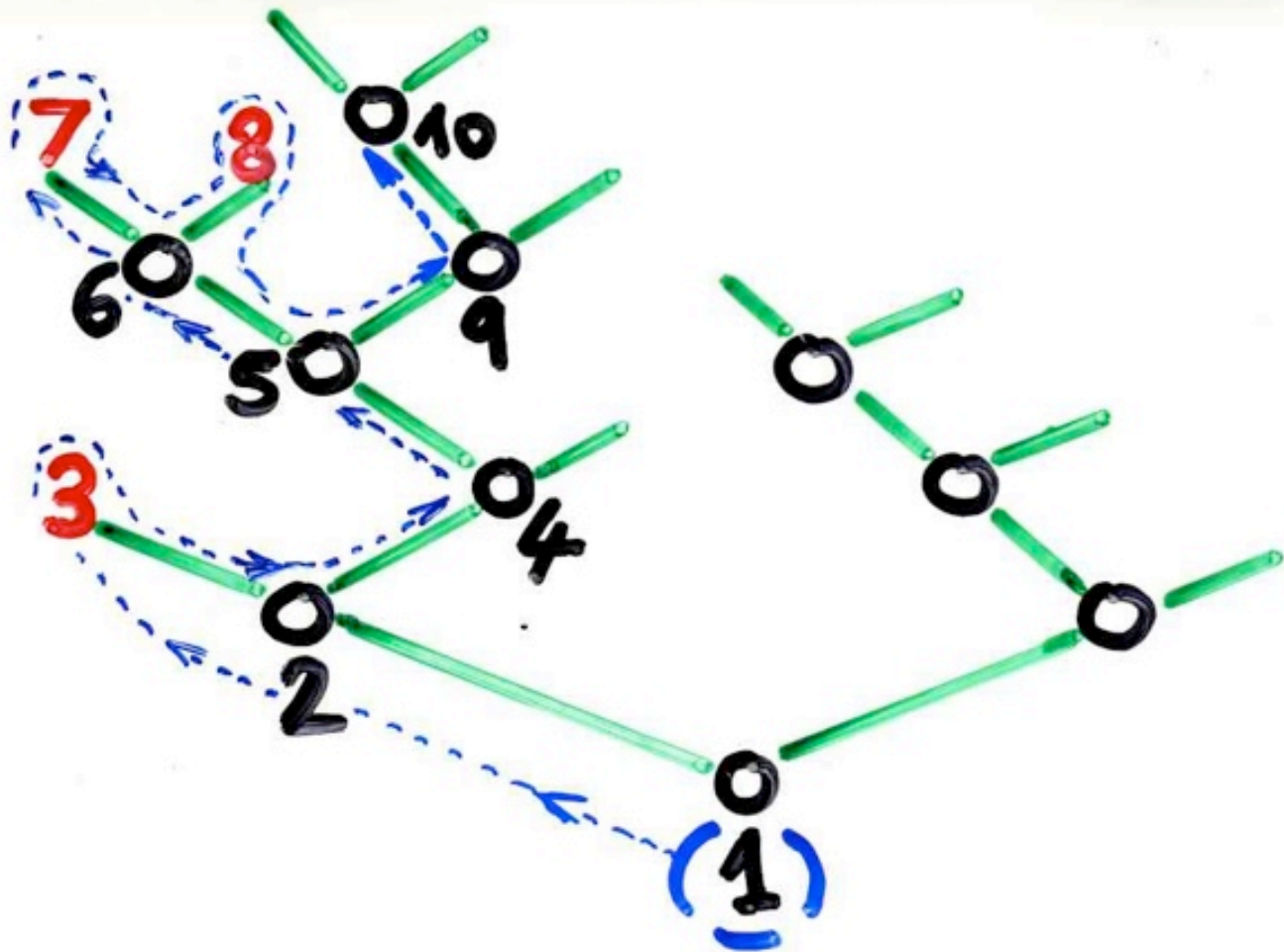


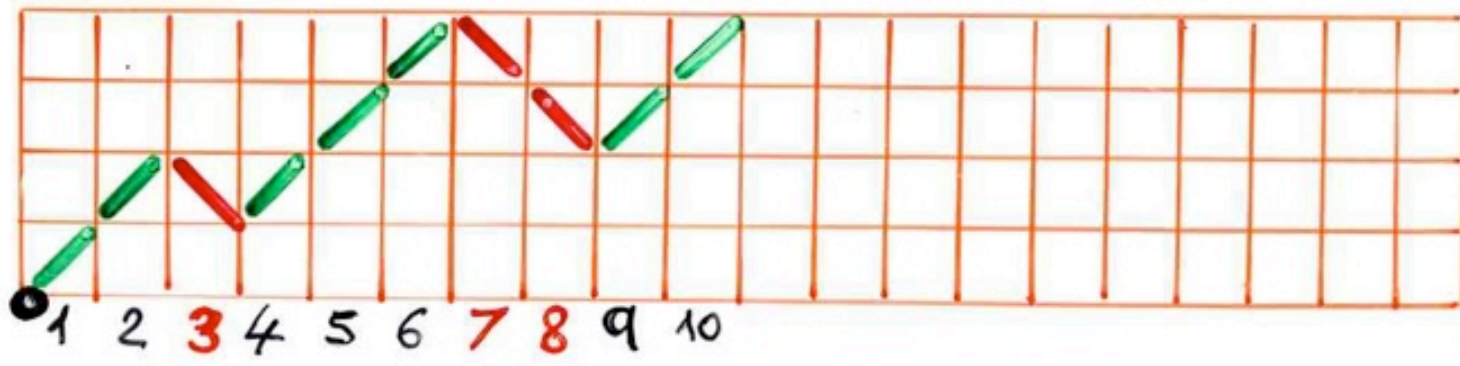
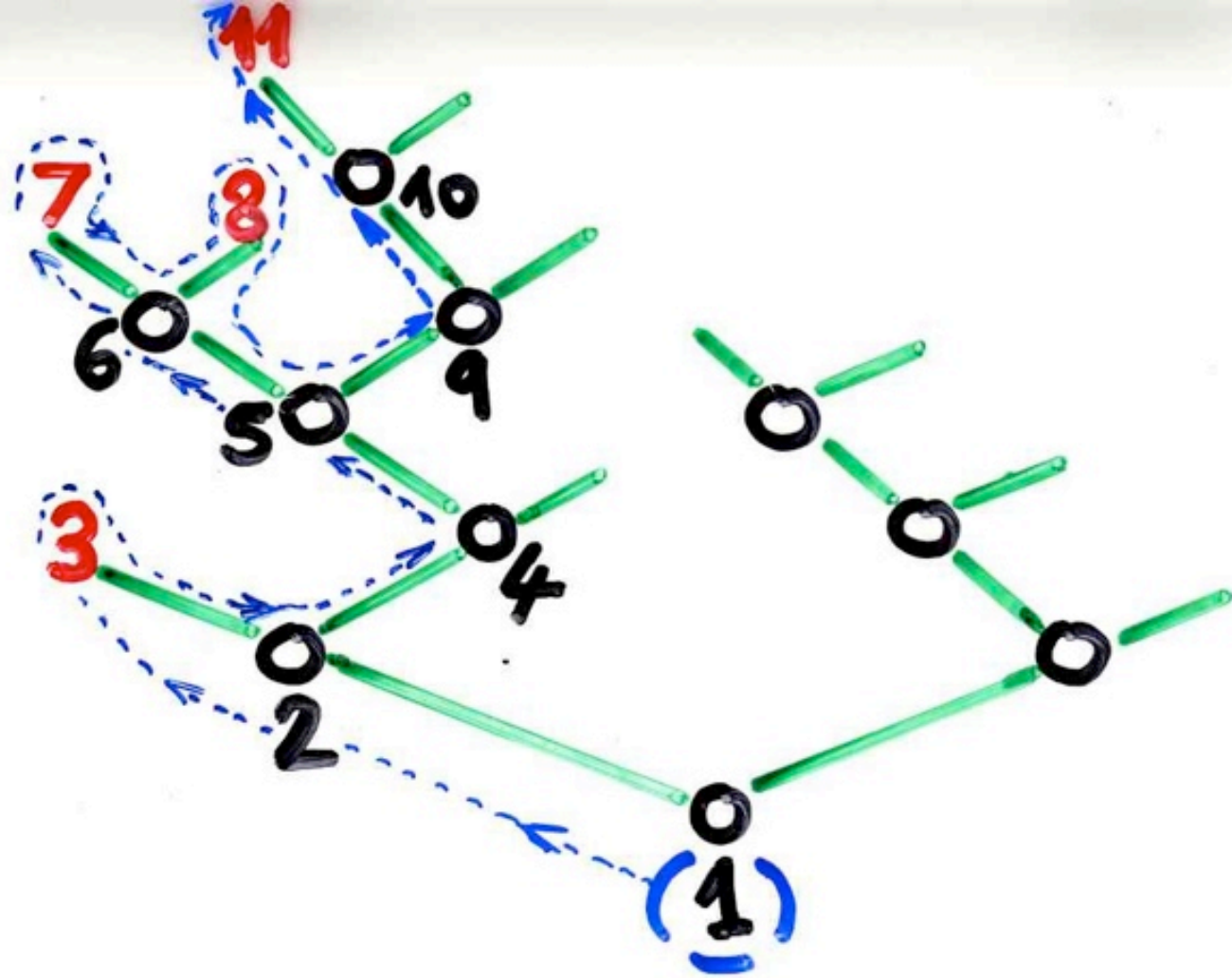


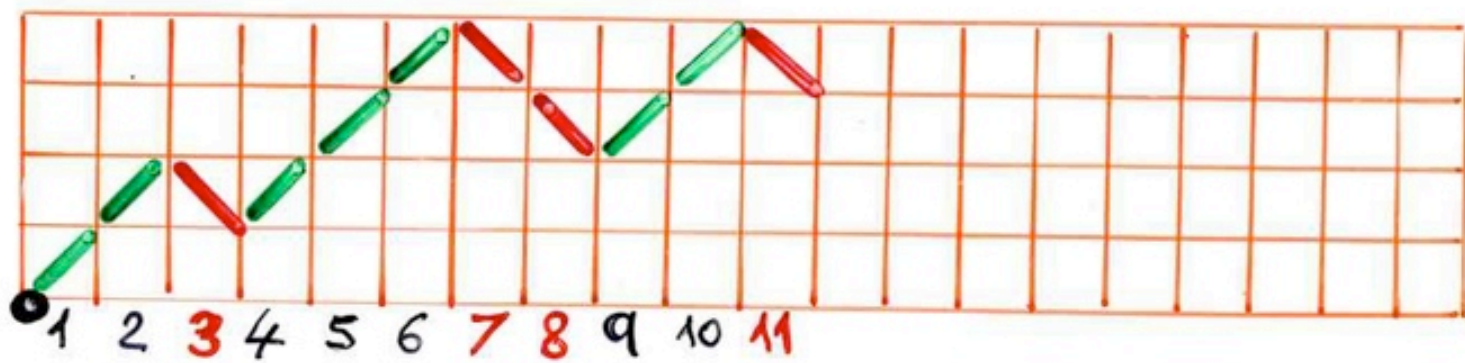
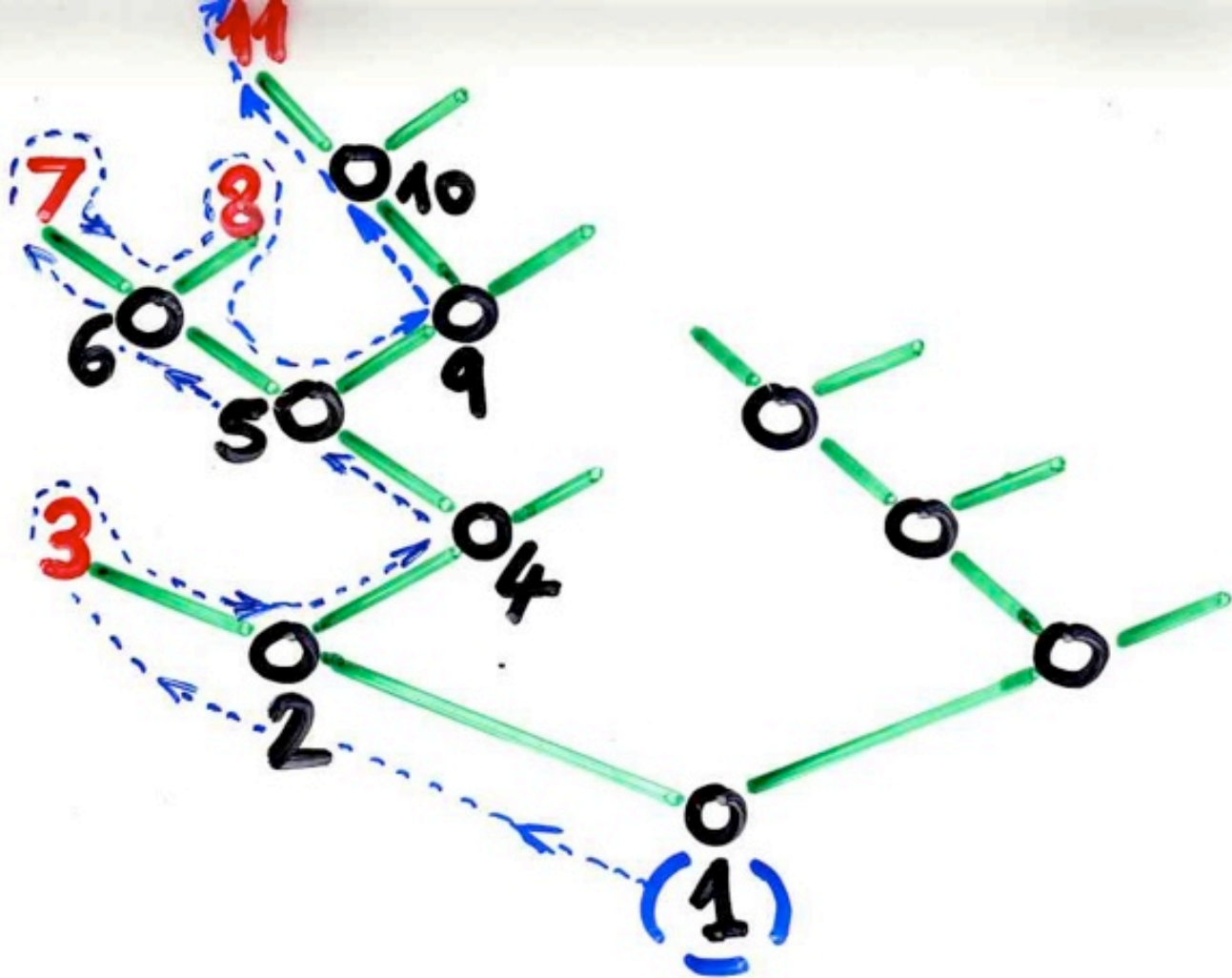


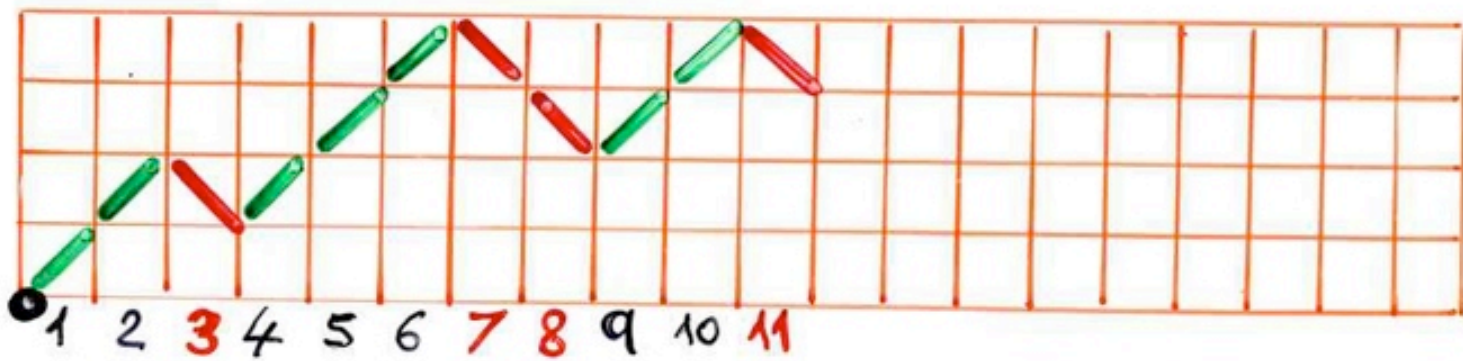
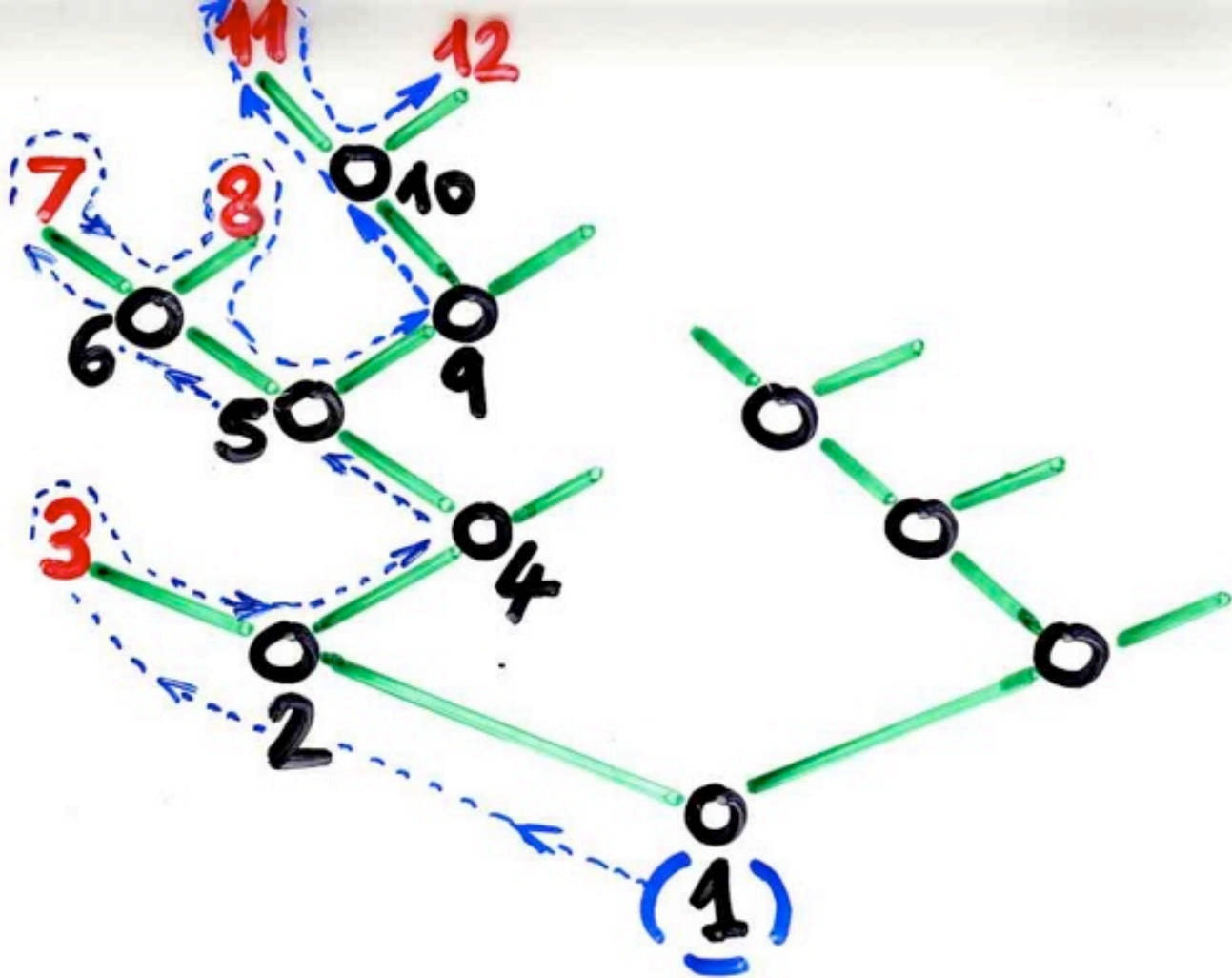


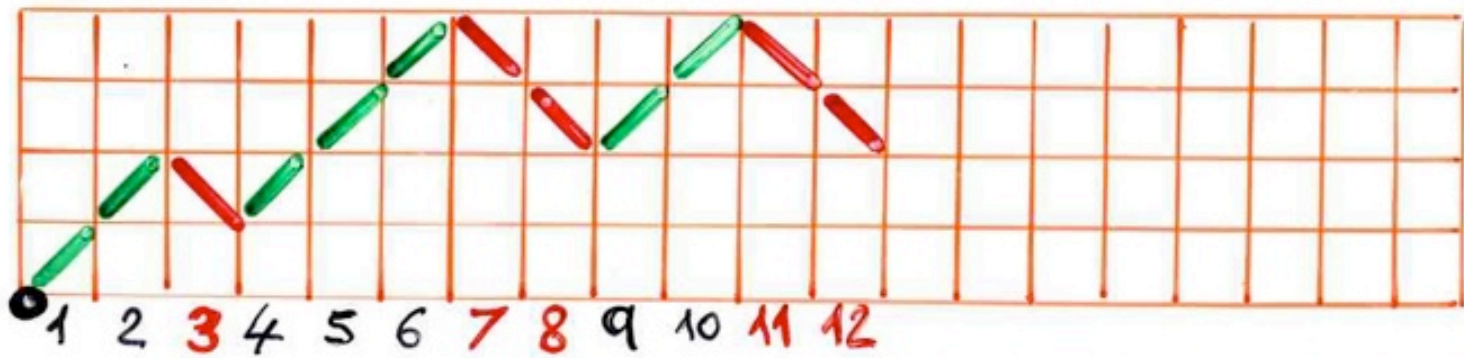
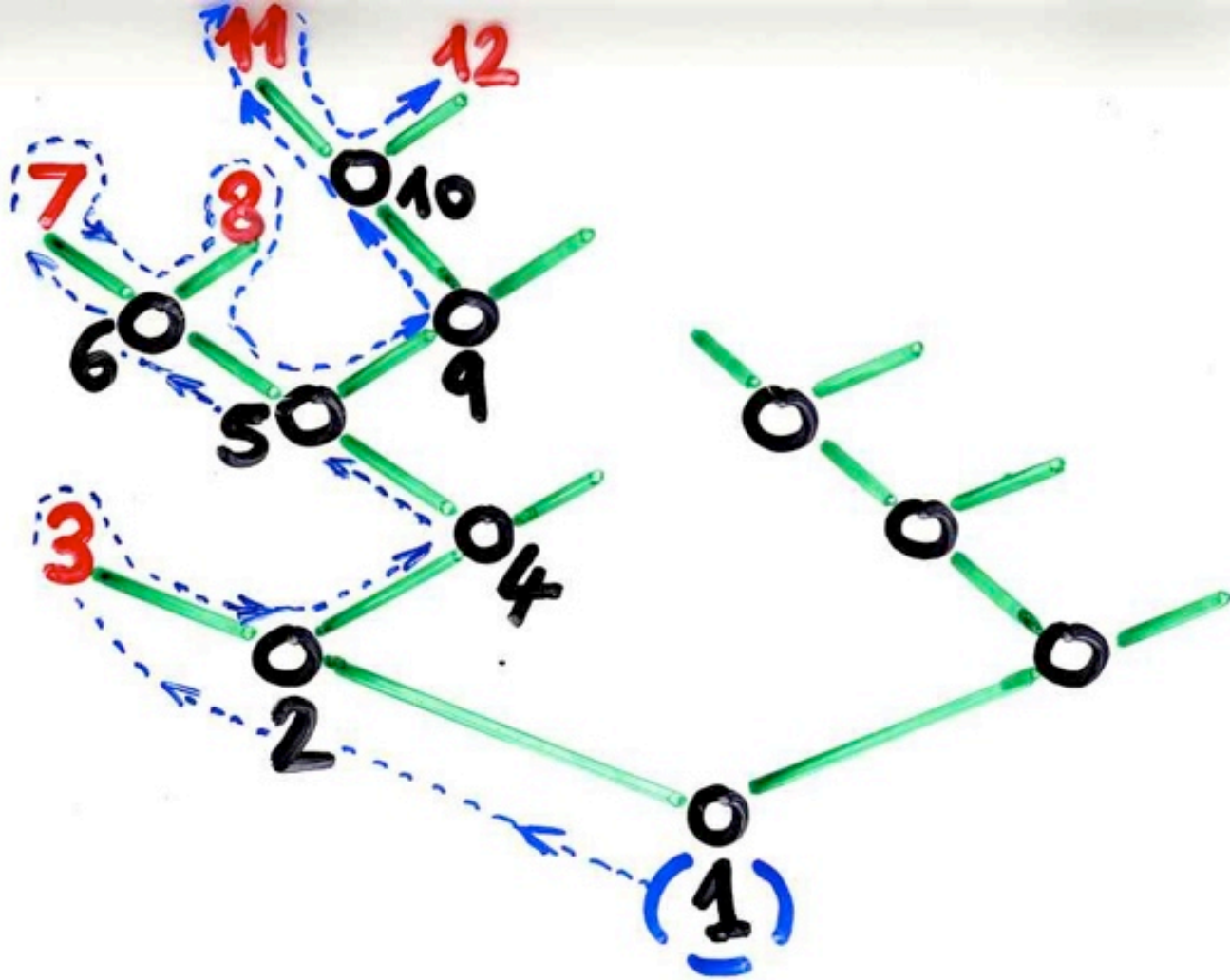


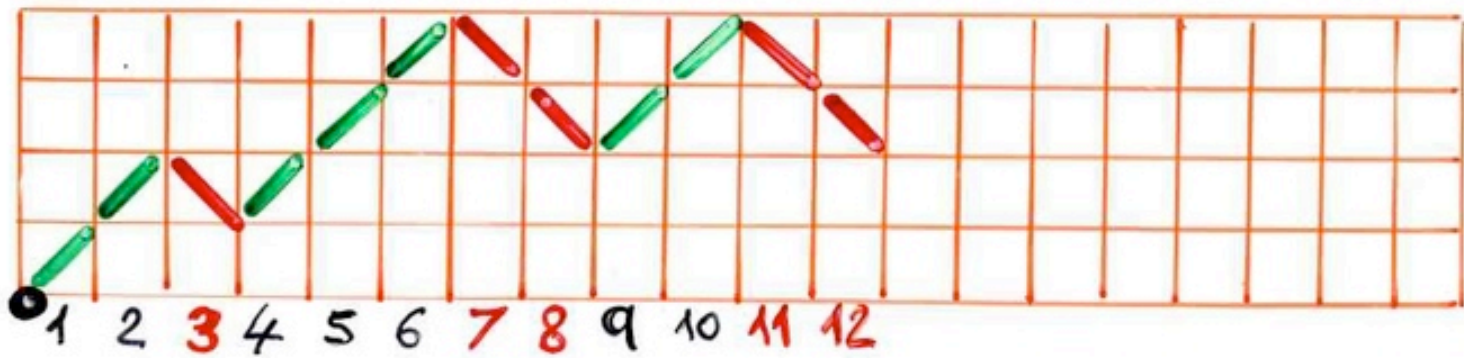
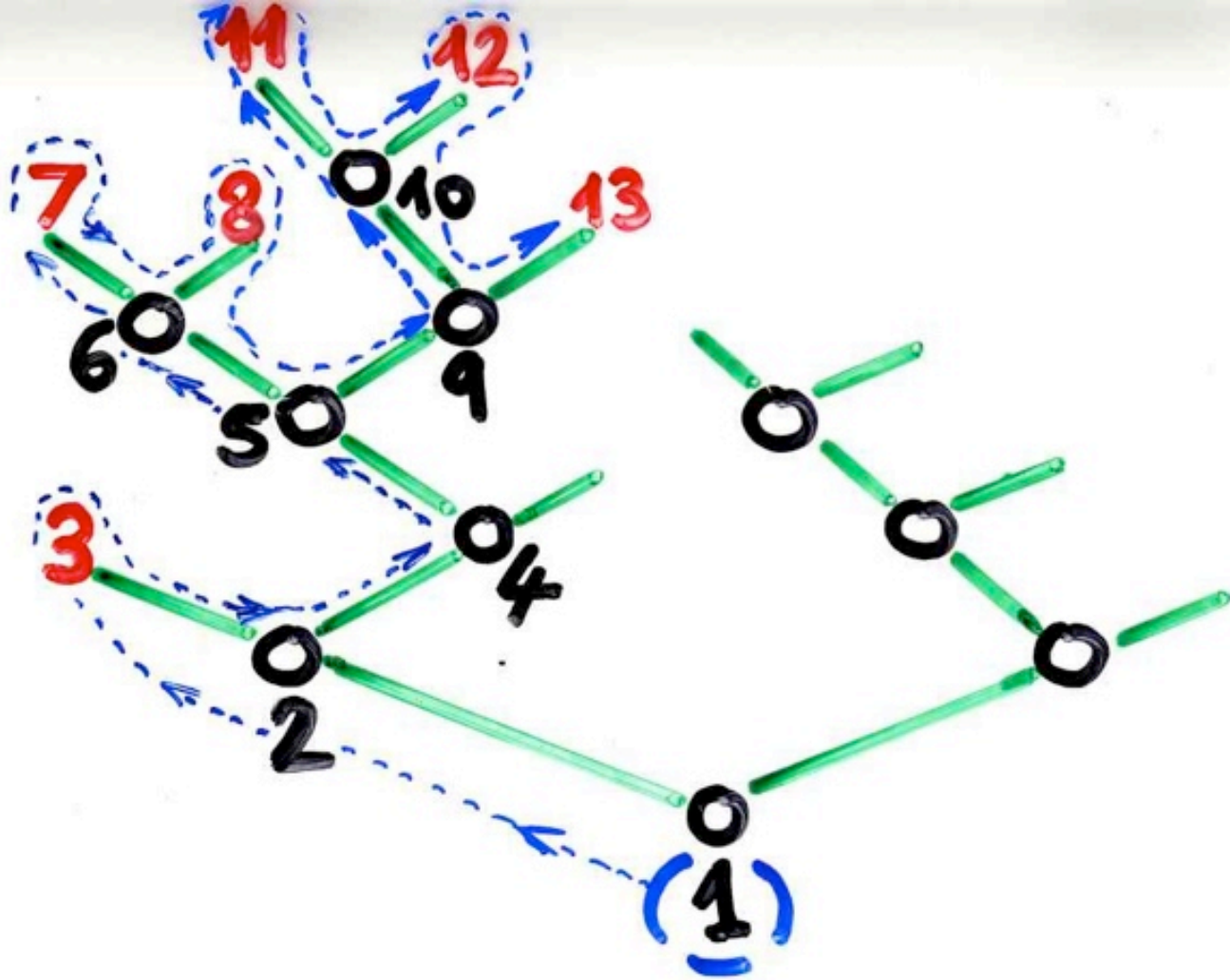


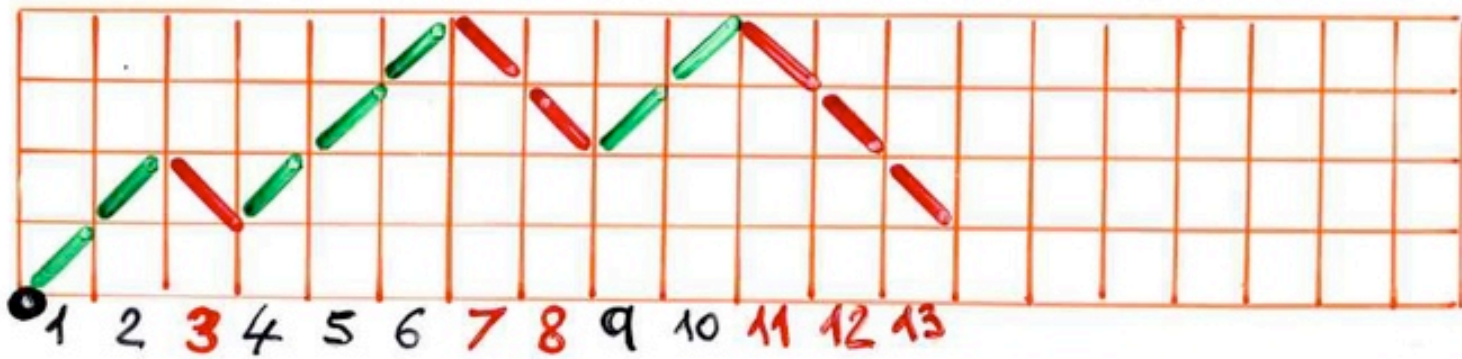
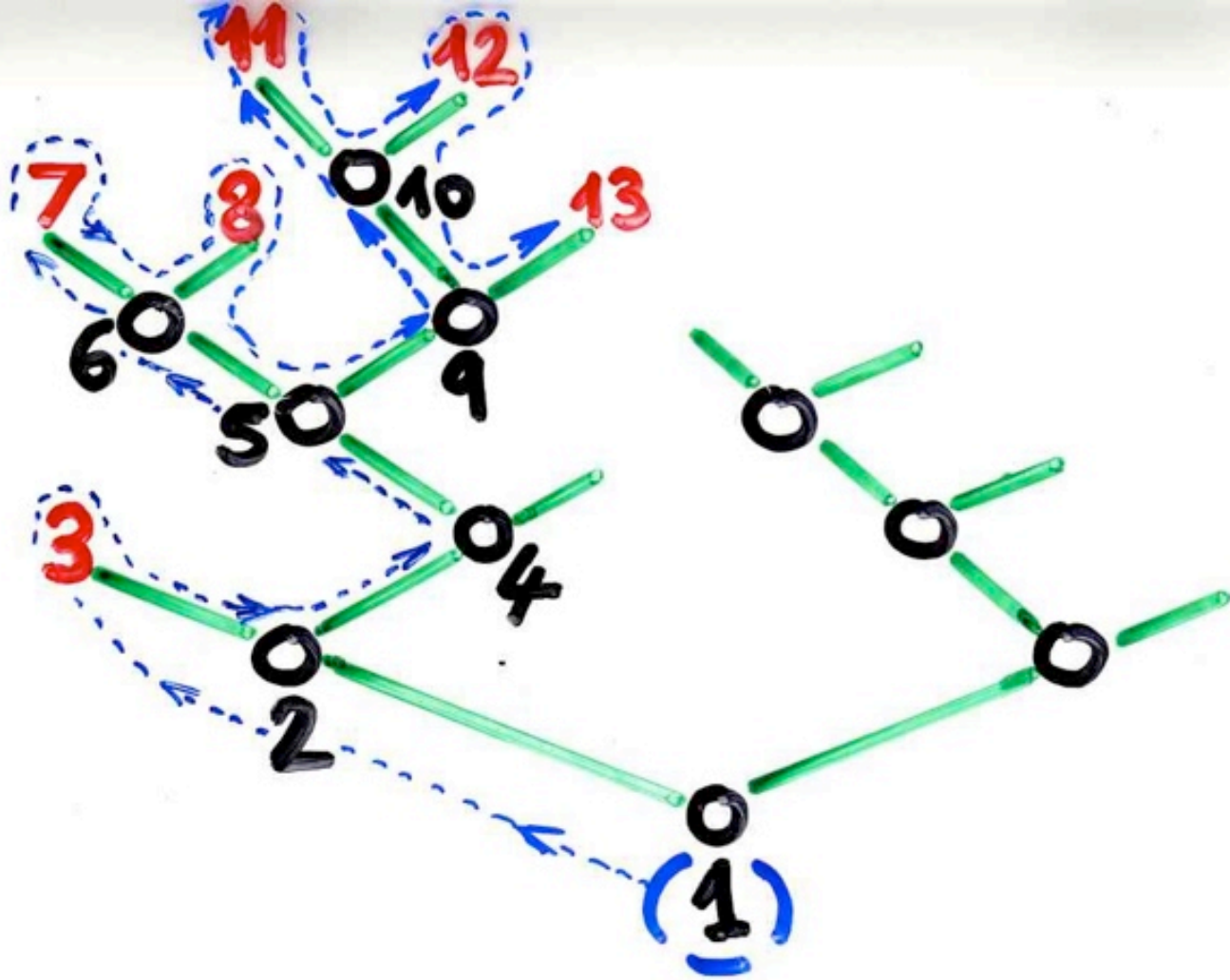


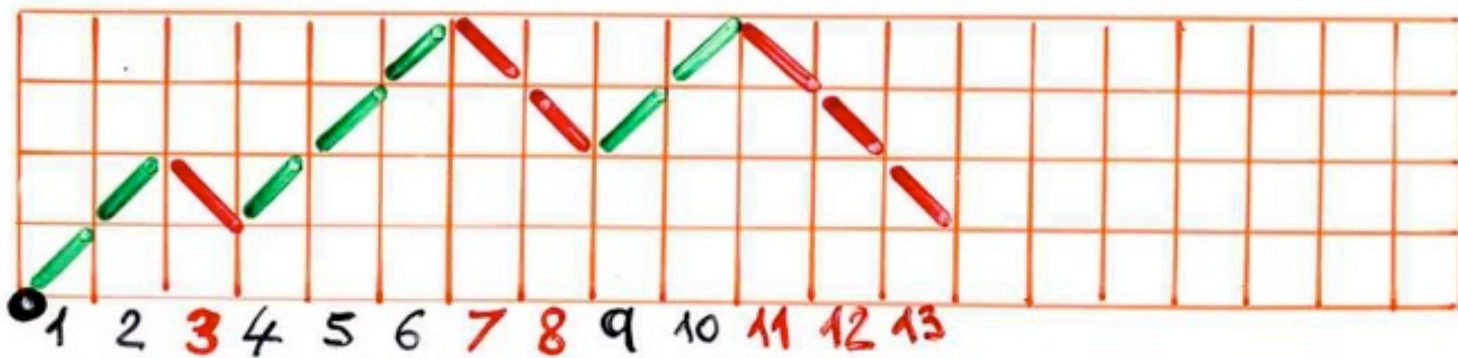
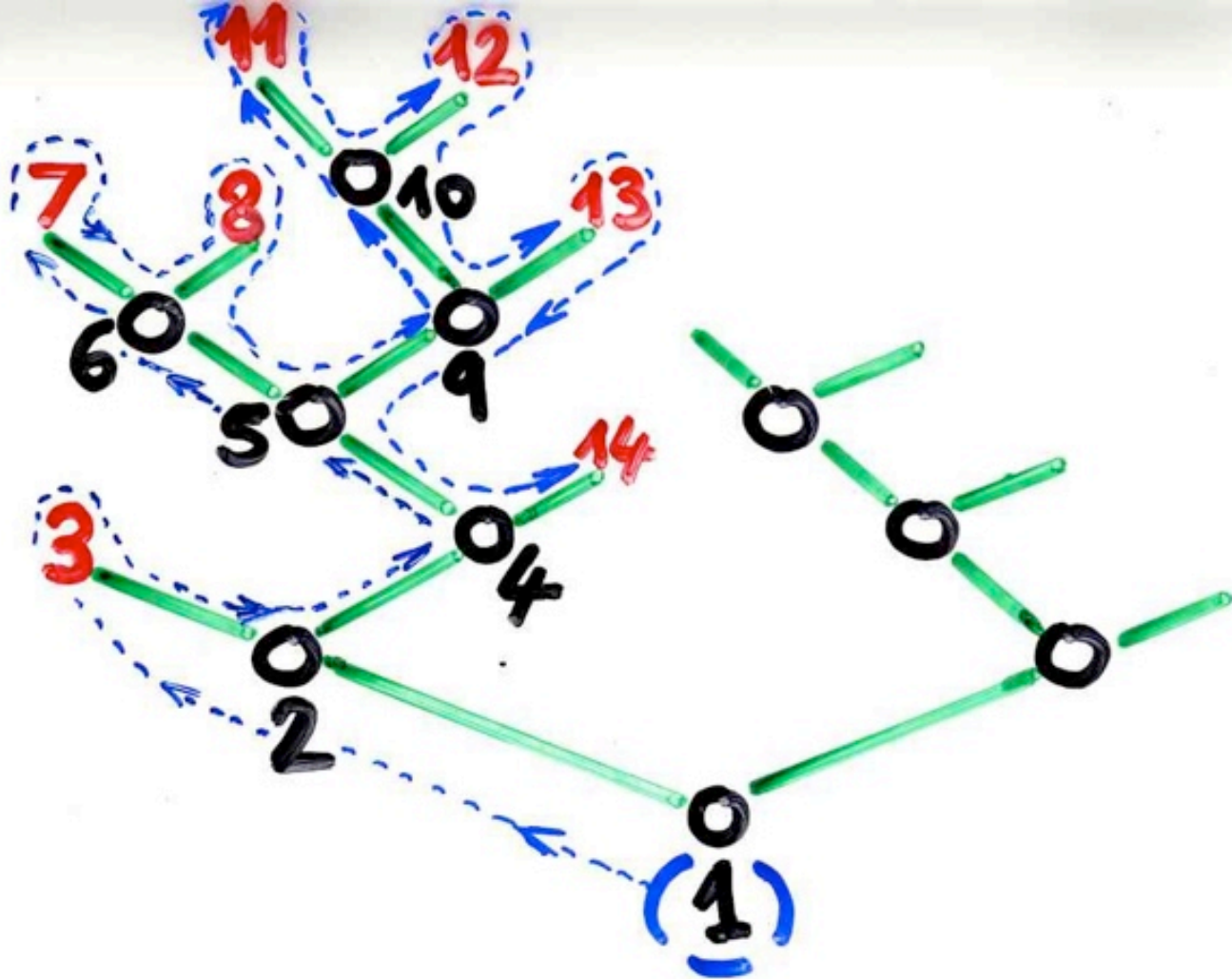


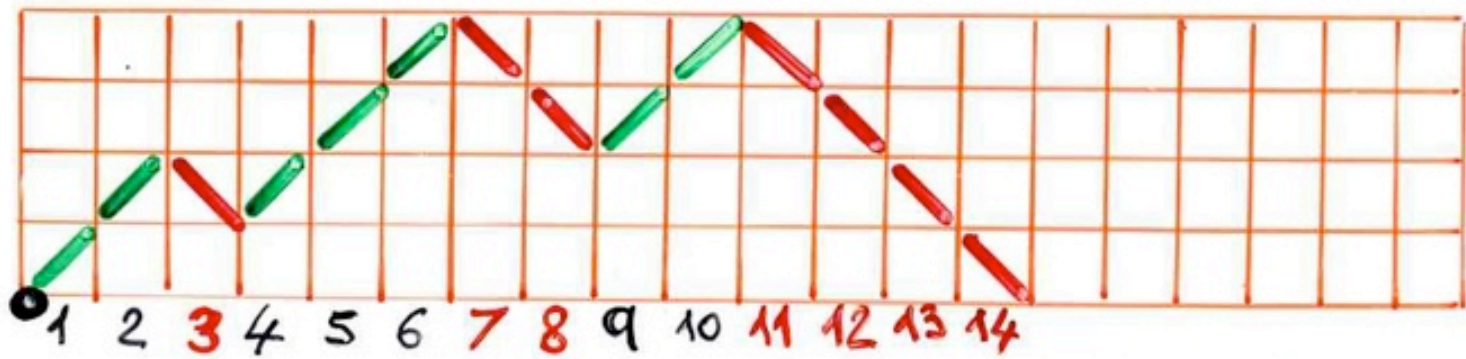
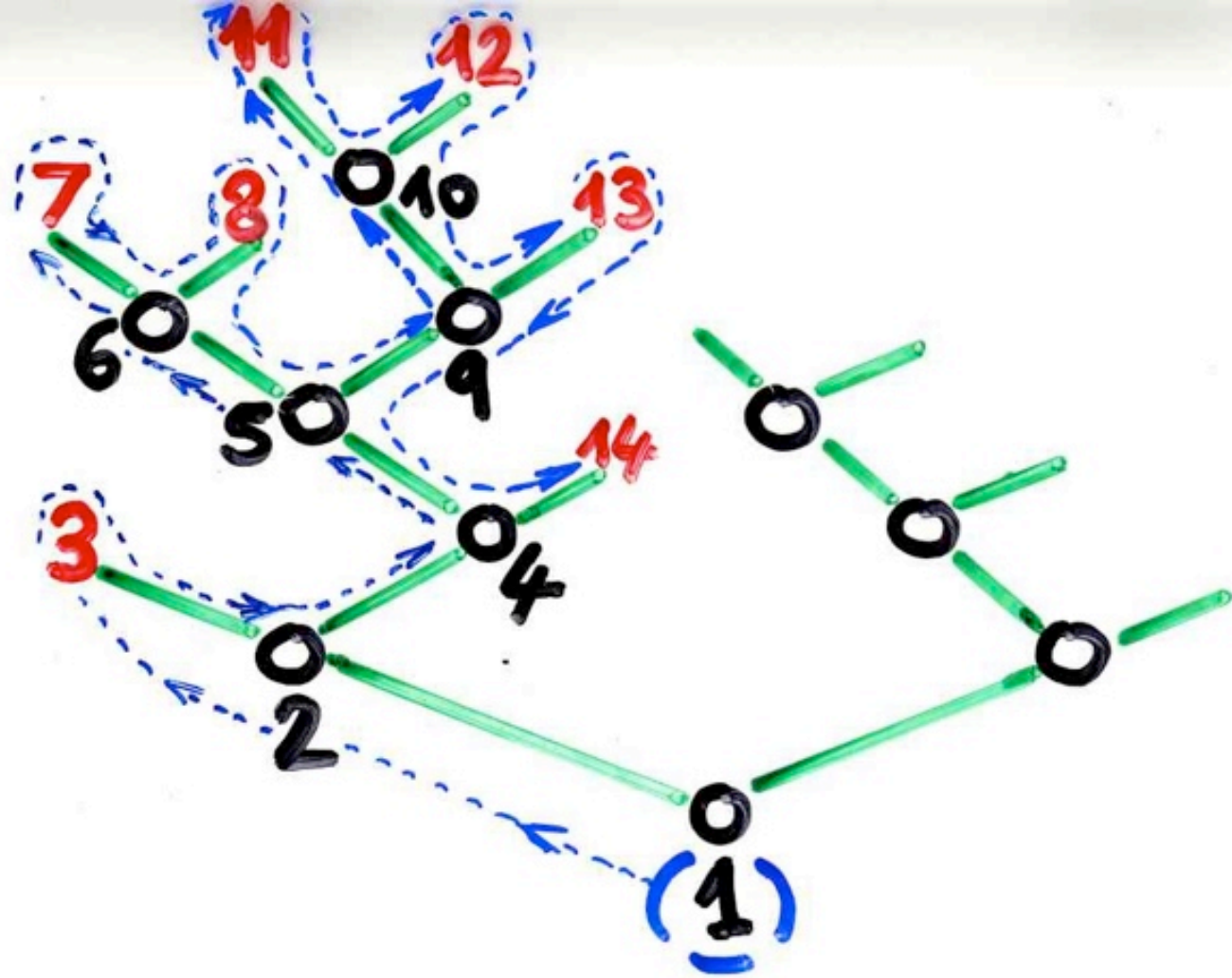


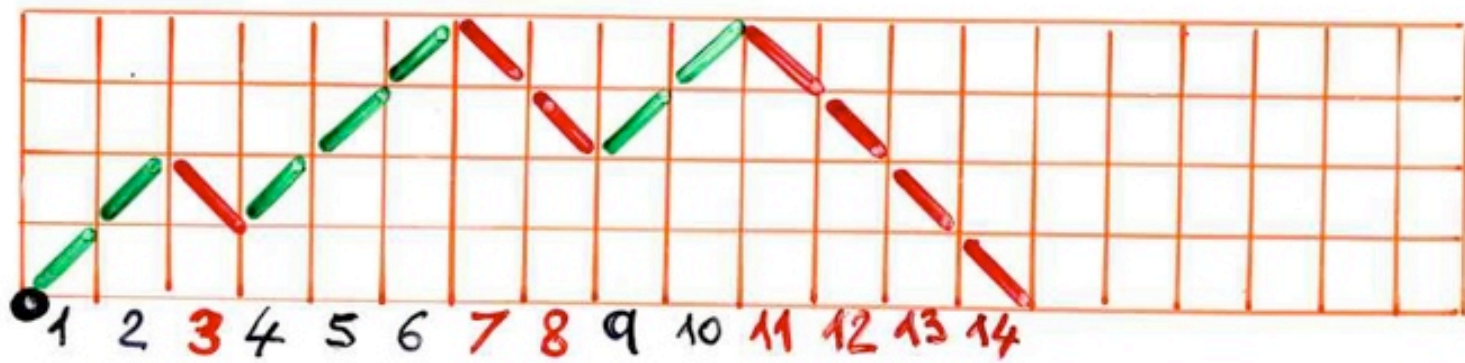
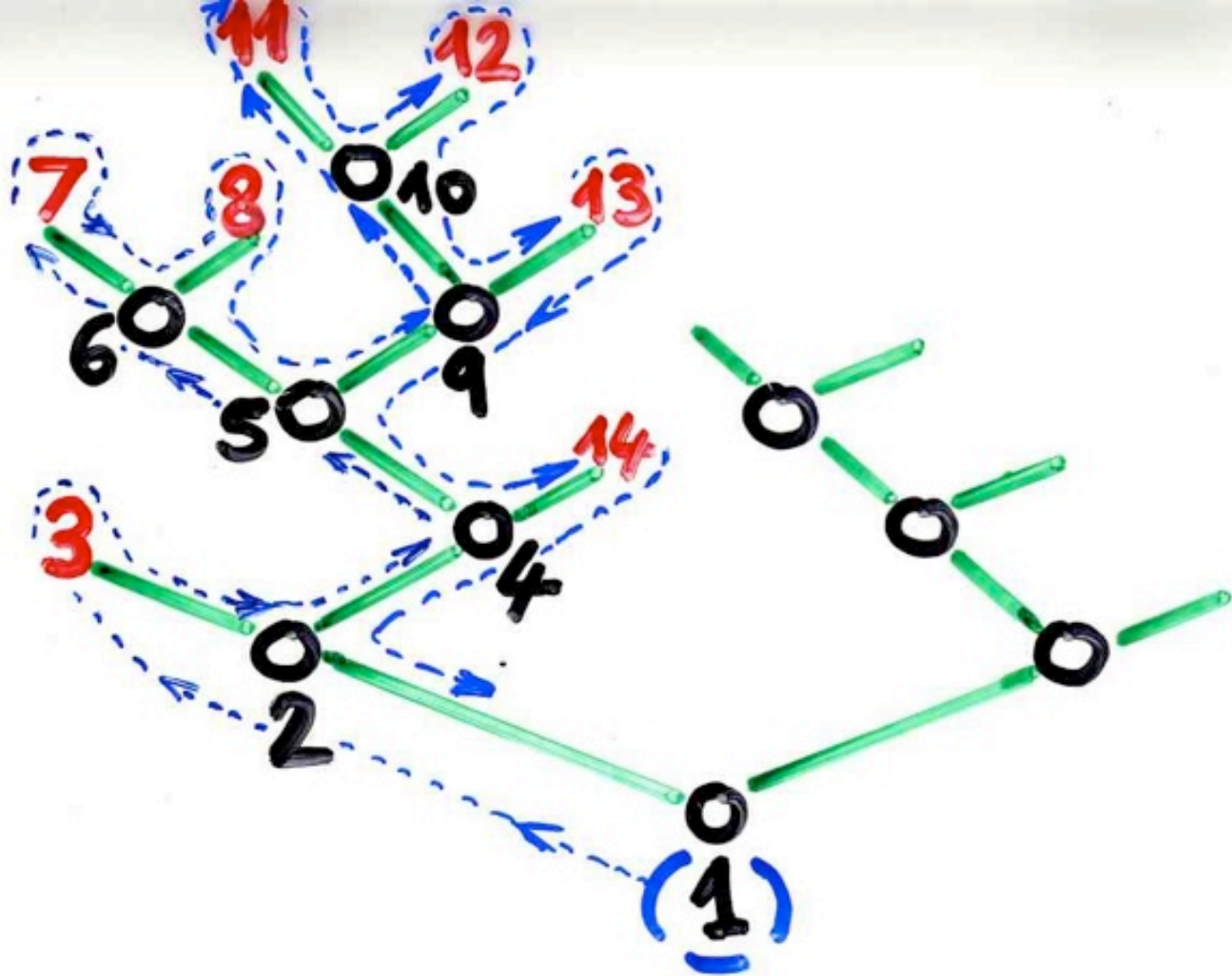


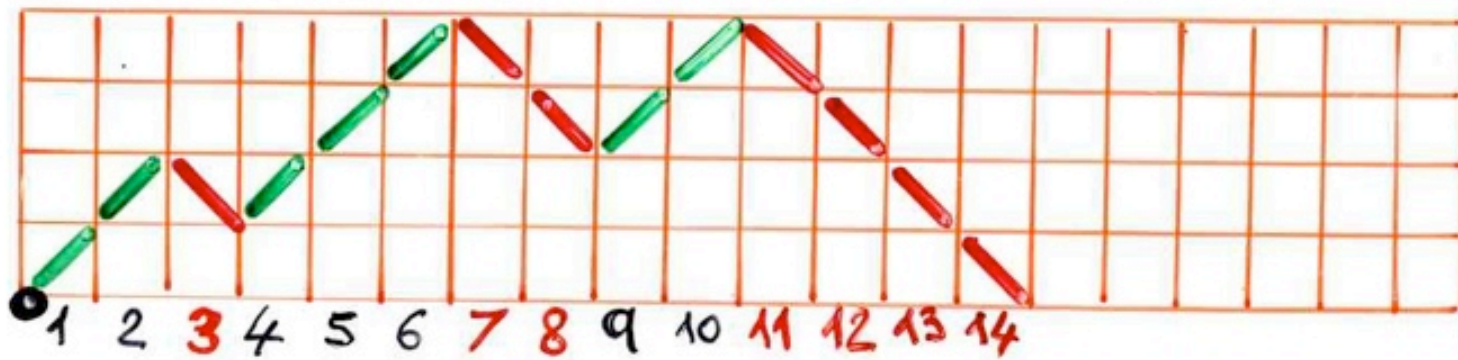
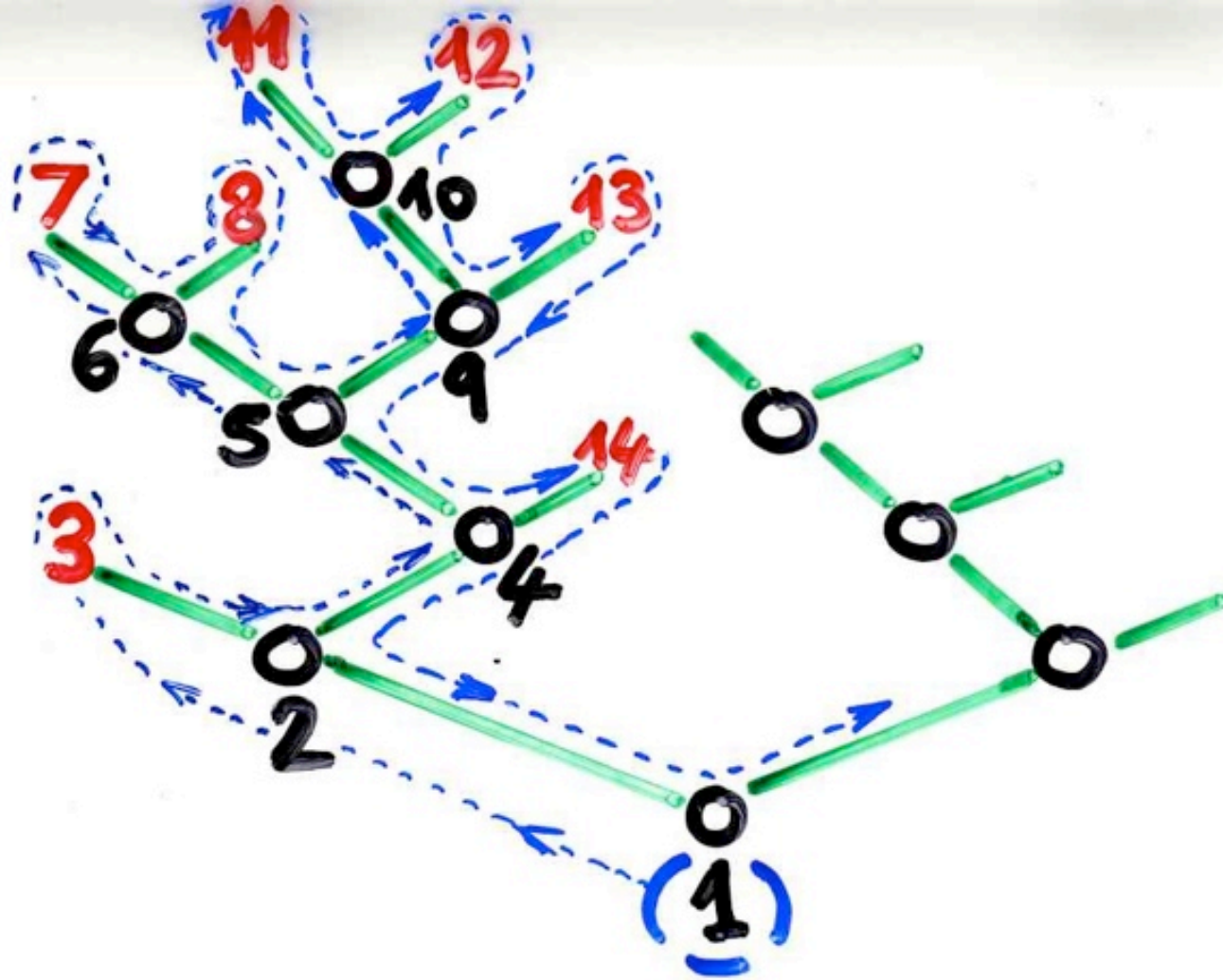


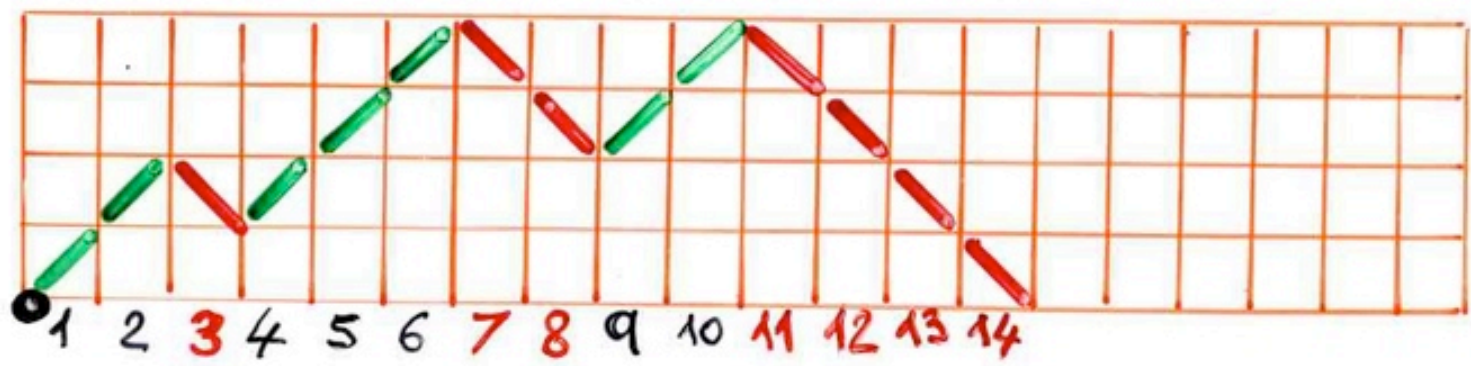
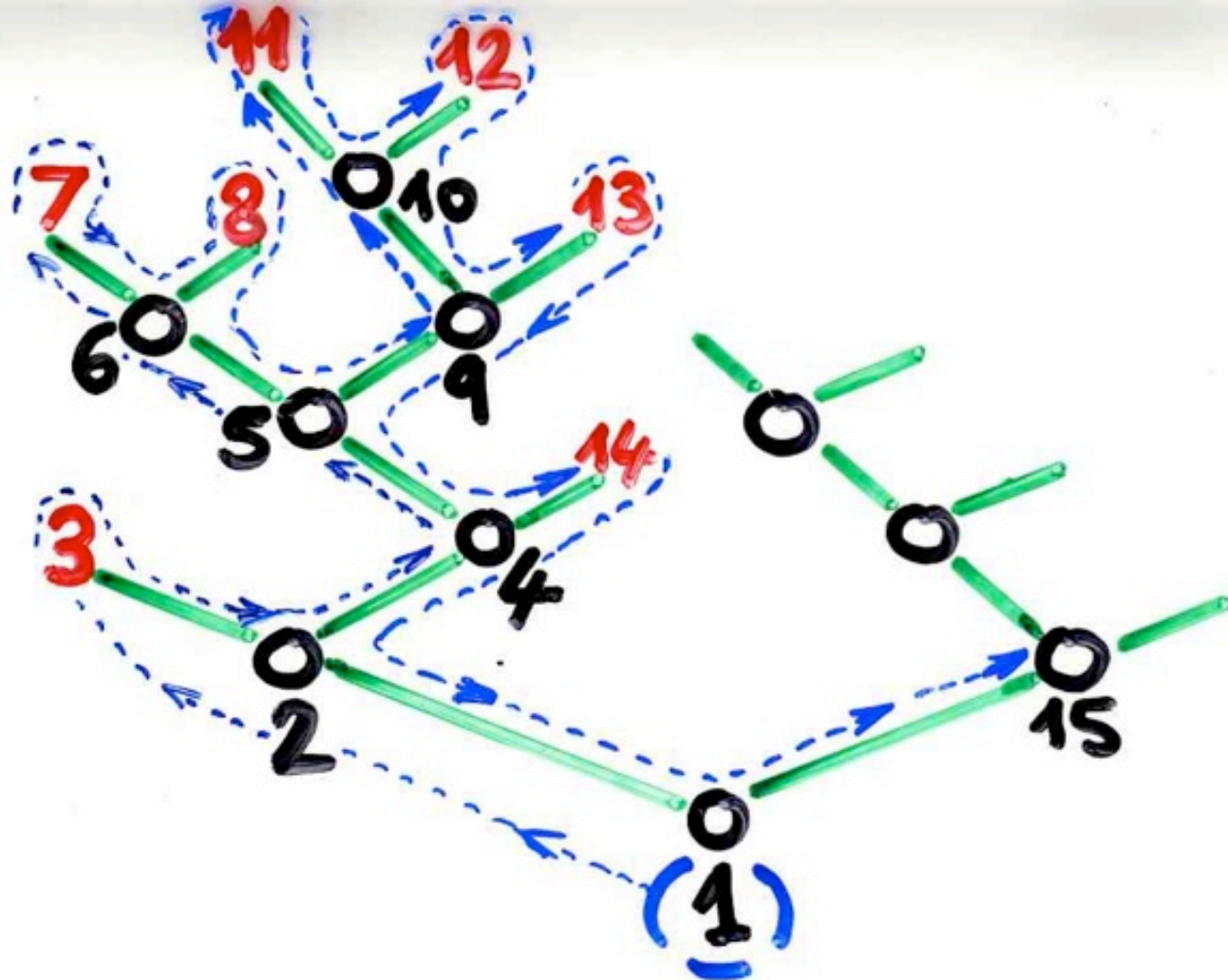


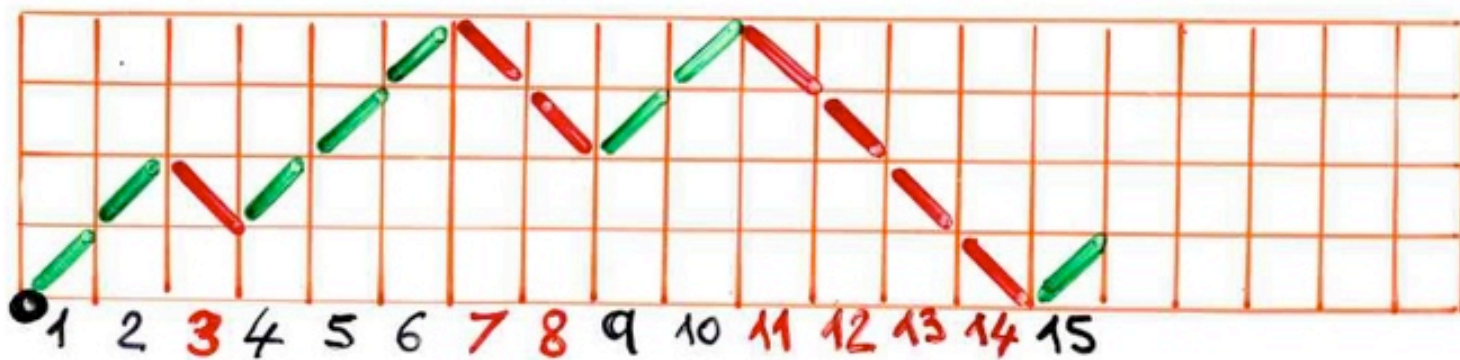
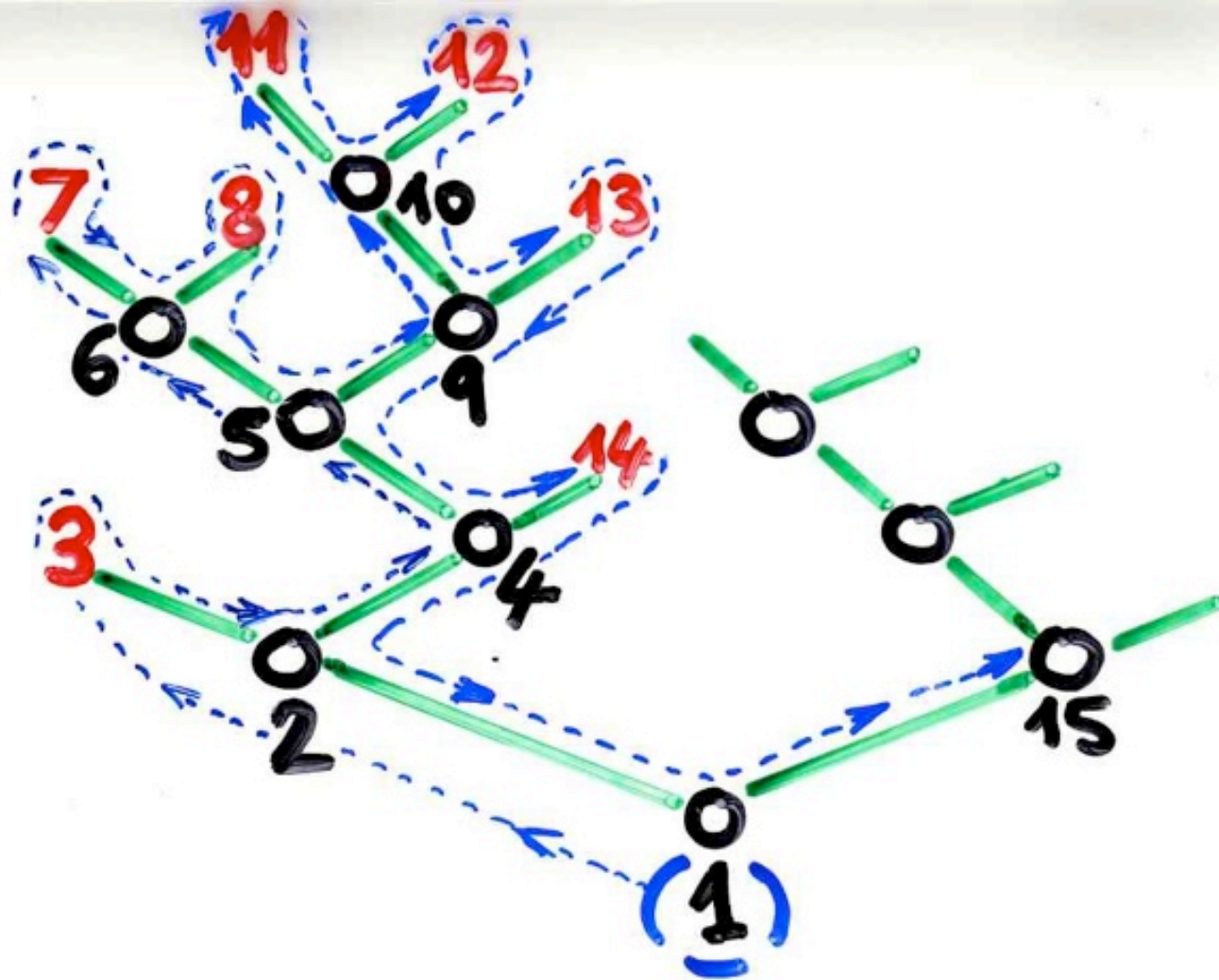


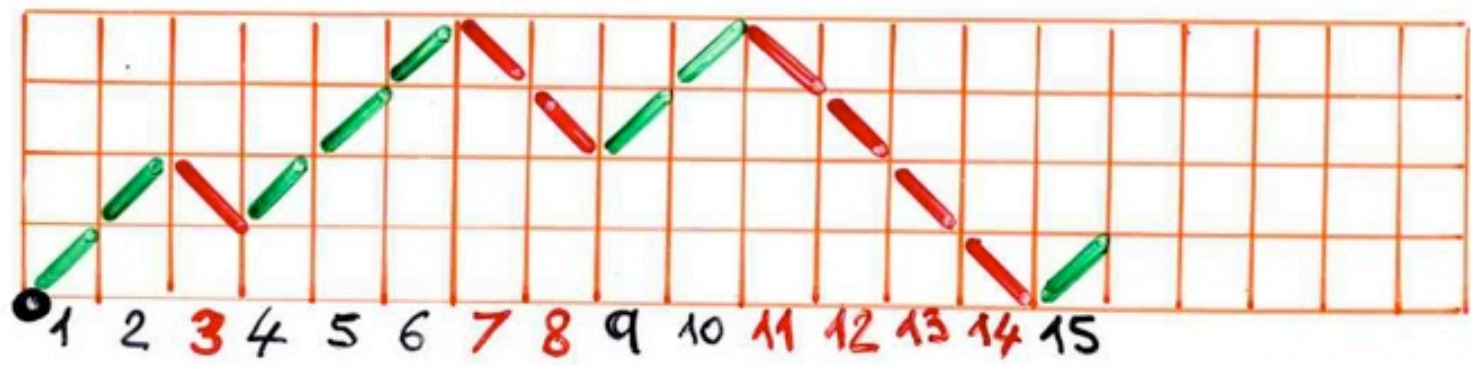
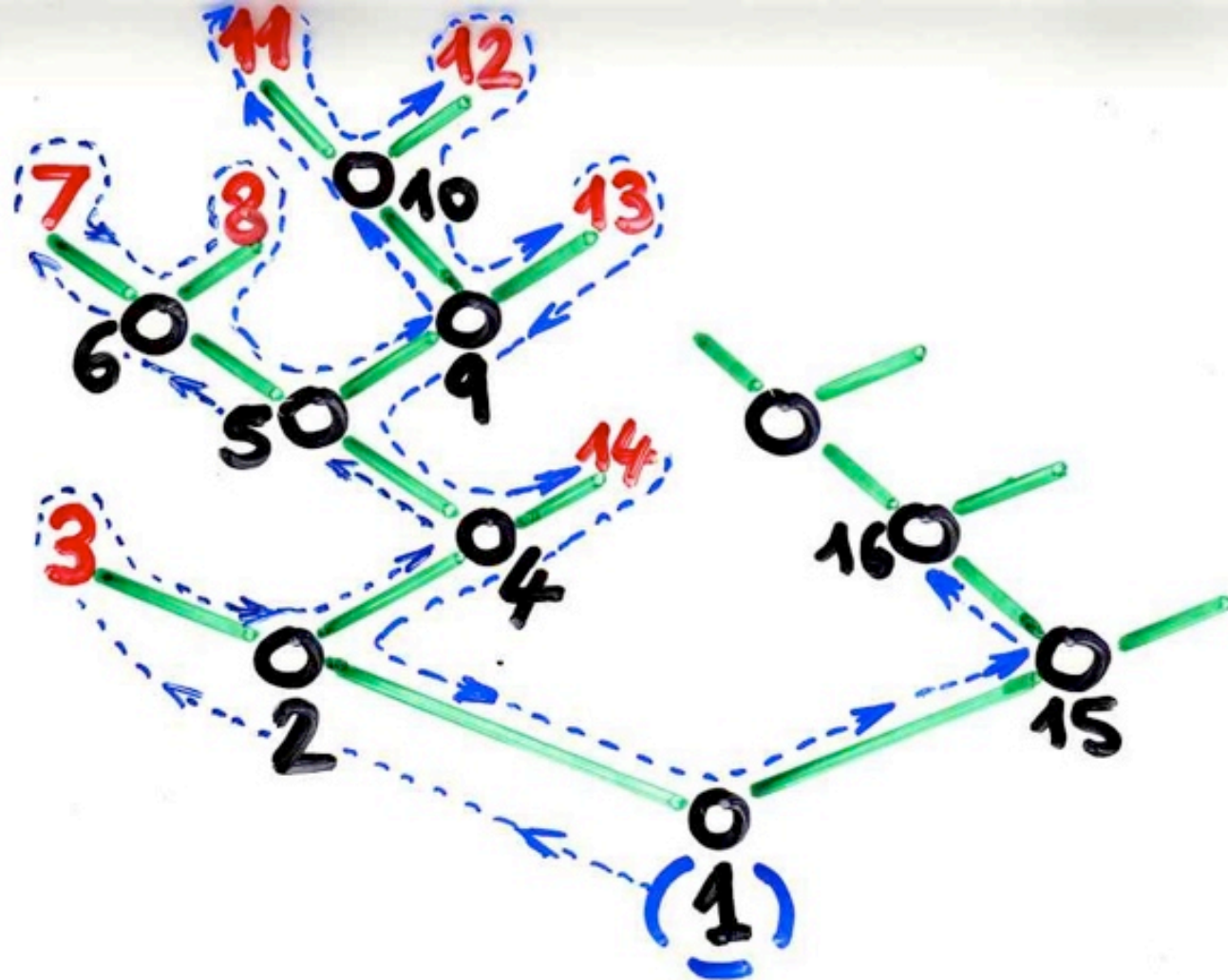


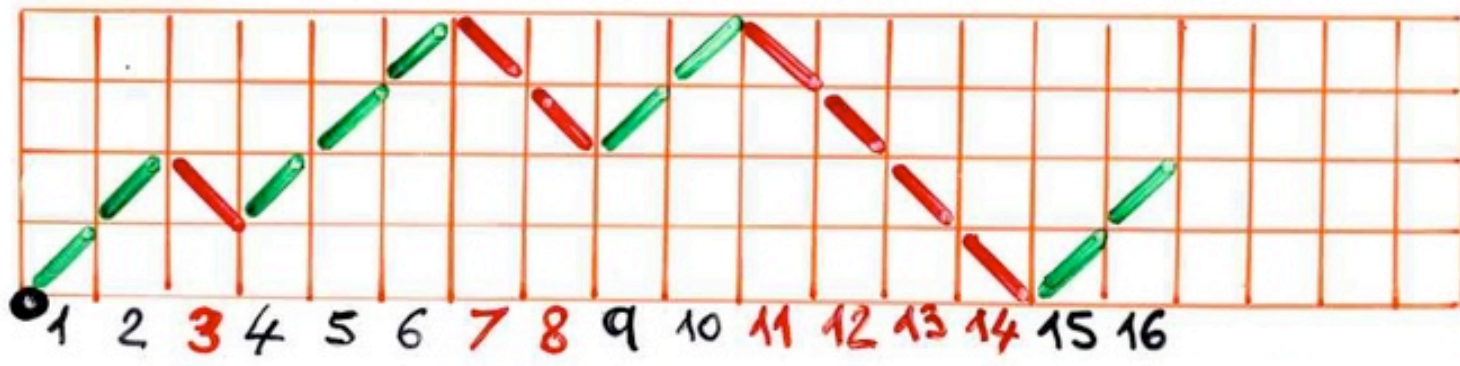
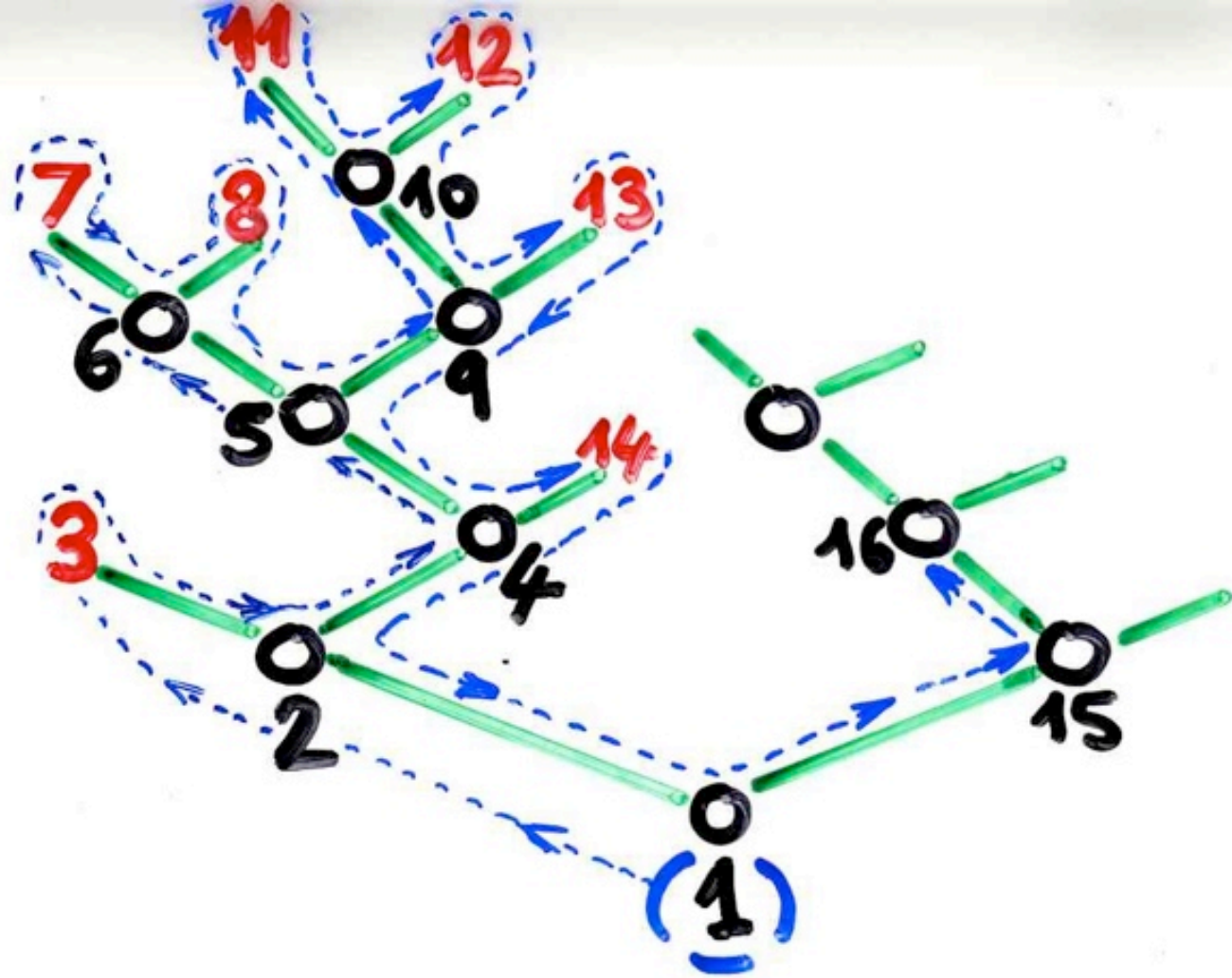


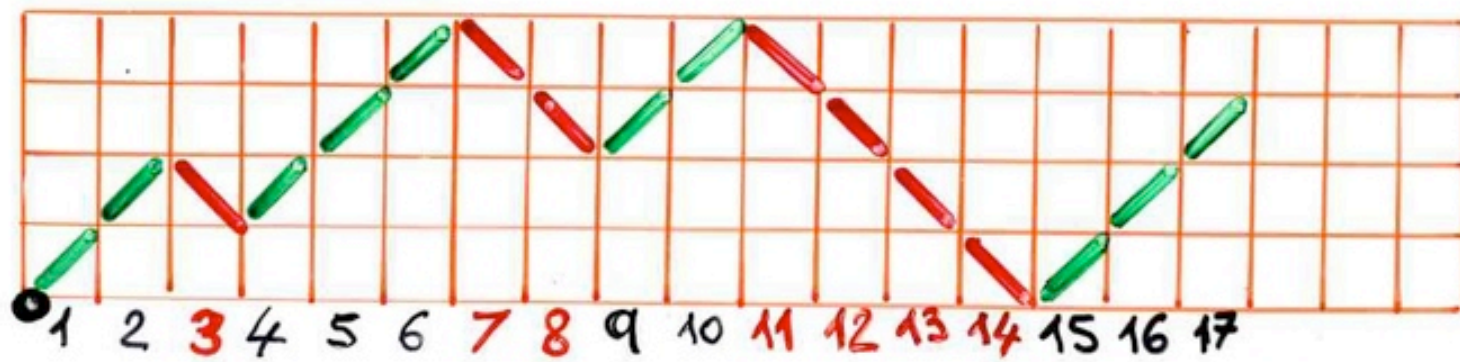
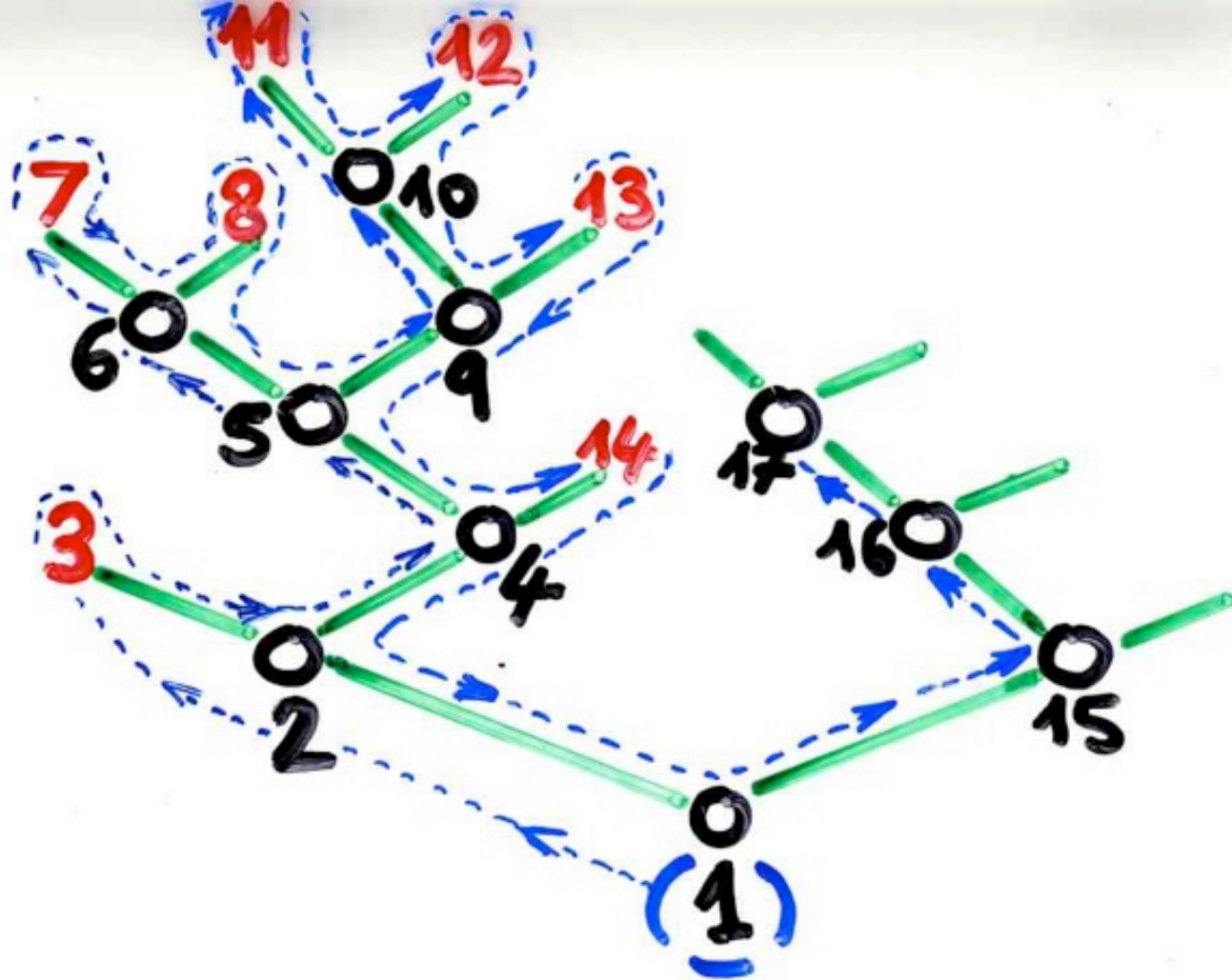


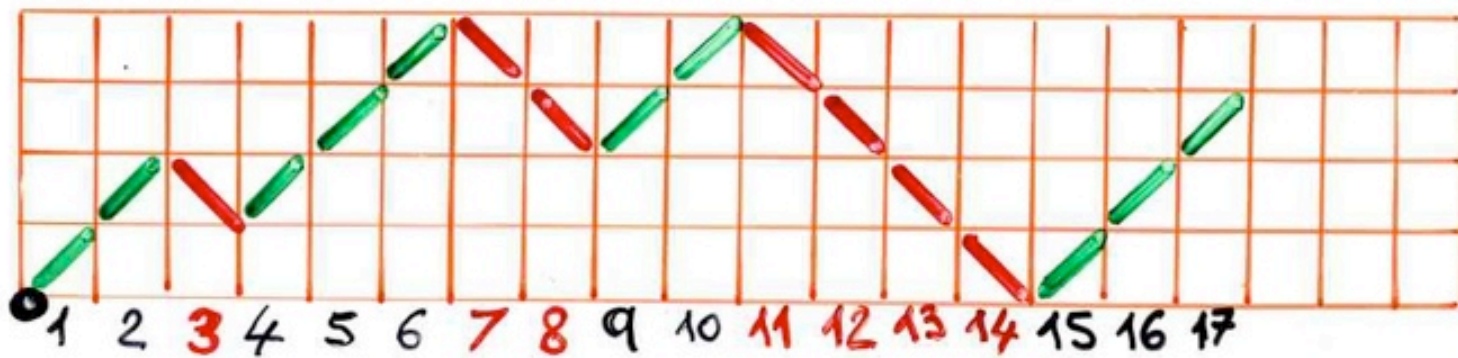
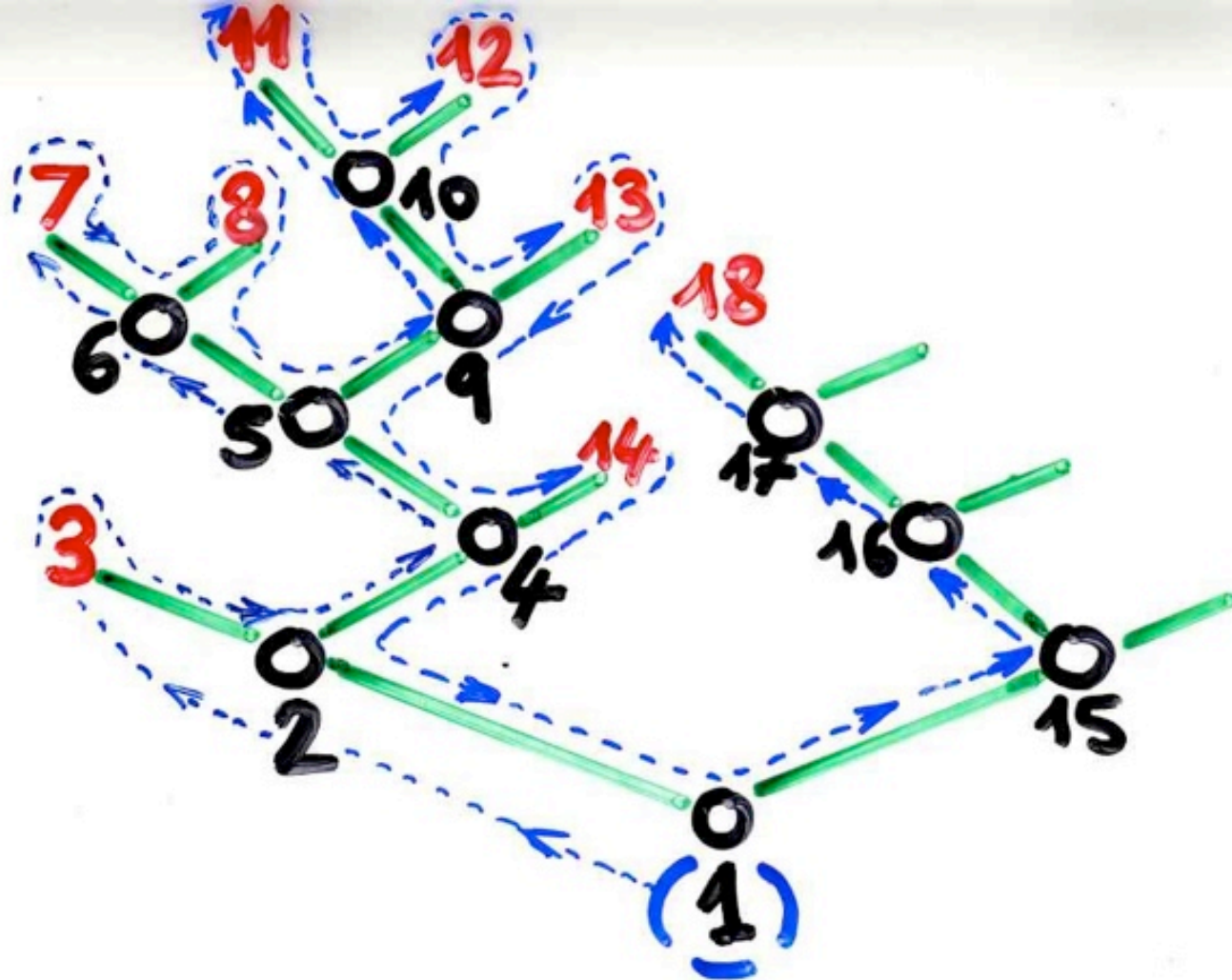


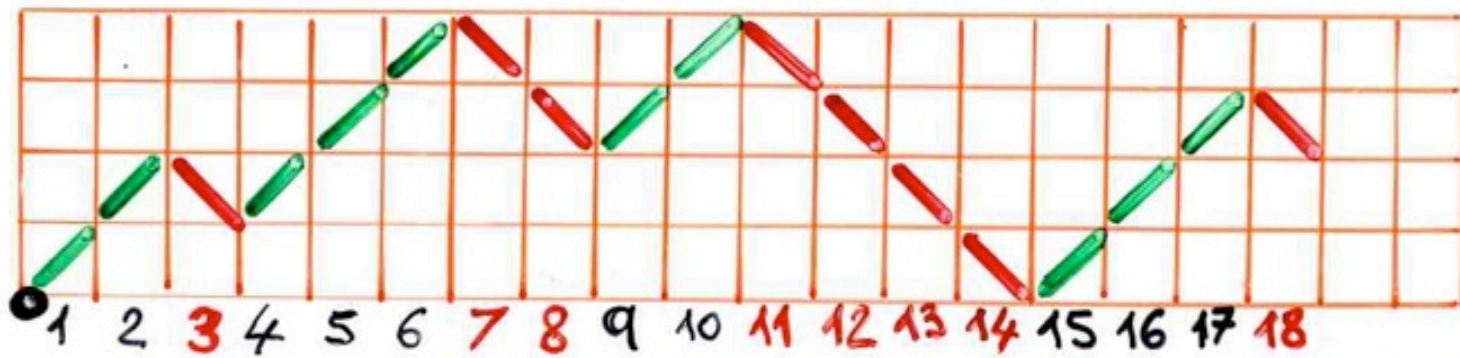
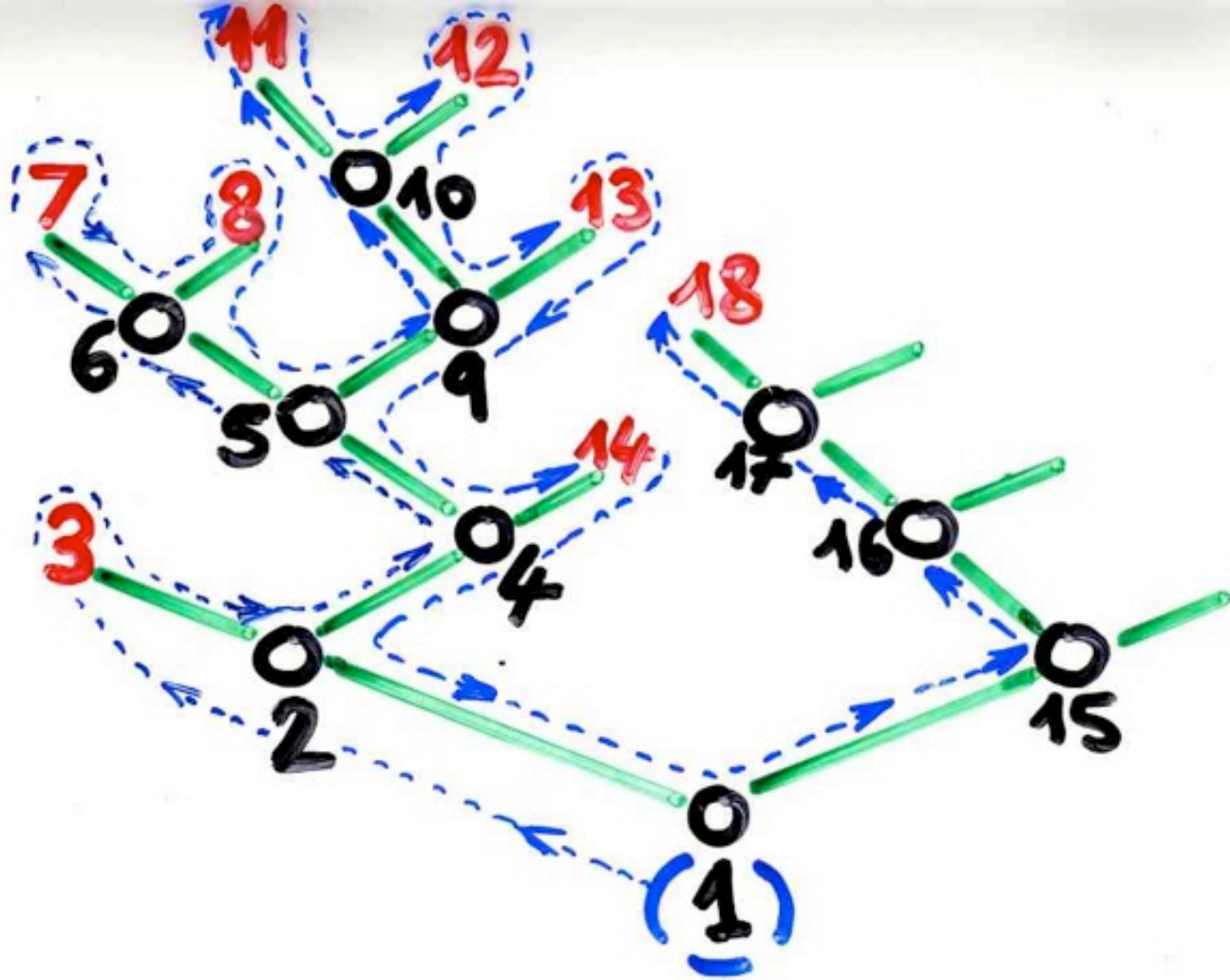


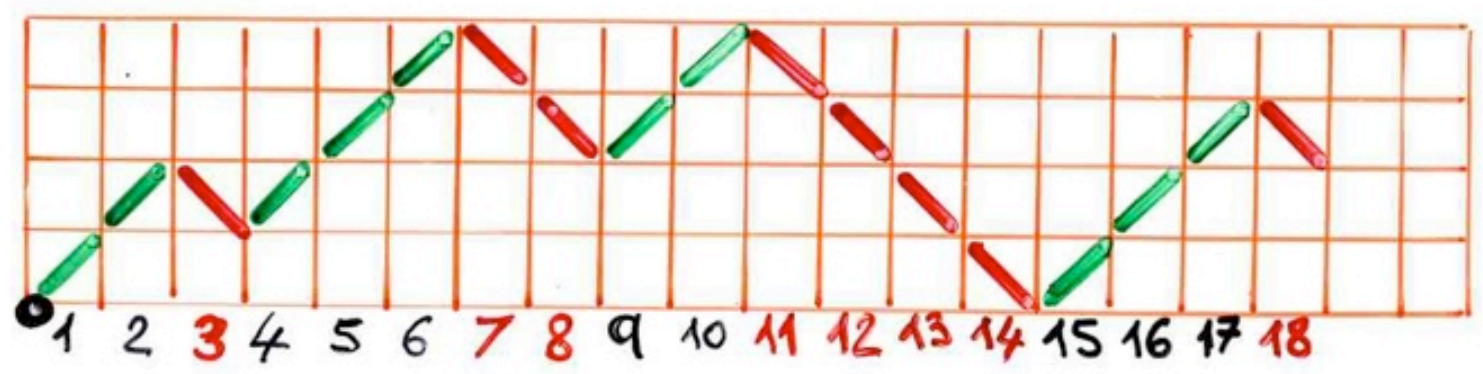
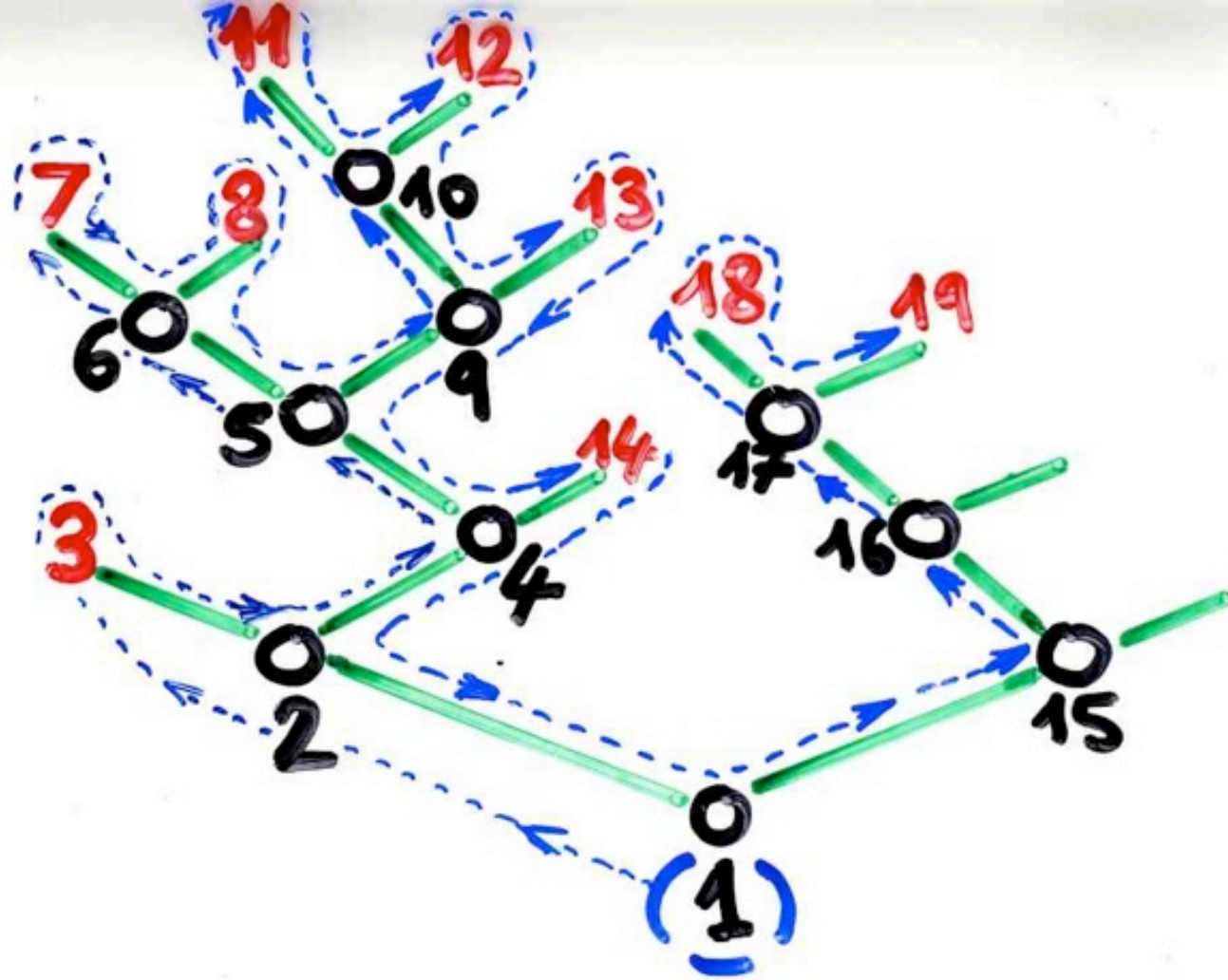


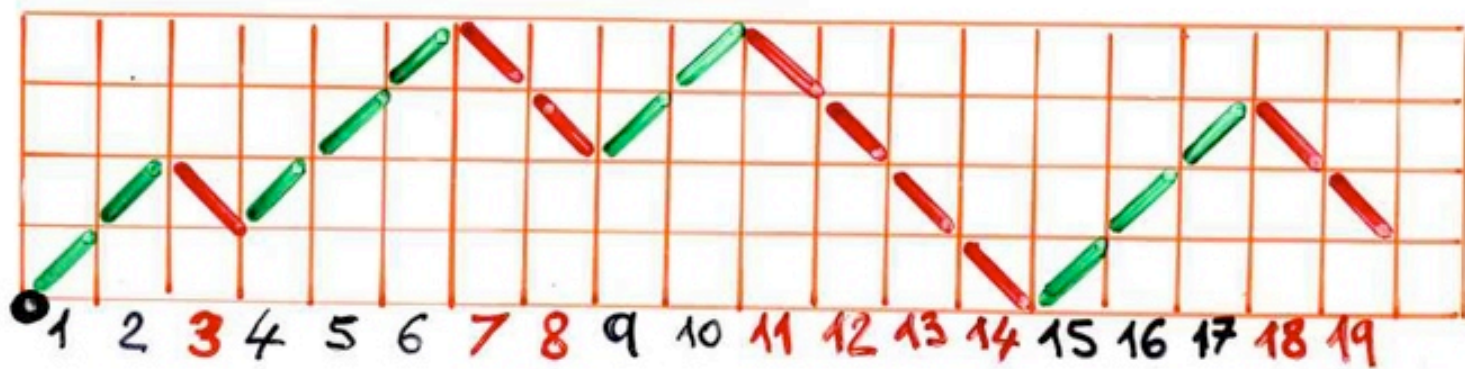
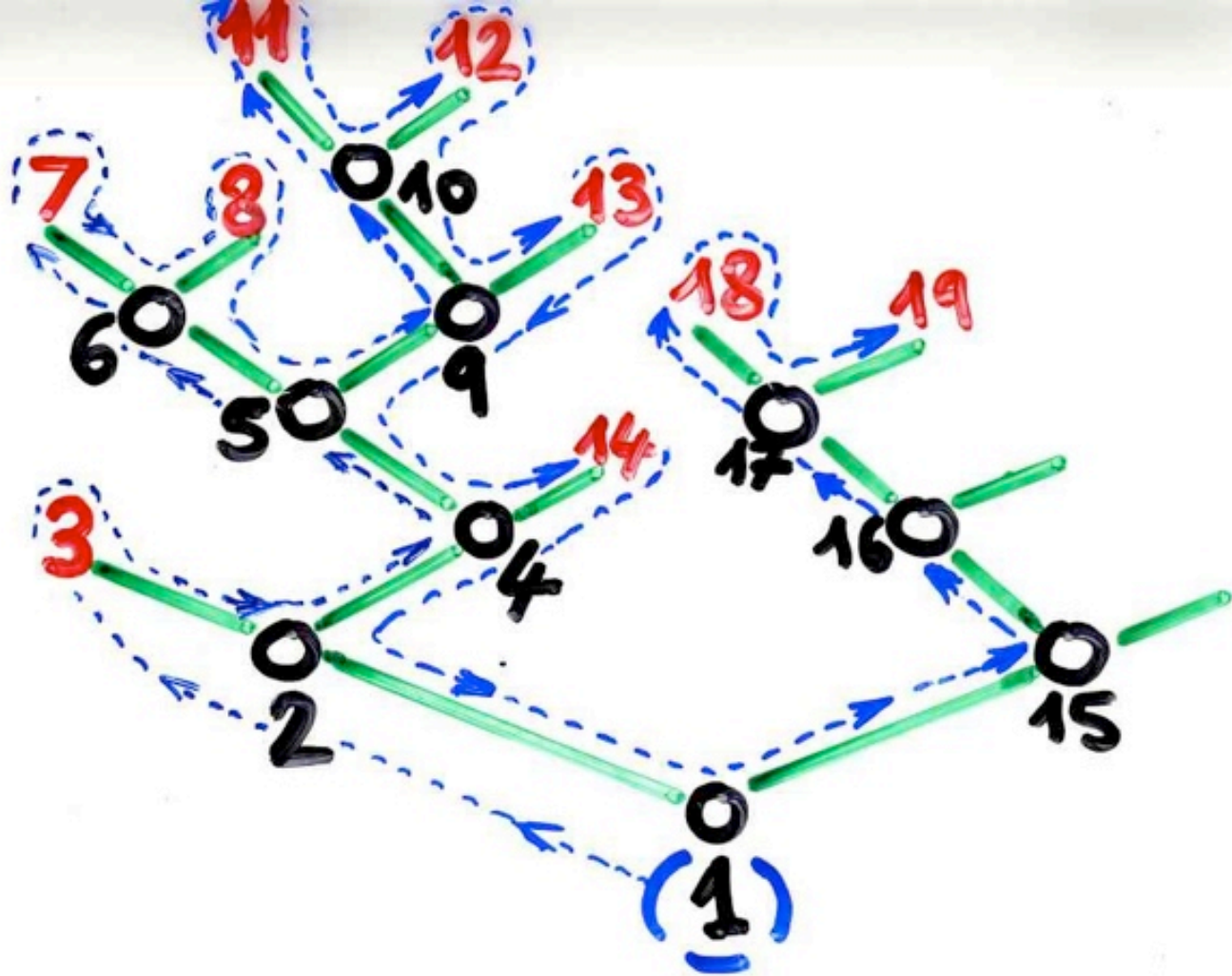


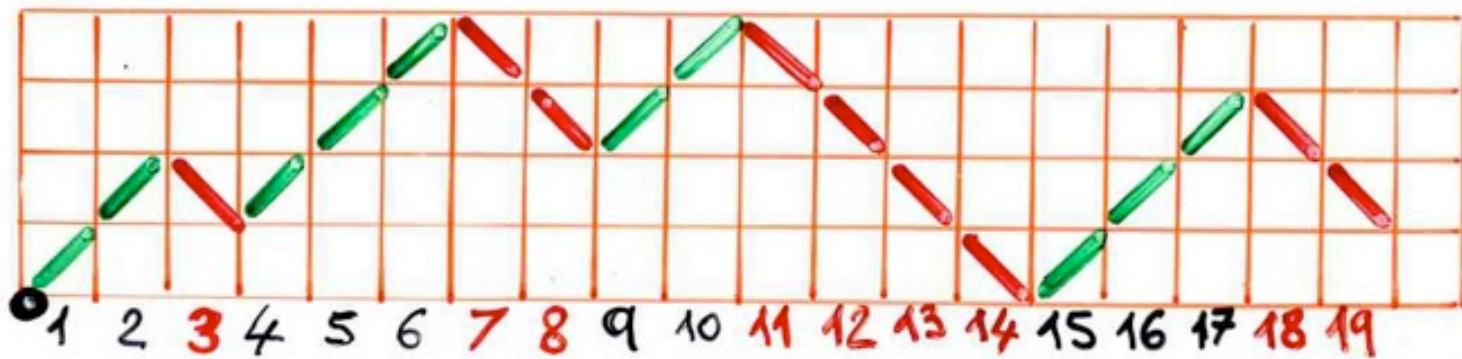
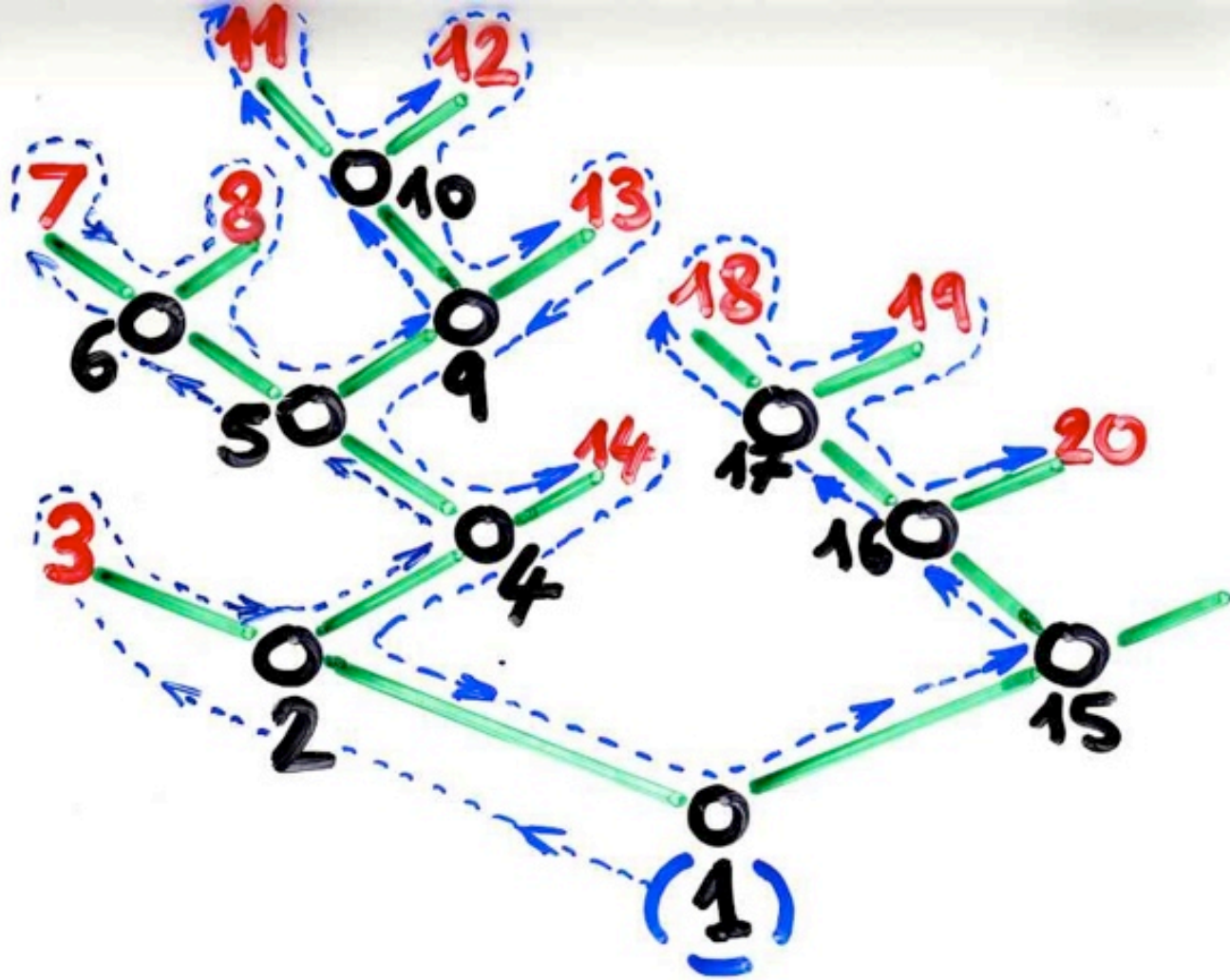


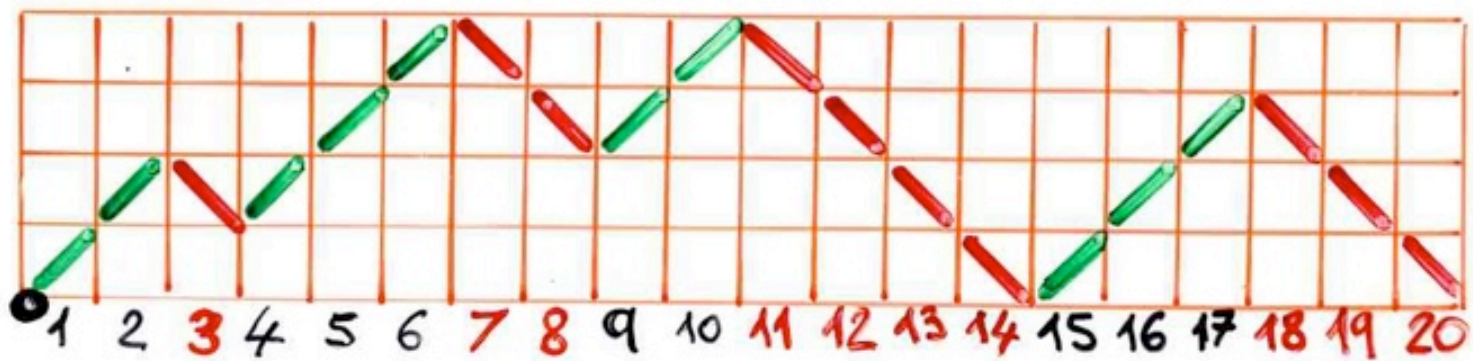
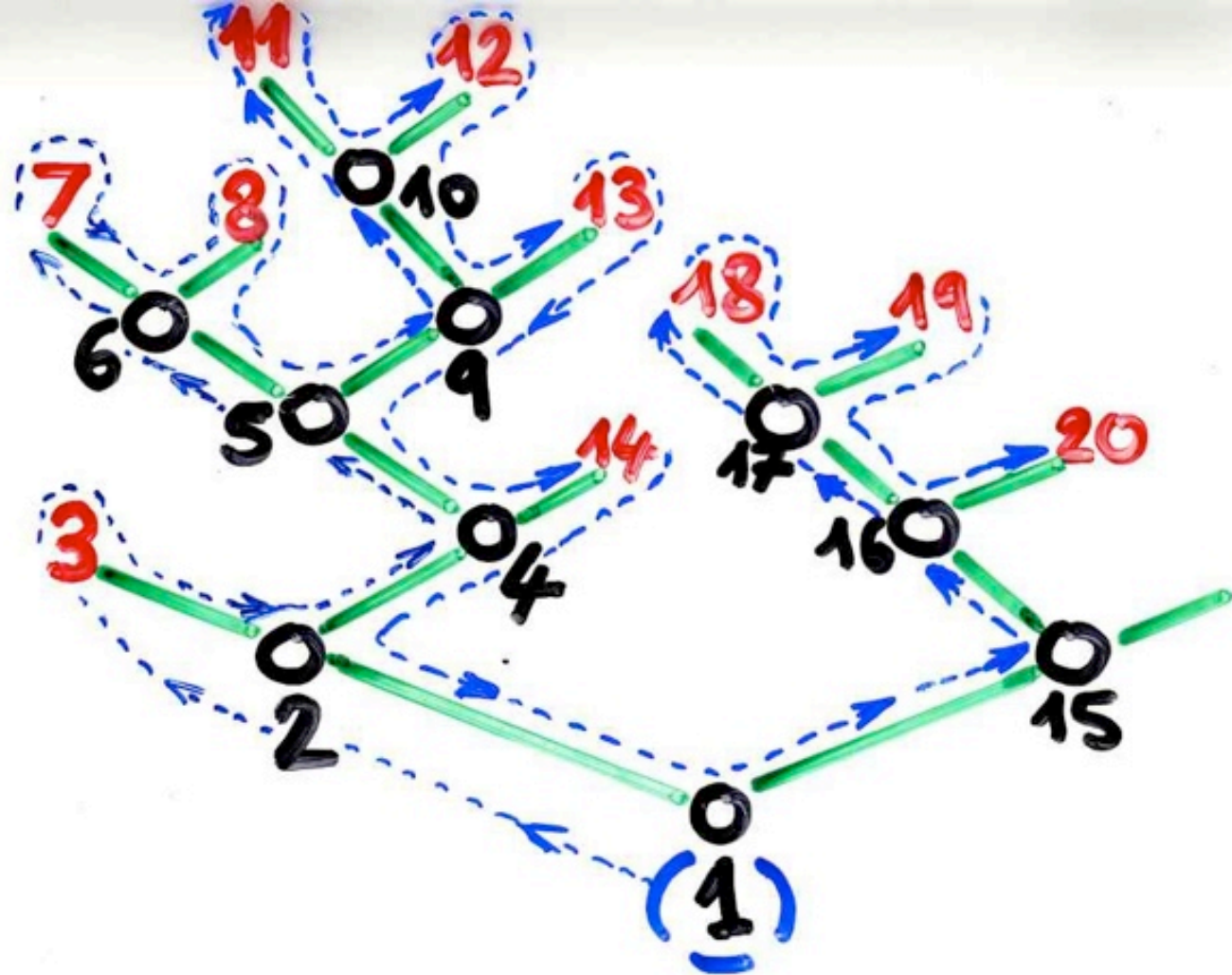


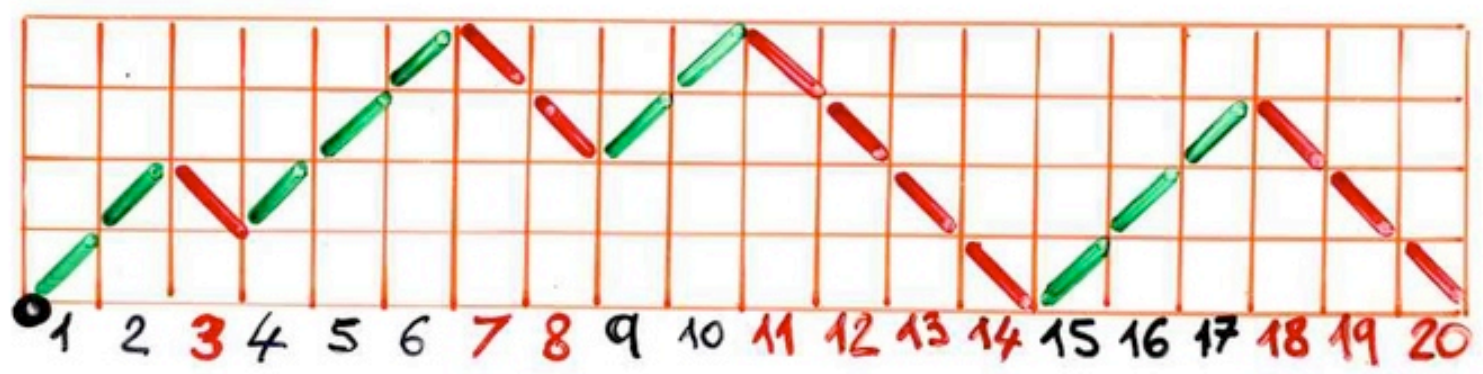
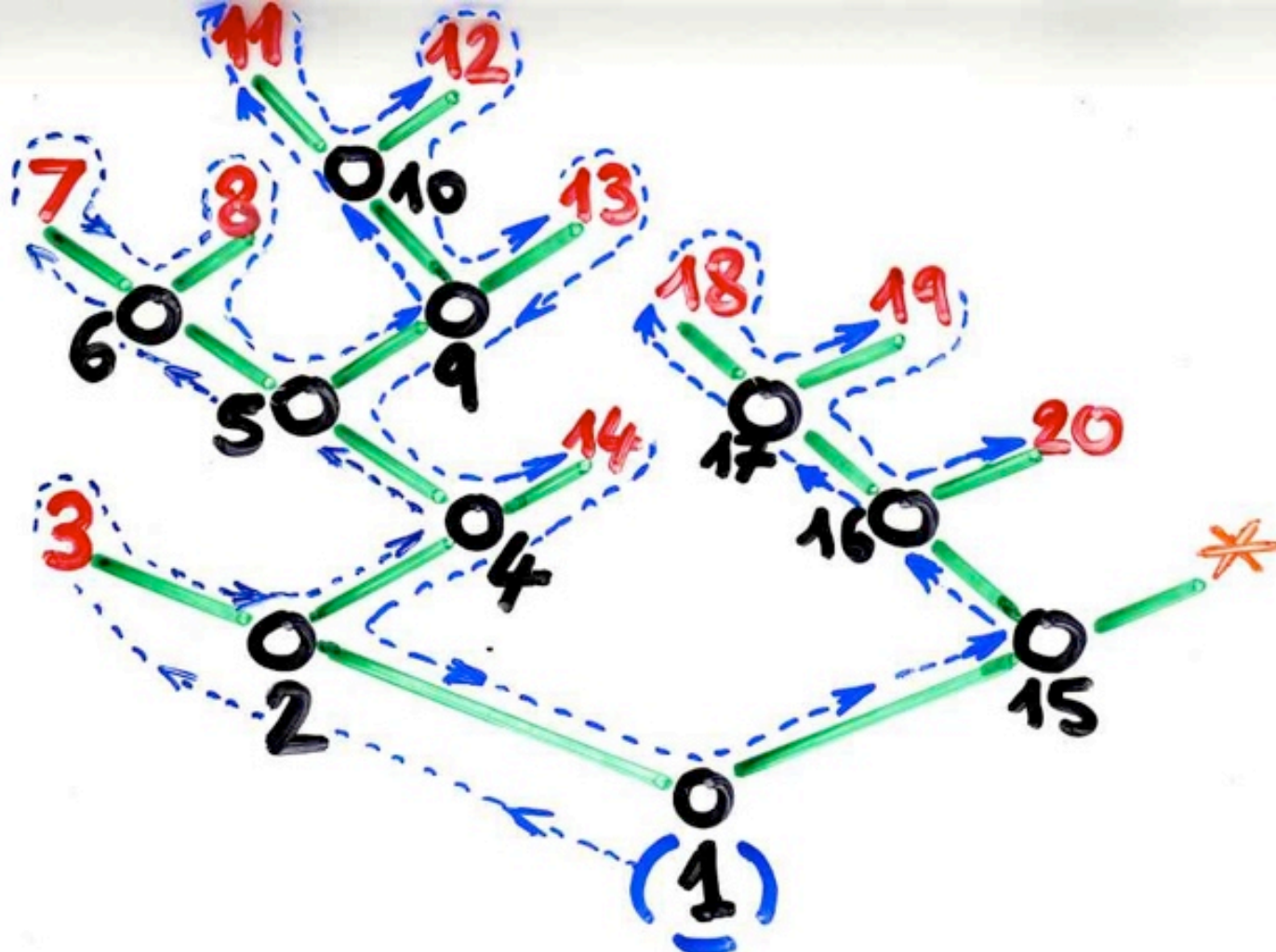




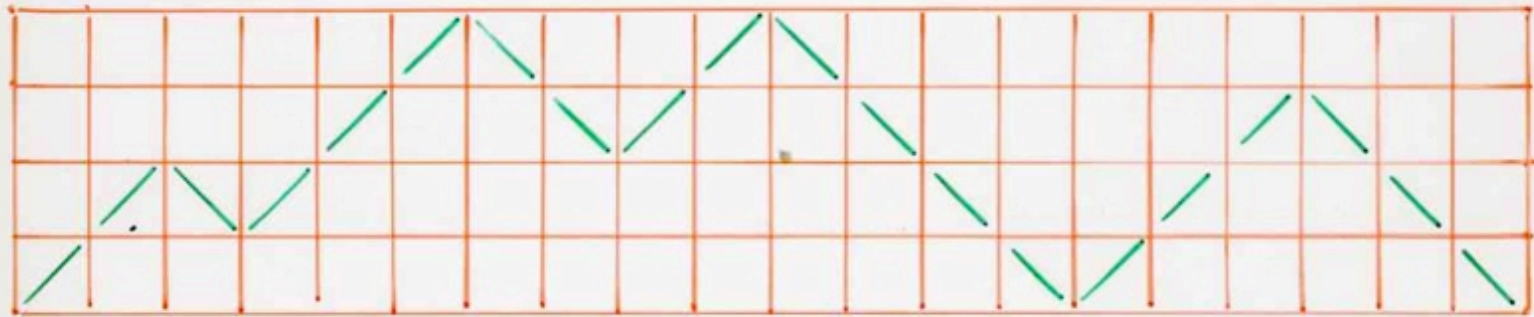


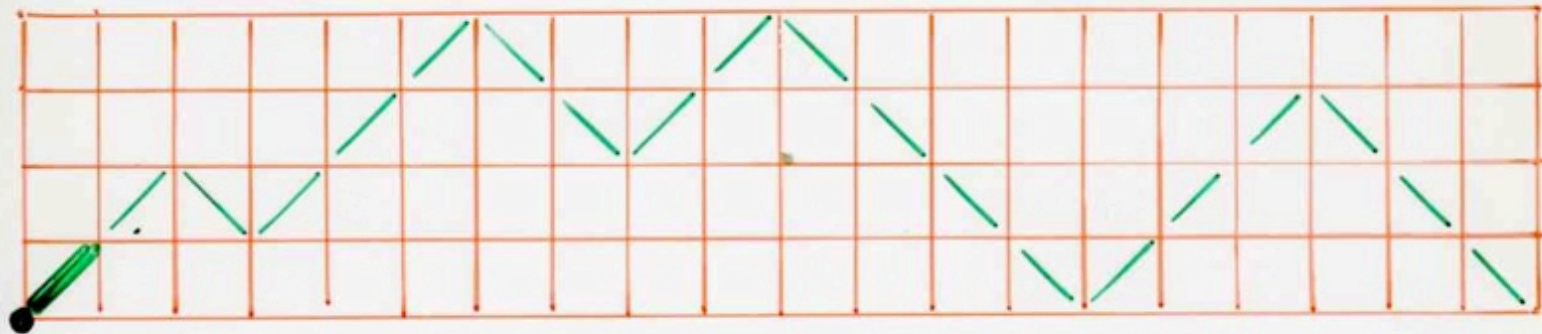


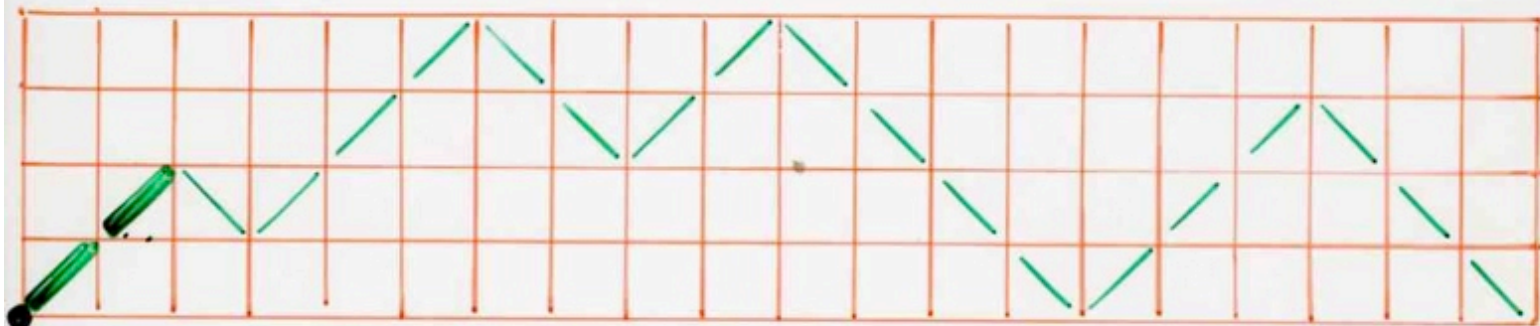
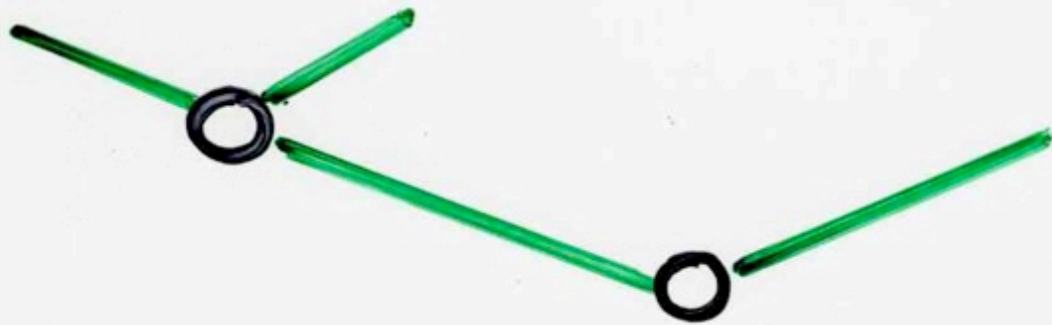


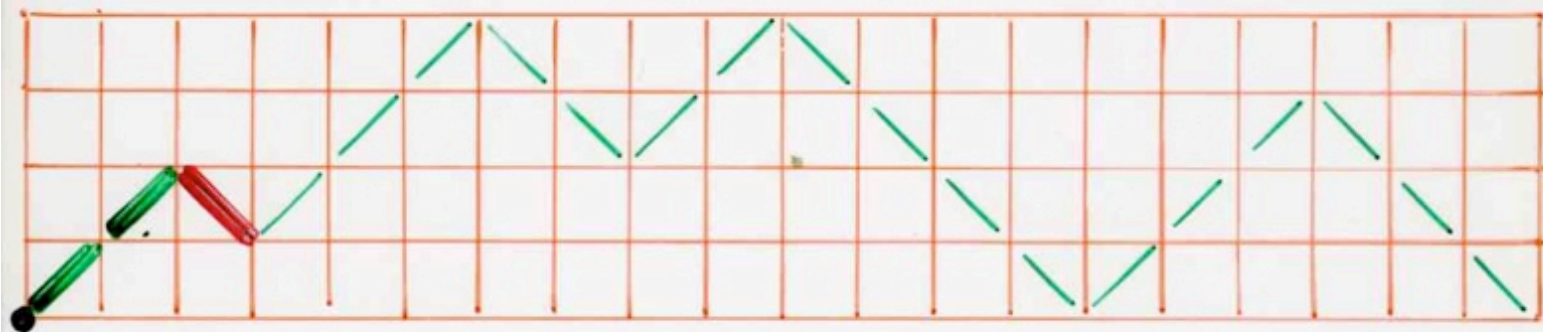
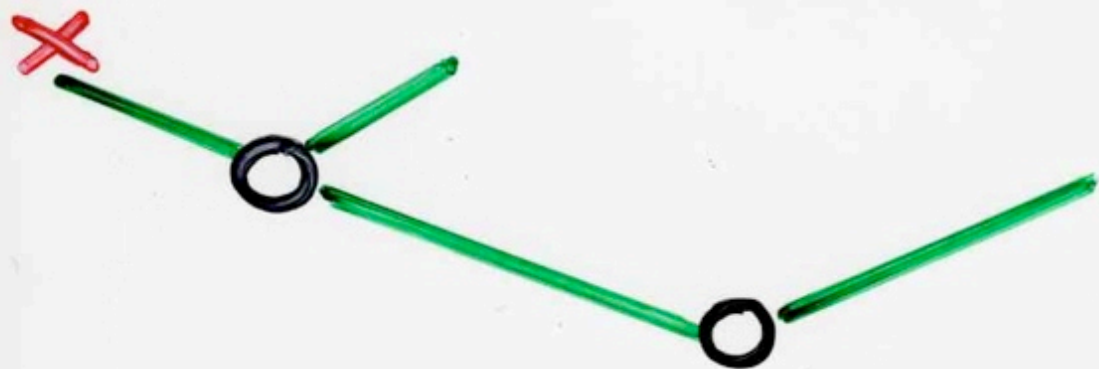


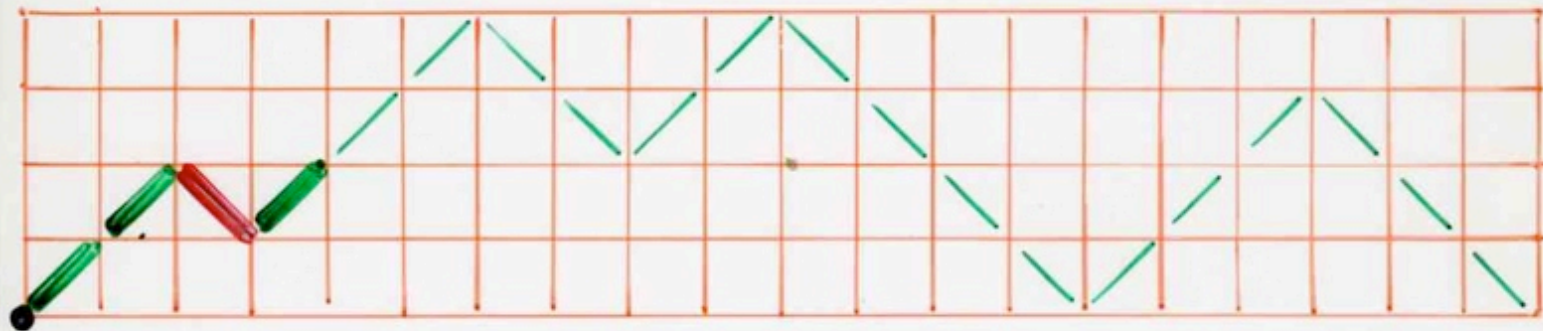
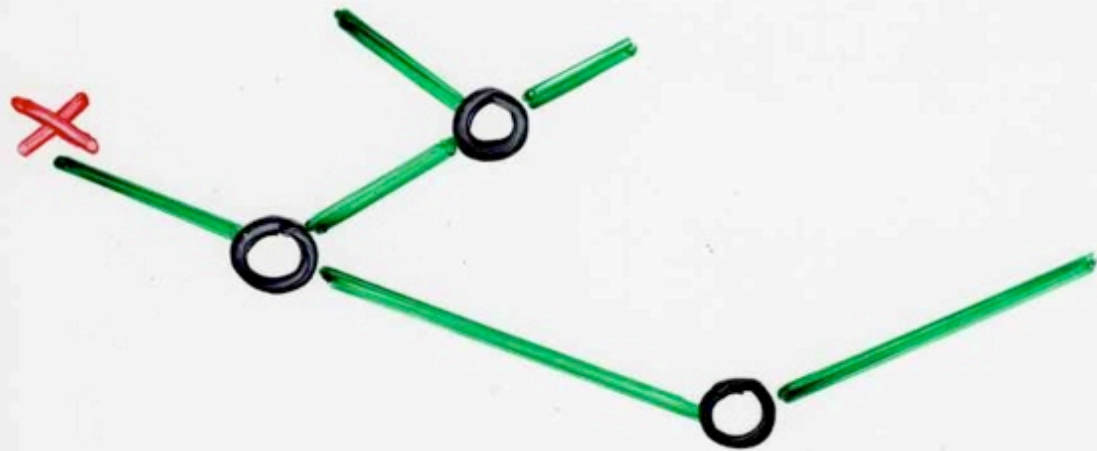
bijection inverse

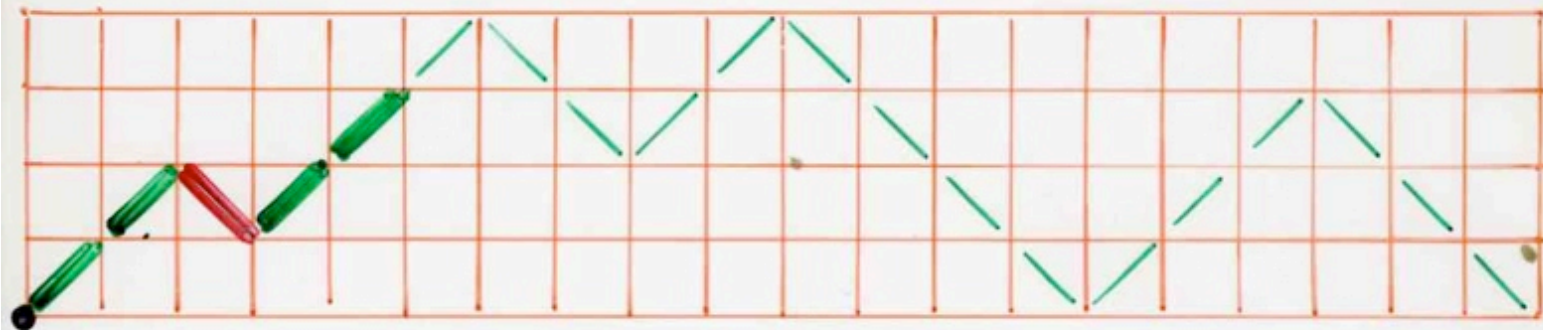
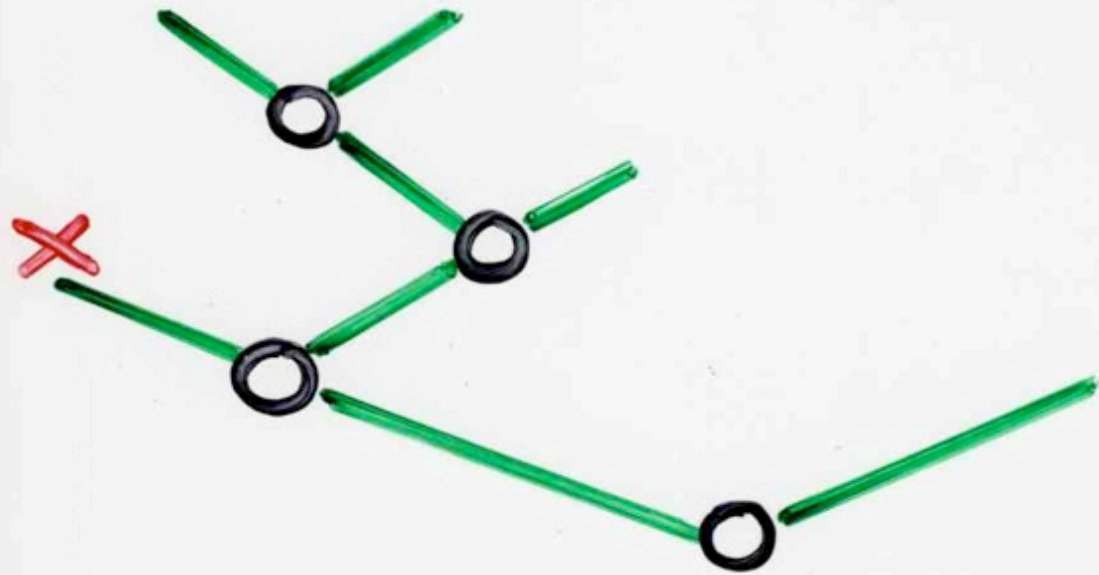


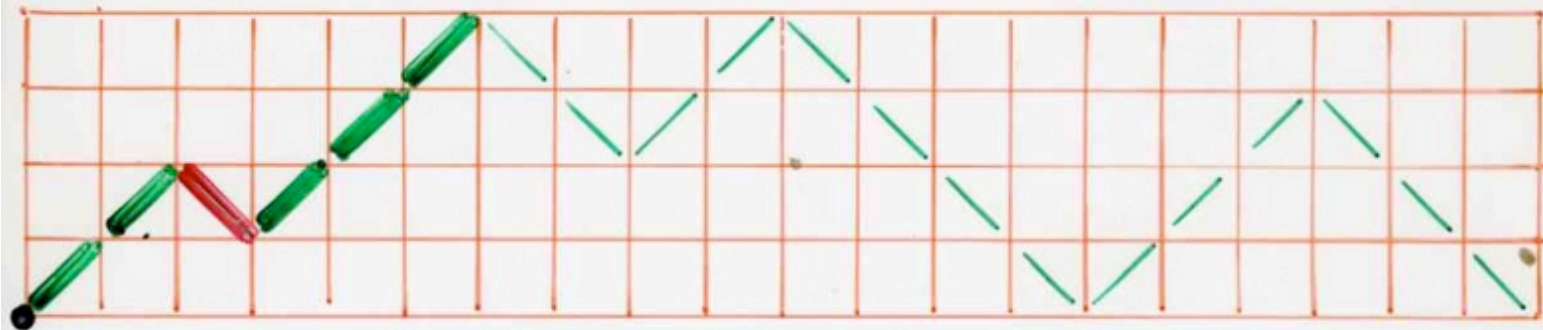
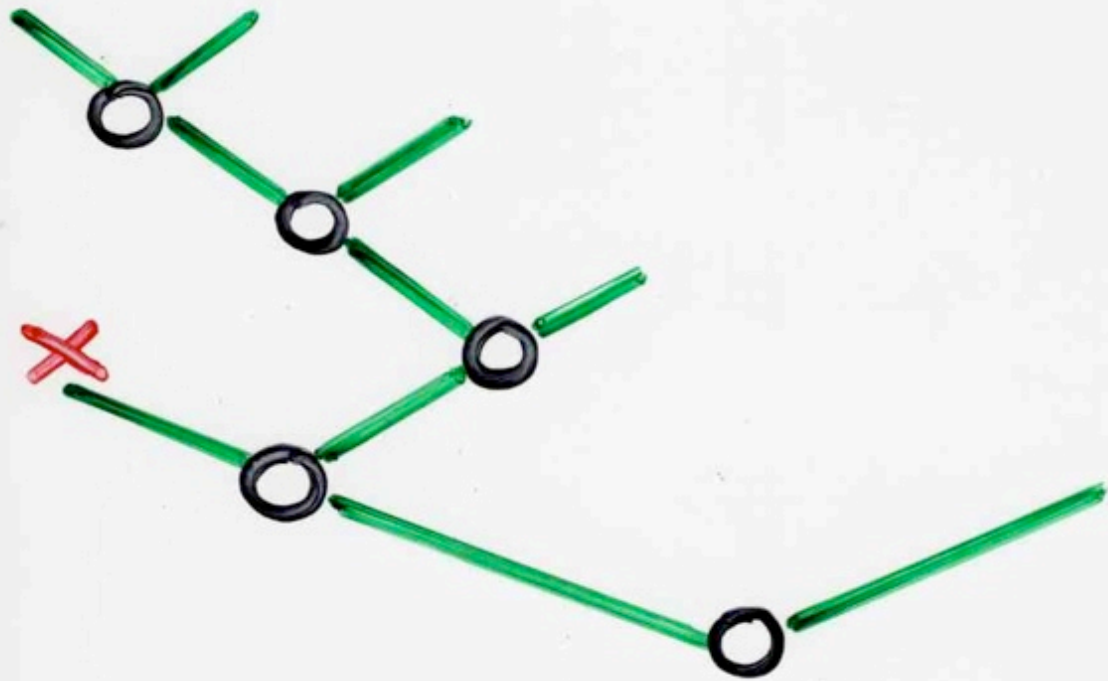


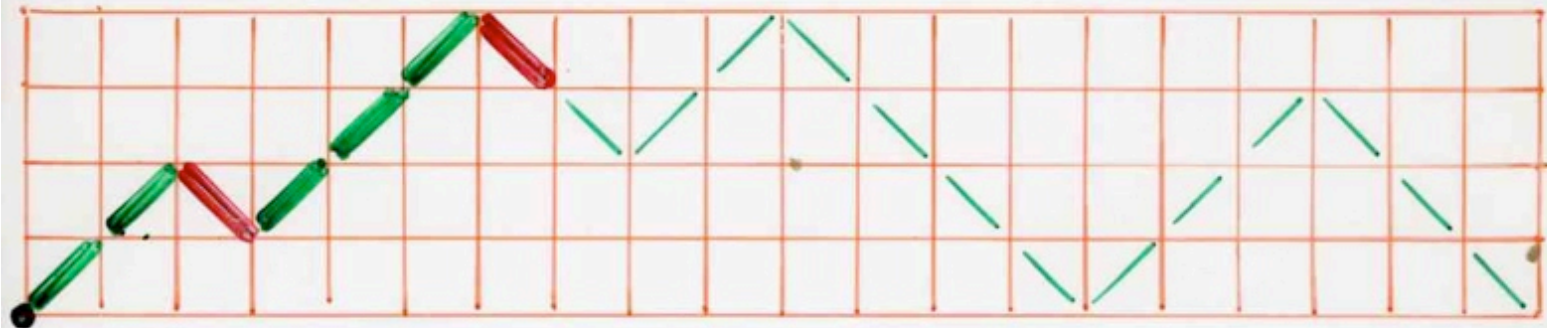
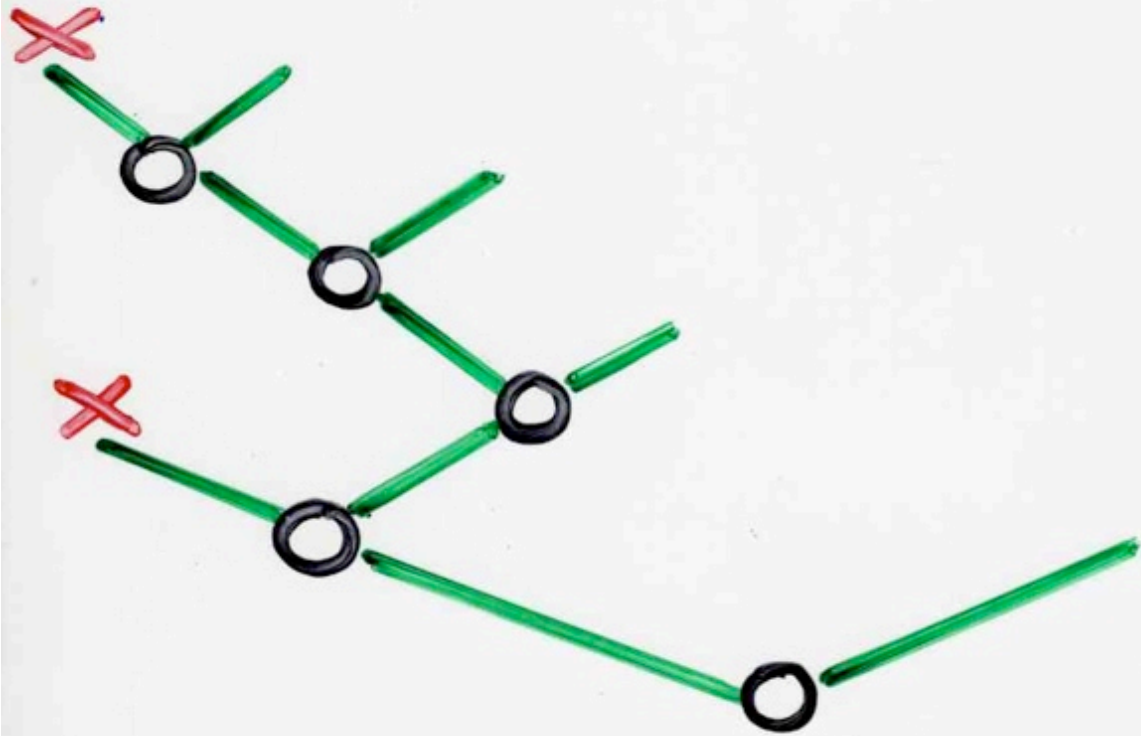


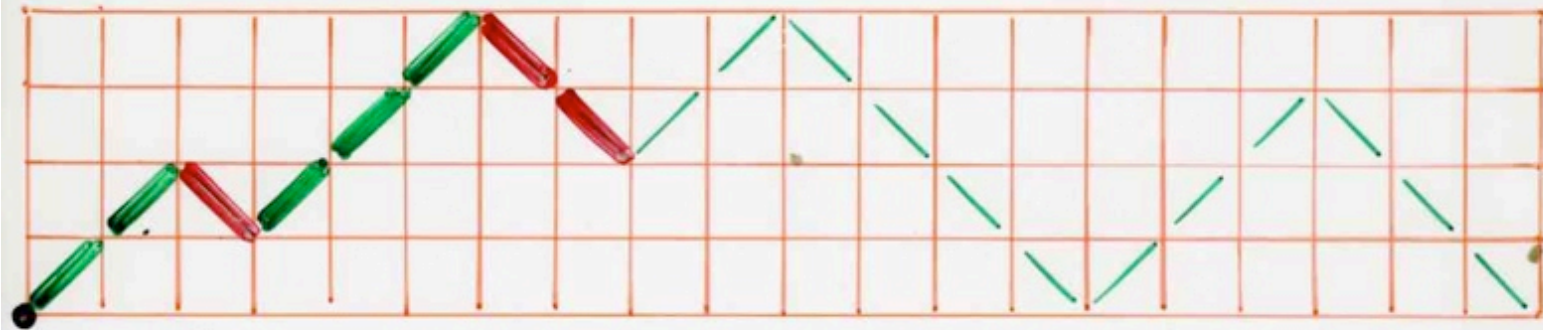
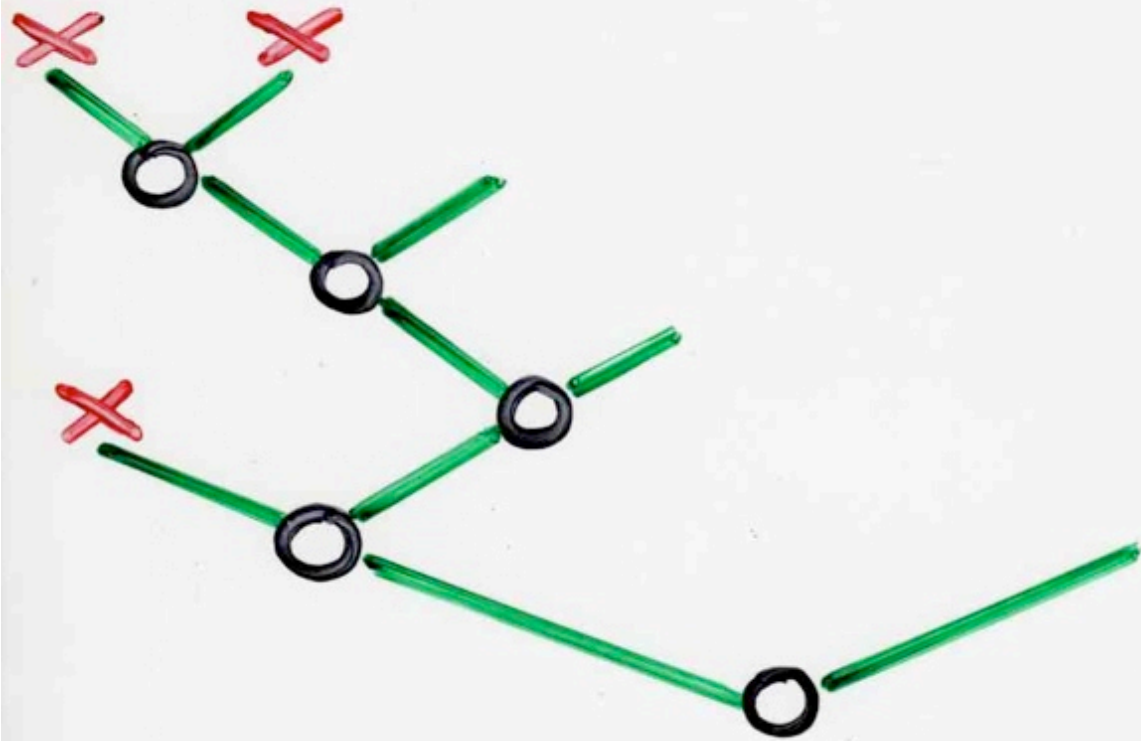


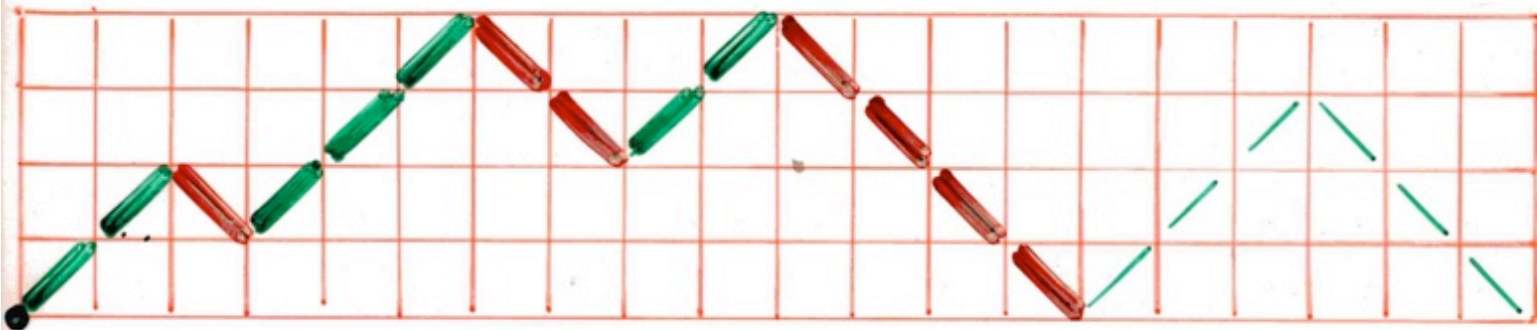
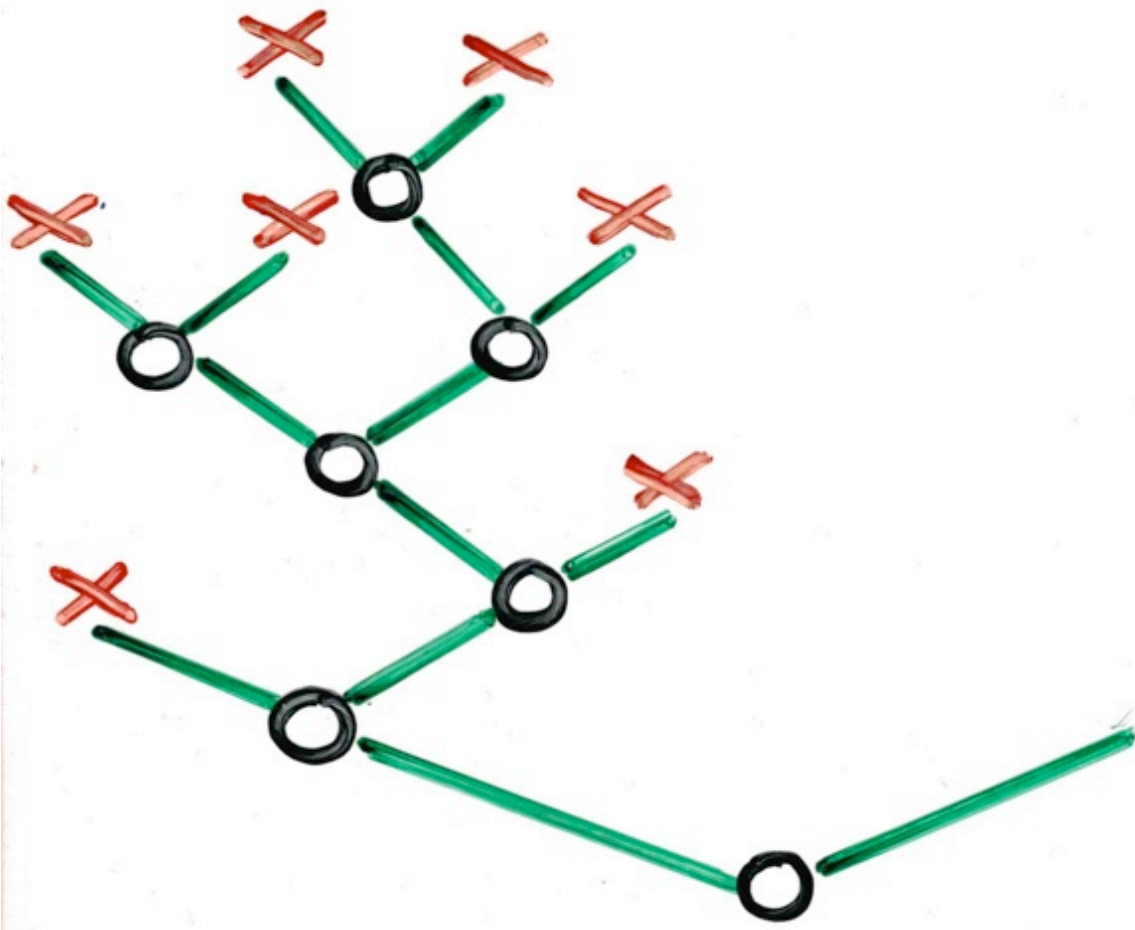


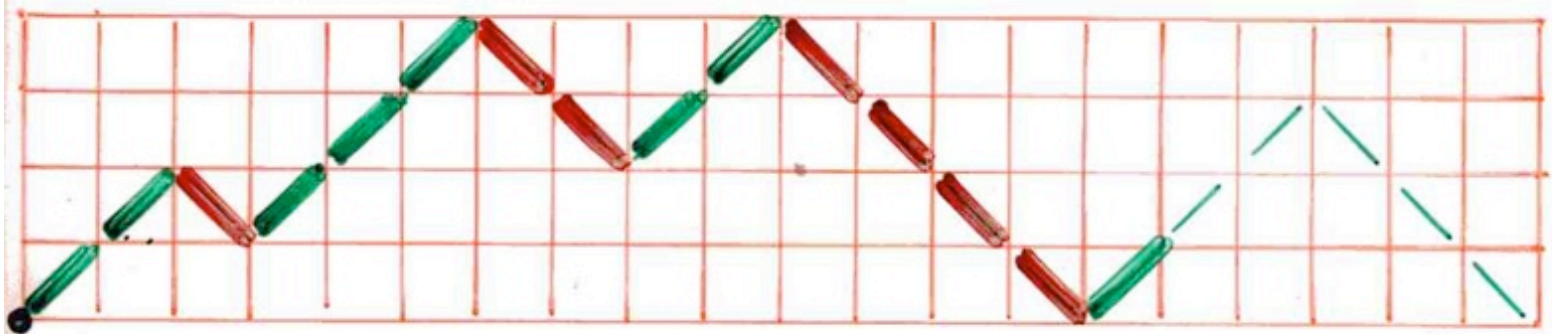
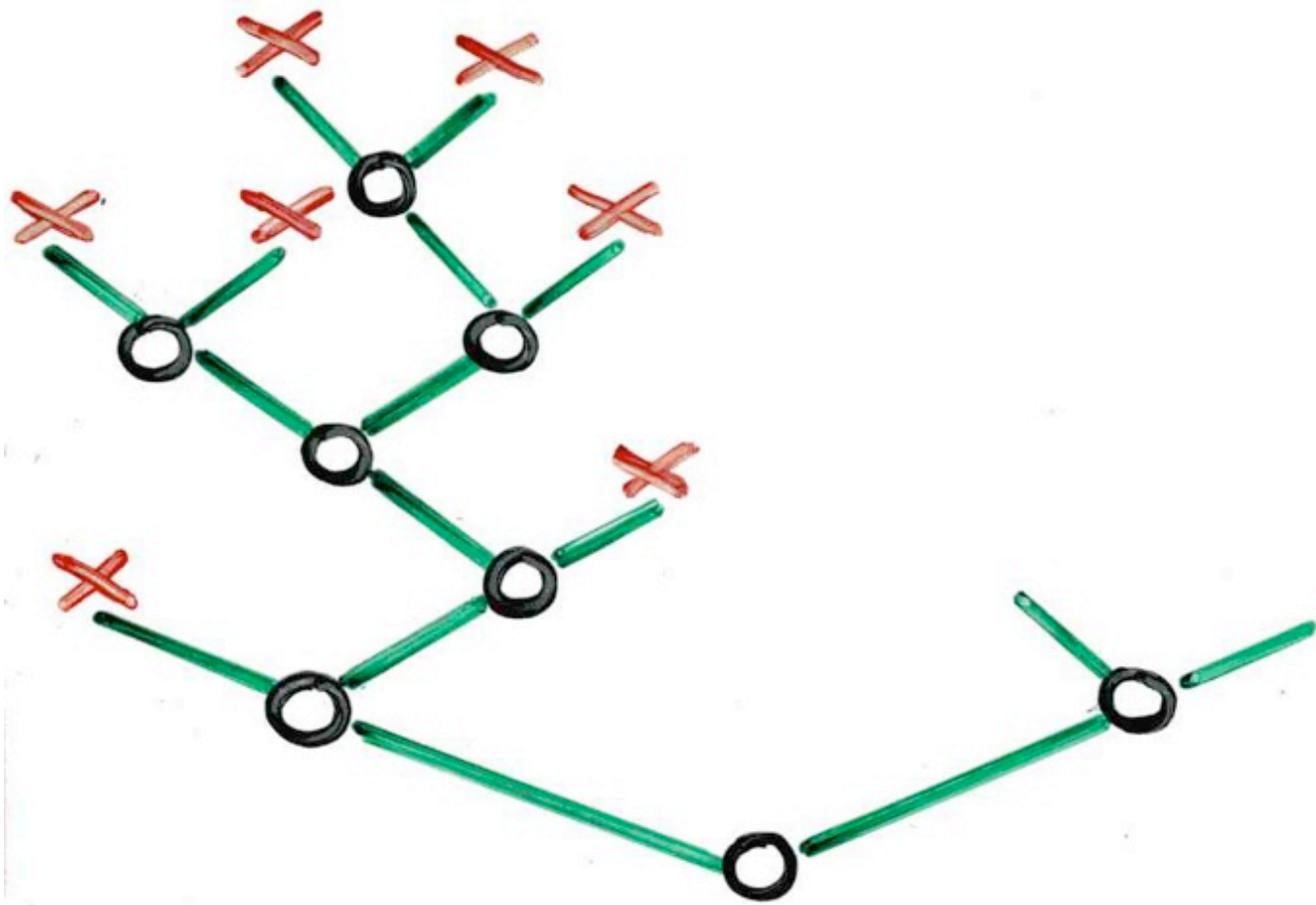


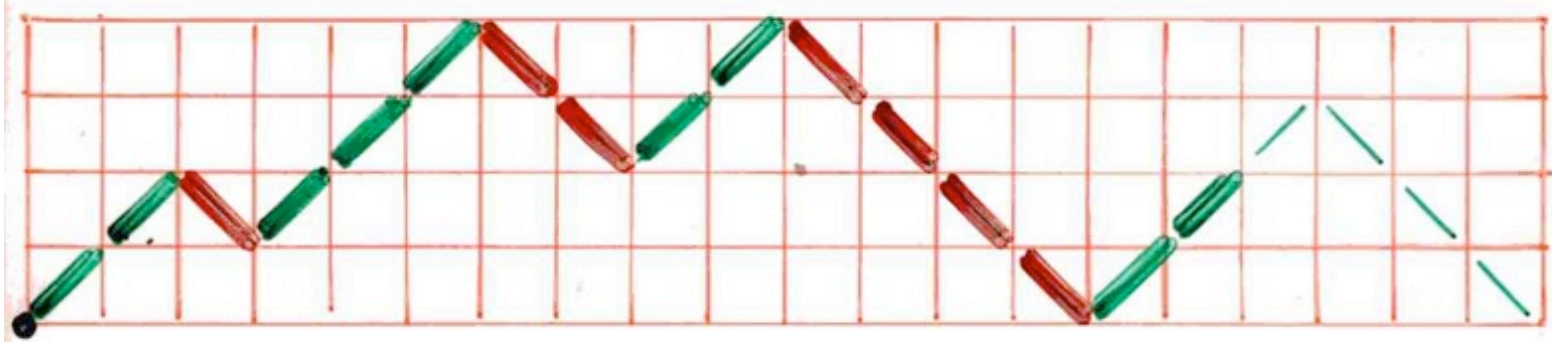
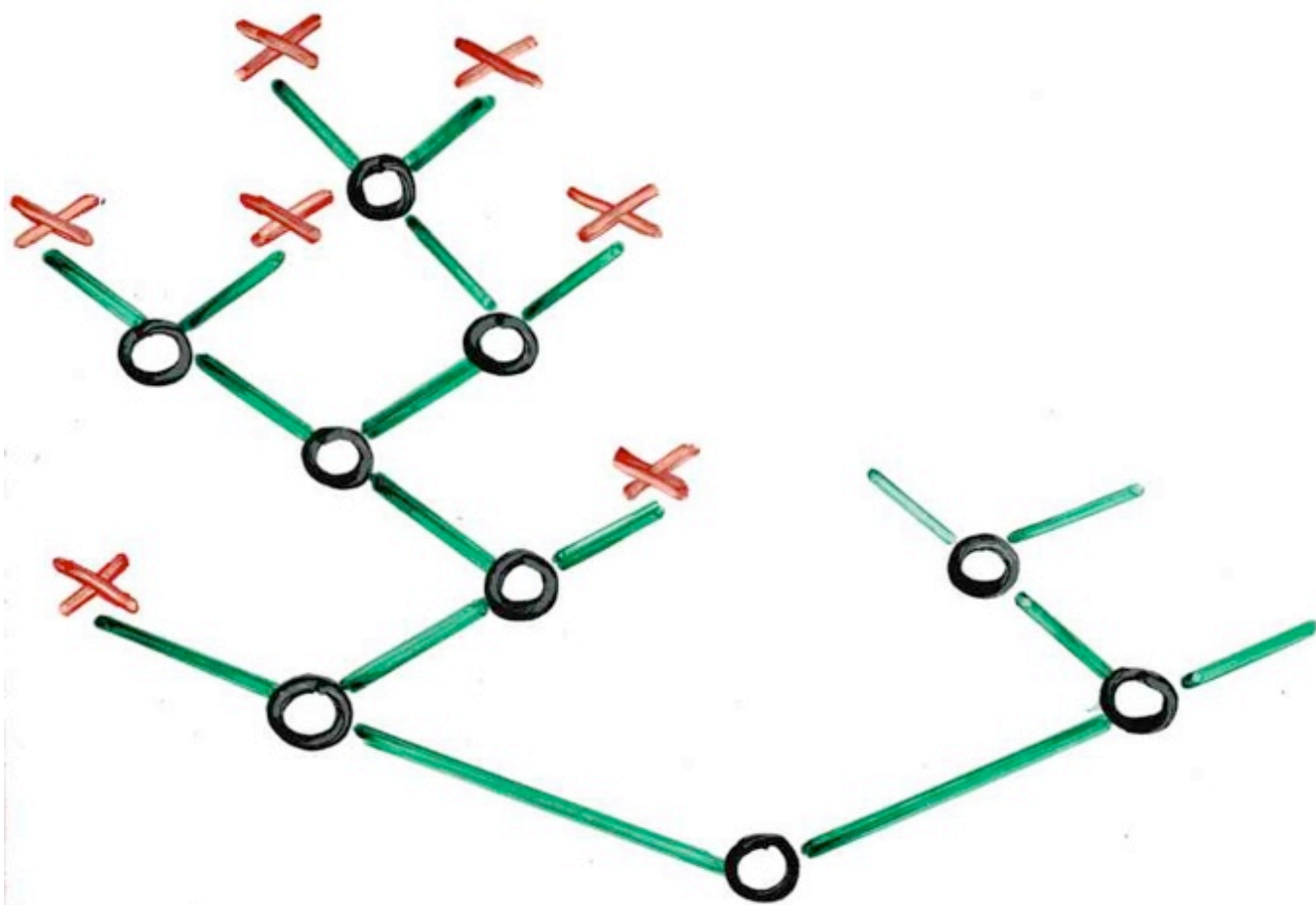


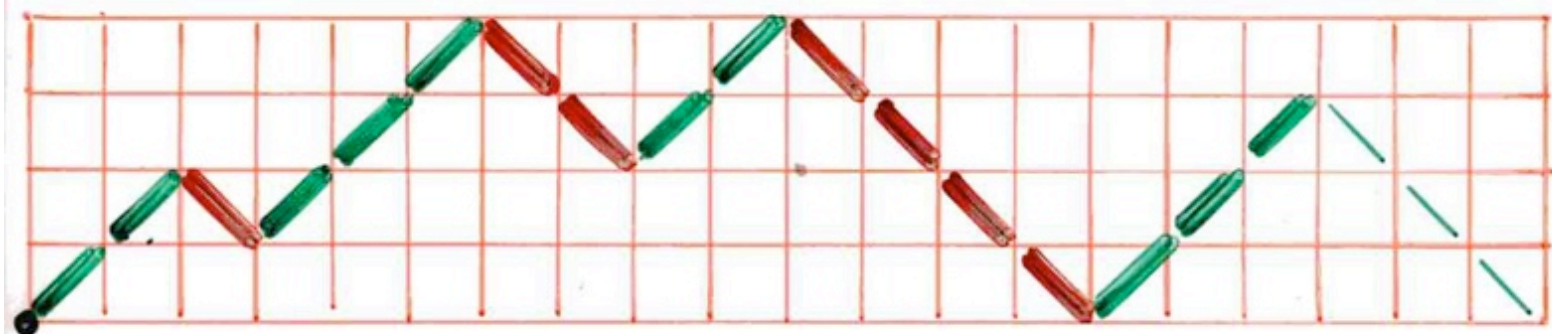
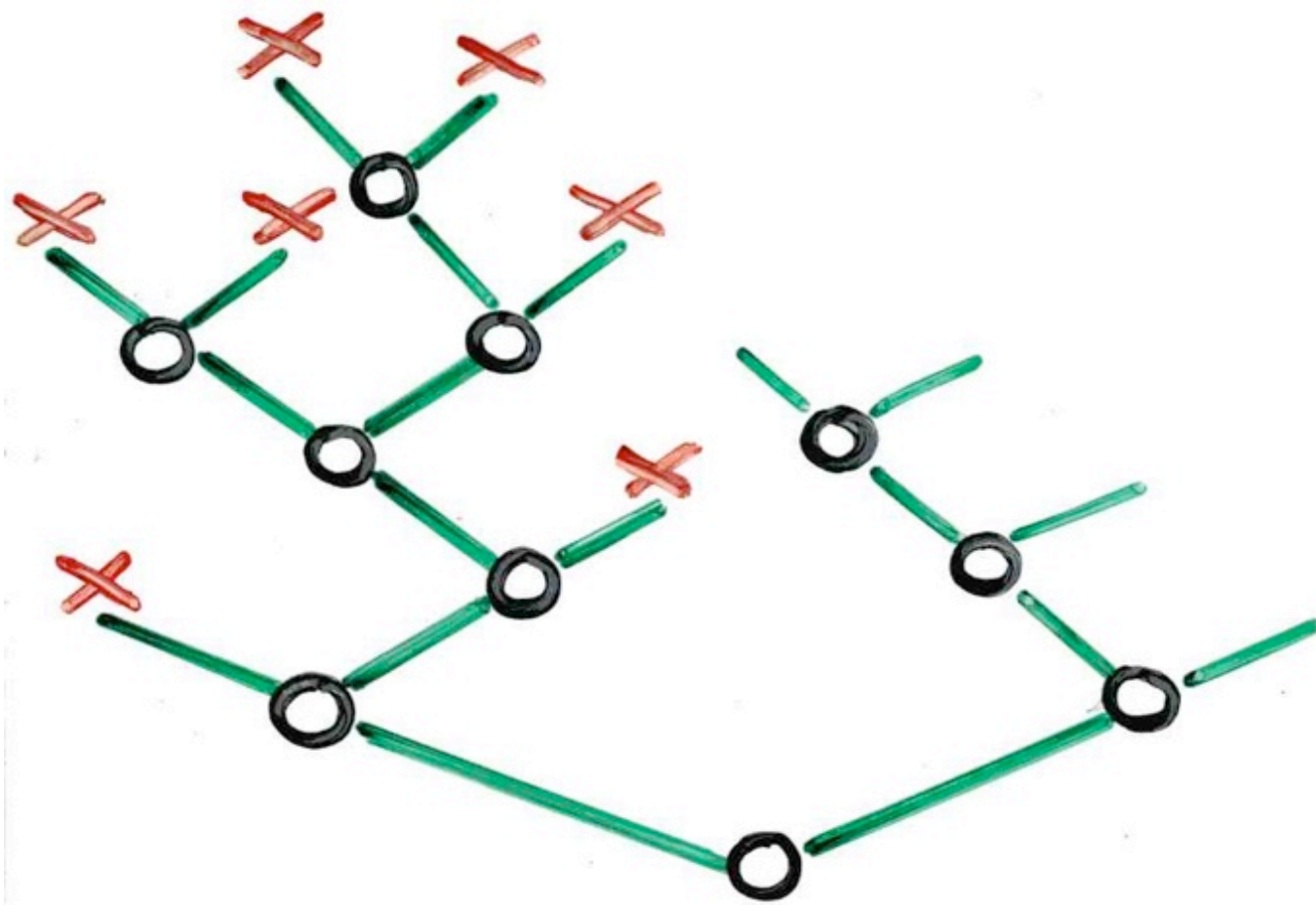


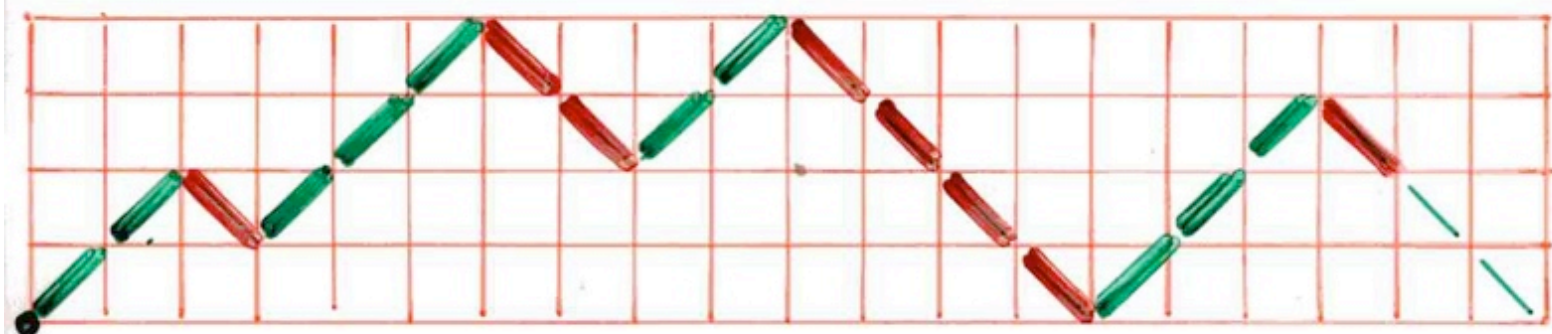
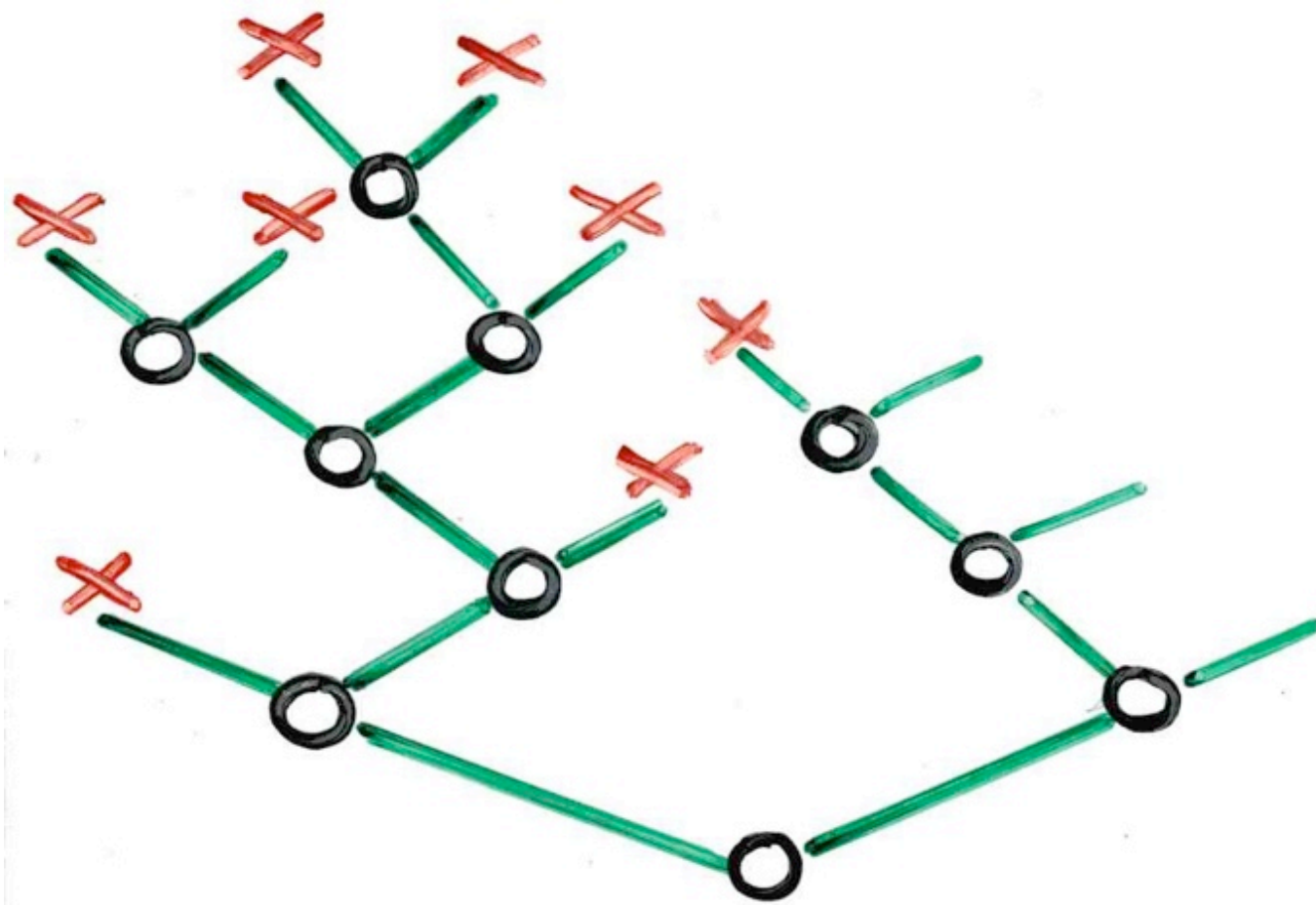


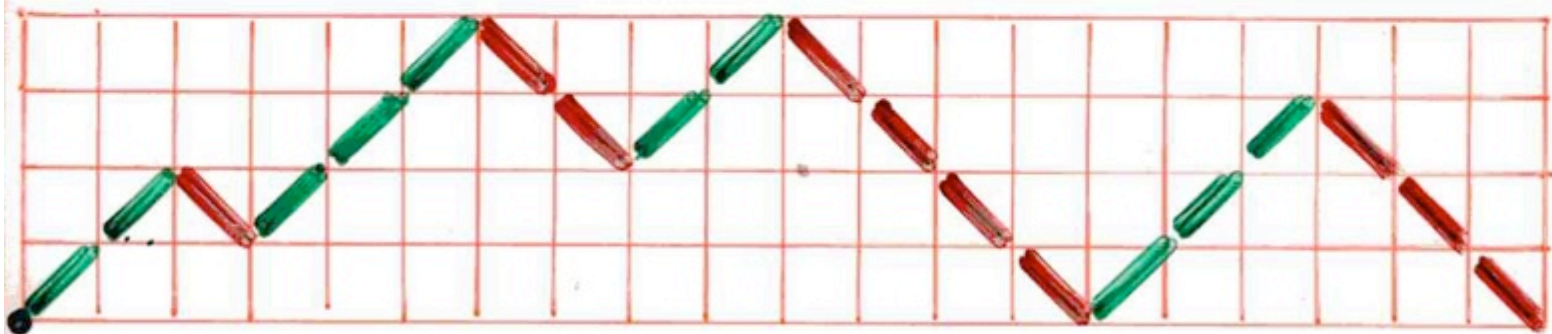
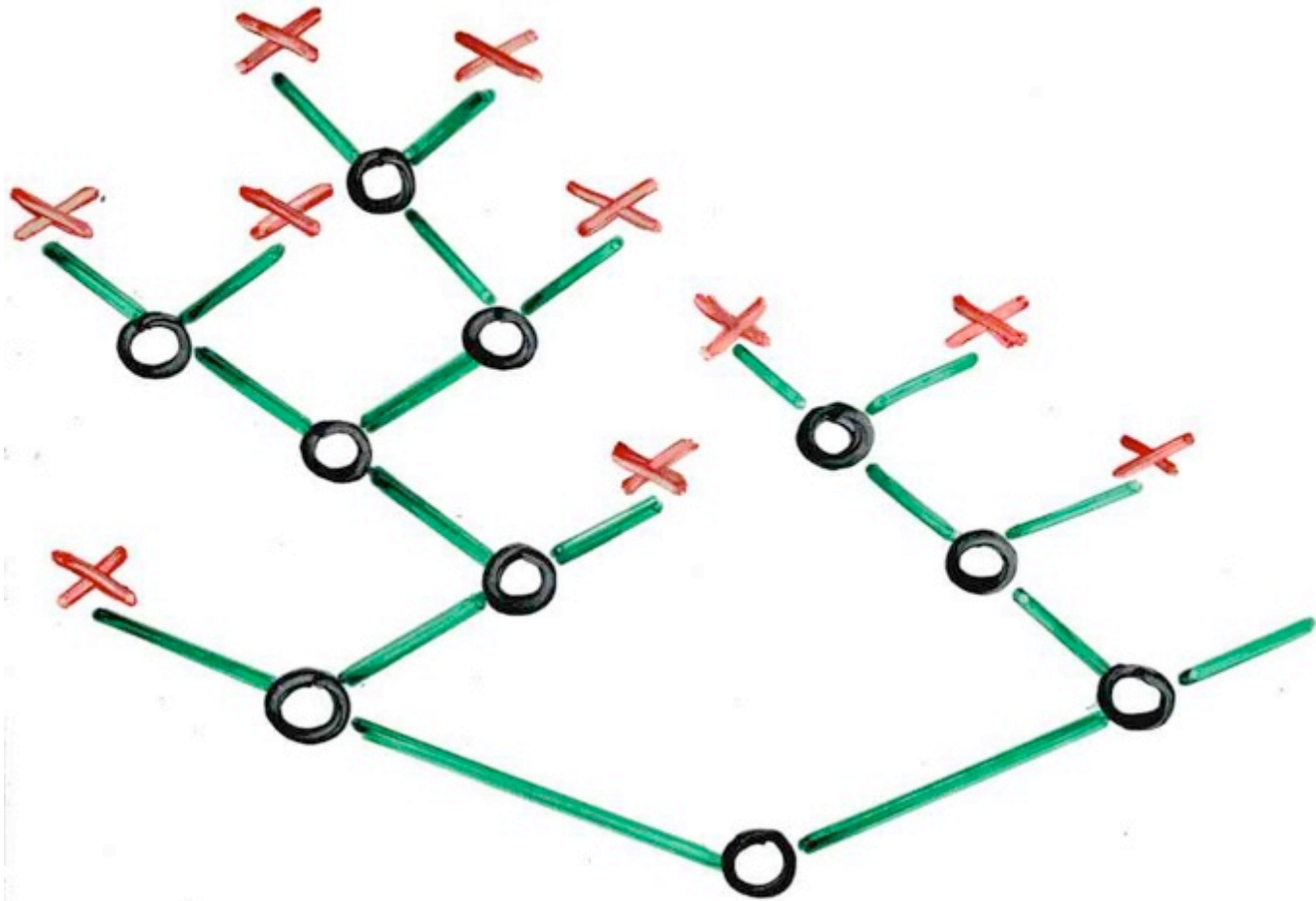


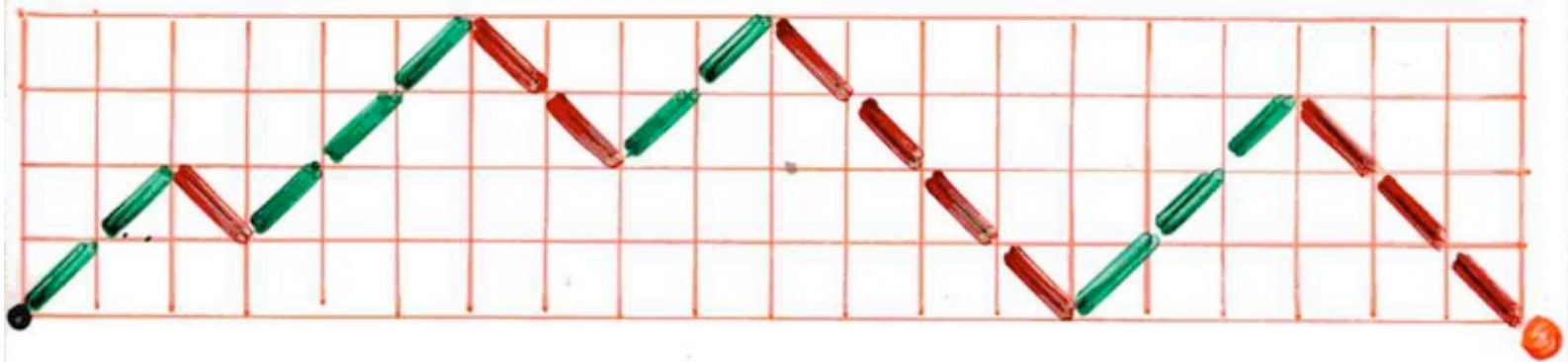
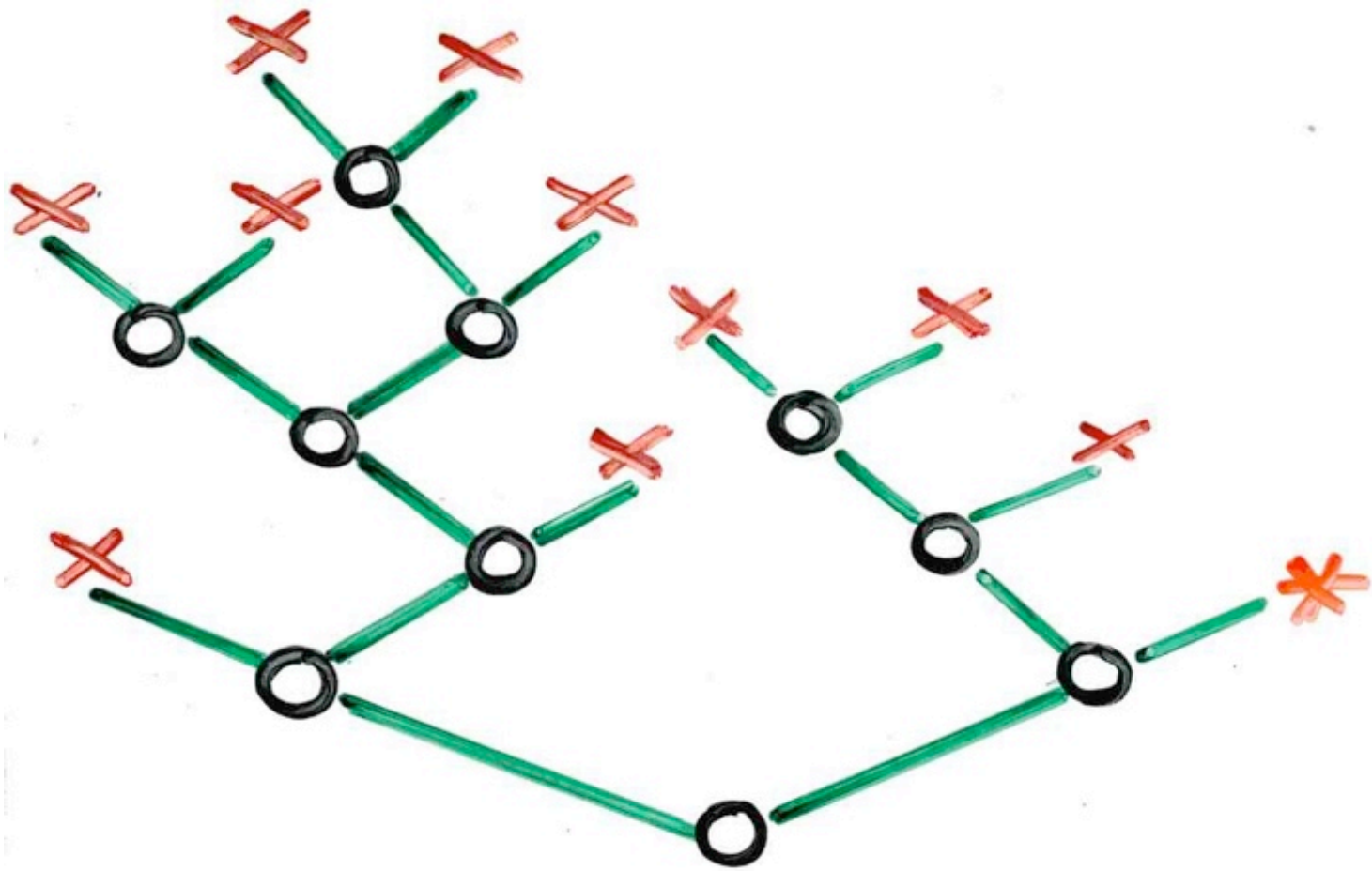












Le majestueux noyer

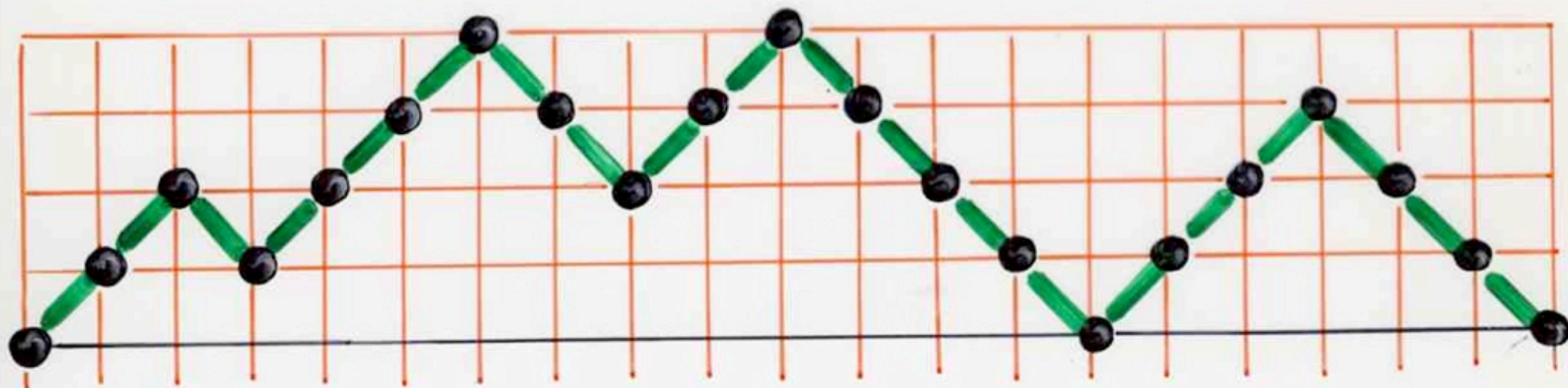


Marcia Pig Lagos

hauteur logarithmique



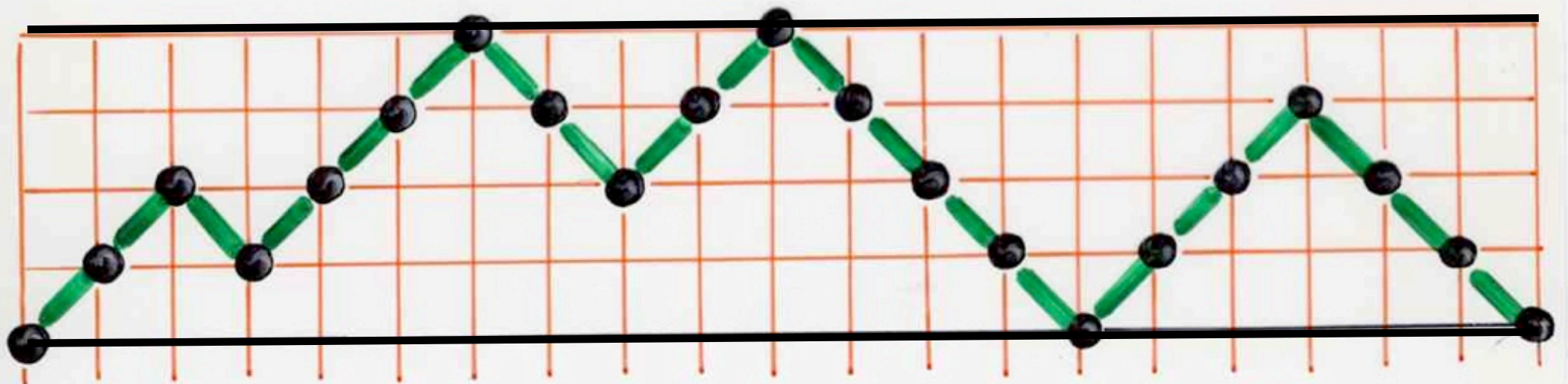
chemin de Dyck w



chemin de Dyck w

hauteur $h(w)$

$= 4$



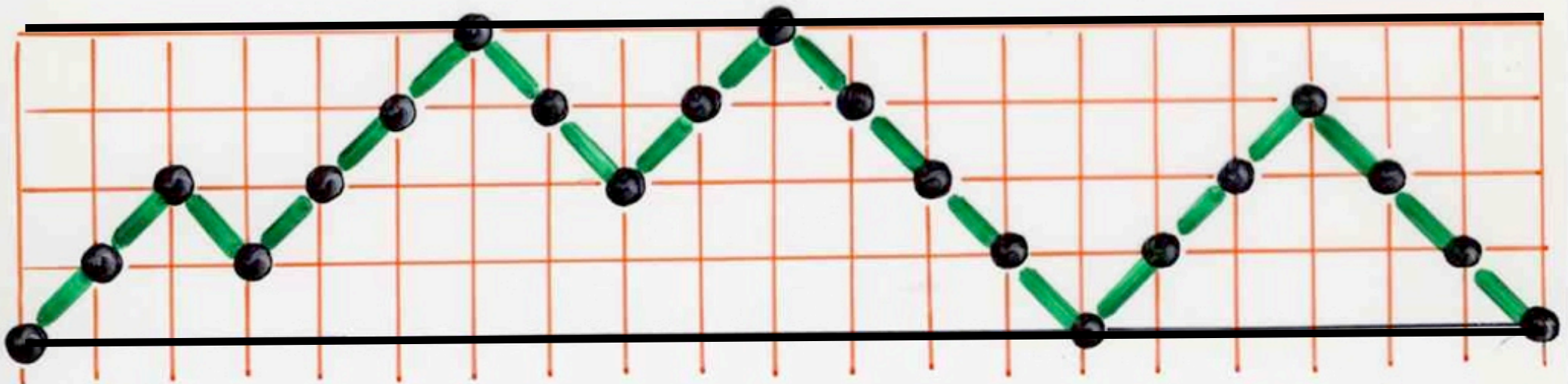
chemin de Dyck w

hauteur $h(w) = 4$

hauteur
logarithmique $lh(w)$

$$= \lfloor \log_2(1 + h(w)) \rfloor$$

$\lfloor \rfloor$ partie entière



arbres binaires B
ayant n sommets

nb de Strahler

$$St(B) = k$$

chemins de Dyck
de longueur $2n^w$

hauteur \log .

$$lh(w) = k$$

←→
Franois
(1984)

même distribution !

Nombre de Strahler

moyen

parmi tous les arbres binaires
ayant n sommets

$$St_n = \log_4 n + f(\log_4 n) + o(1)$$

Flejolet, Raoult, Vuillemin (1979)
Kemp (1979)

f fonction
périodique

matrices de ramification

comme

une analyse mathématique de la forme
des structures arborescentes

Comment «mesurer» la forme d'un arbre ?

BERNARD
GANTNER





ARBRES AUX CORBEAUX

MUSÉE DU LOUVRE

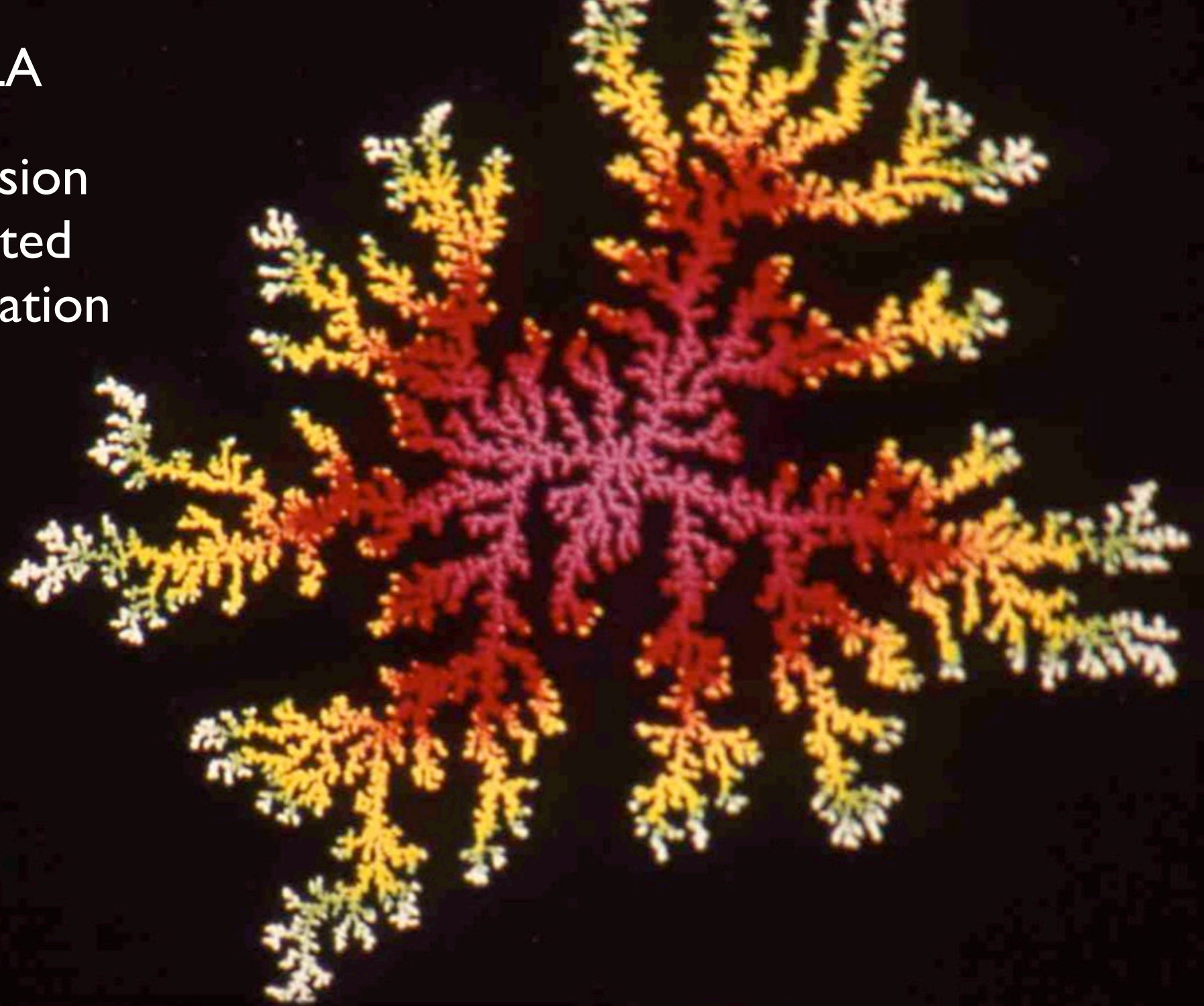
matrices
de ramification
en physique

digitation
visqueuse



DLA

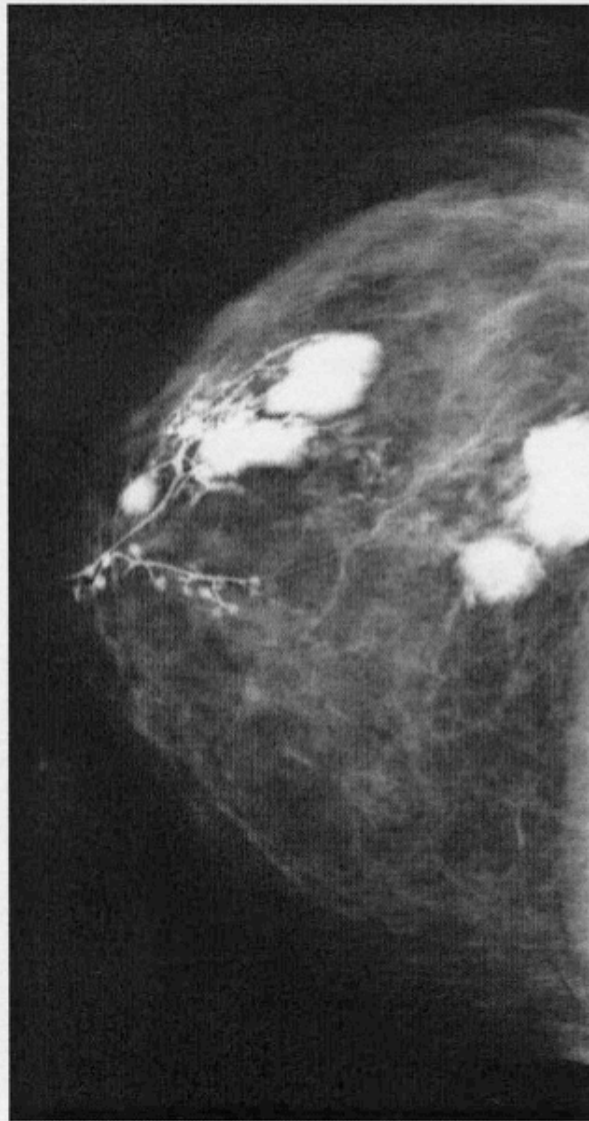
Diffusion
Limited
Aggregation



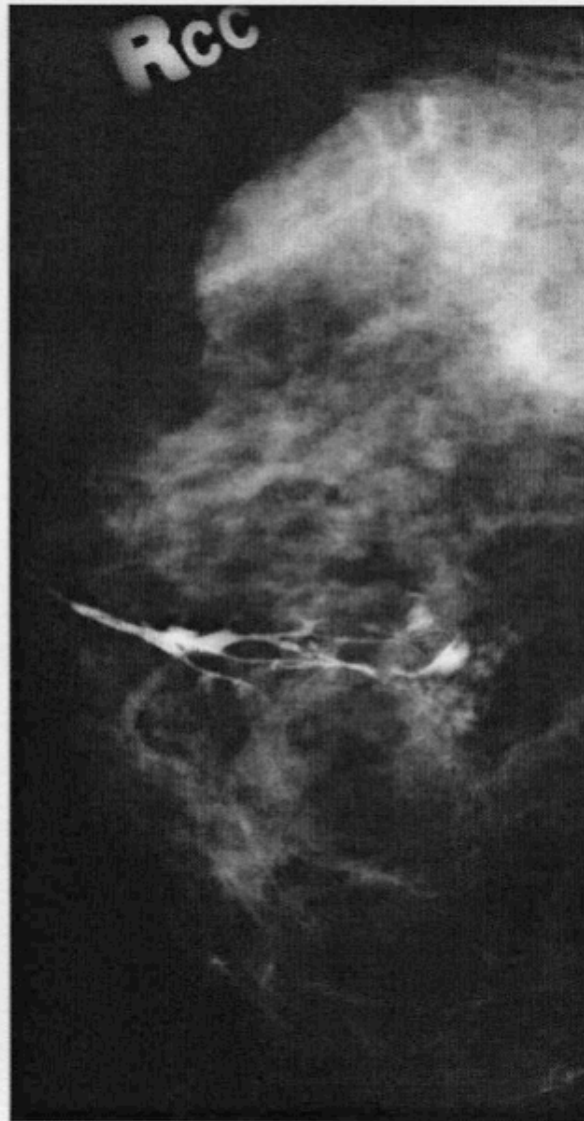
classification des galactogrammes
avec des matrices de ramification

P. Bakic, M. Albert, A. Maidment (2003)

galactographie digitale
mammographie



a.



b.

Figure 4. Two examples of galactograms that have been correctly classified by means of R matrices. **(a)** Galactogram with no reported findings (patient age, 45 years; right CC view; $r_{3,2} = 0.5$ and $r_{3,3} = 0.19$). (Large bright regions seen in this galactogram are due to extravasation, which did not affect the segmentation of the ductal tree.) **(b)** Galactogram with a reported finding of cysts (patient age, 55 years; right CC view; $r_{3,2} = 0.33$ and $r_{3,3} = 0.67$).

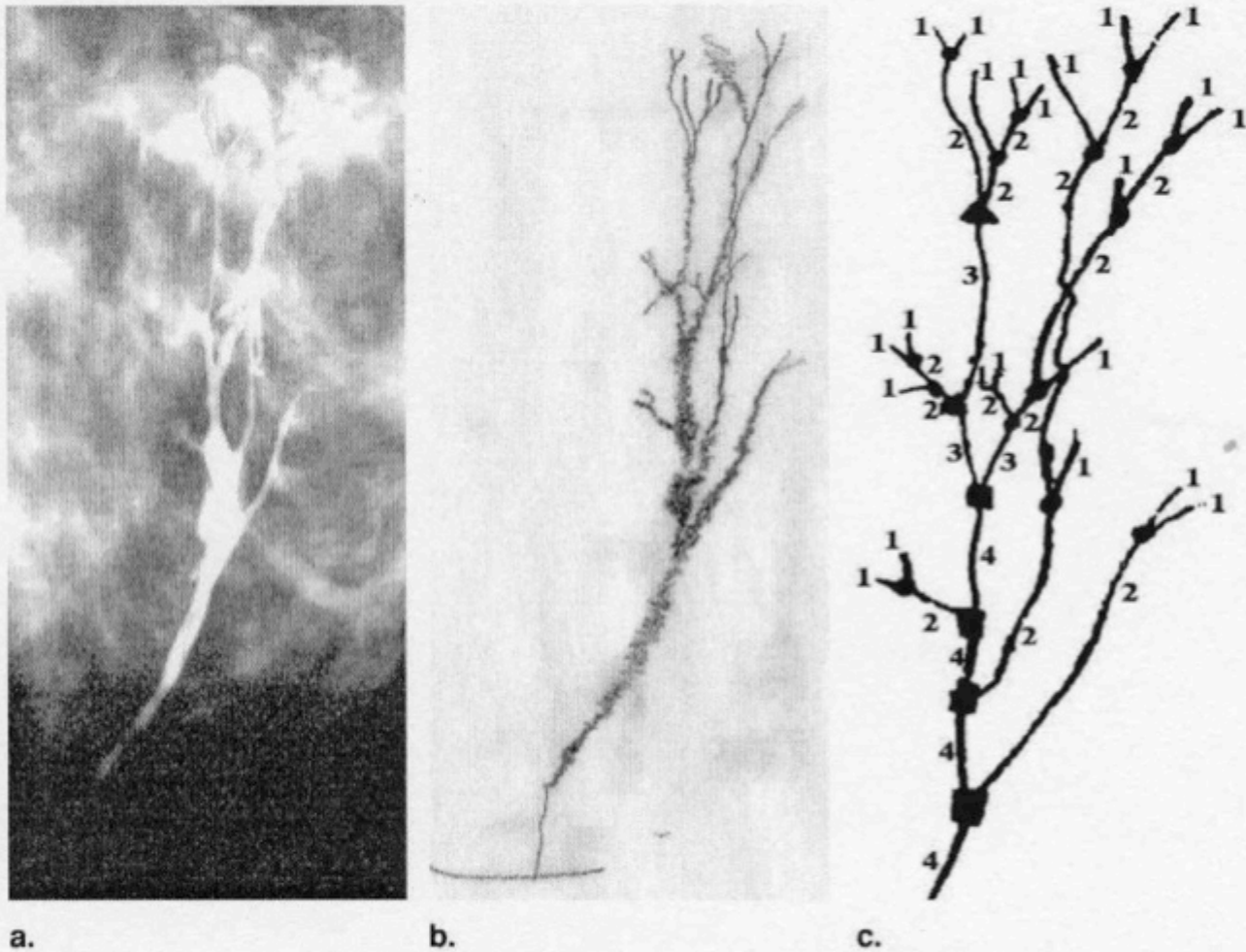


Figure 1. Segmentation of a ductal tree, showing (a) part of a galactogram with a contrast-enhanced ductal network, (b) the manually traced network of larger ducts from the contrast-enhanced portion of the galactogram, (c) numeric labeling of branches in the ductal network, and (d) the R matrix computed from the branching pattern. The dots, triangles, and squares denote branching points of different levels of the tree.

$$R = \begin{bmatrix} r_{2,1} & r_{2,2} & \cdot & \cdot \\ r_{3,1} & r_{3,2} & r_{3,3} & \cdot \\ r_{4,1} & r_{4,2} & r_{4,3} & r_{4,4} \end{bmatrix} = \begin{bmatrix} 0.43 & 0.57 & \cdot & \cdot \\ 0 & 0.33 & 0.67 & \cdot \\ 0 & 0.75 & 0 & 0.25 \end{bmatrix}$$

d.

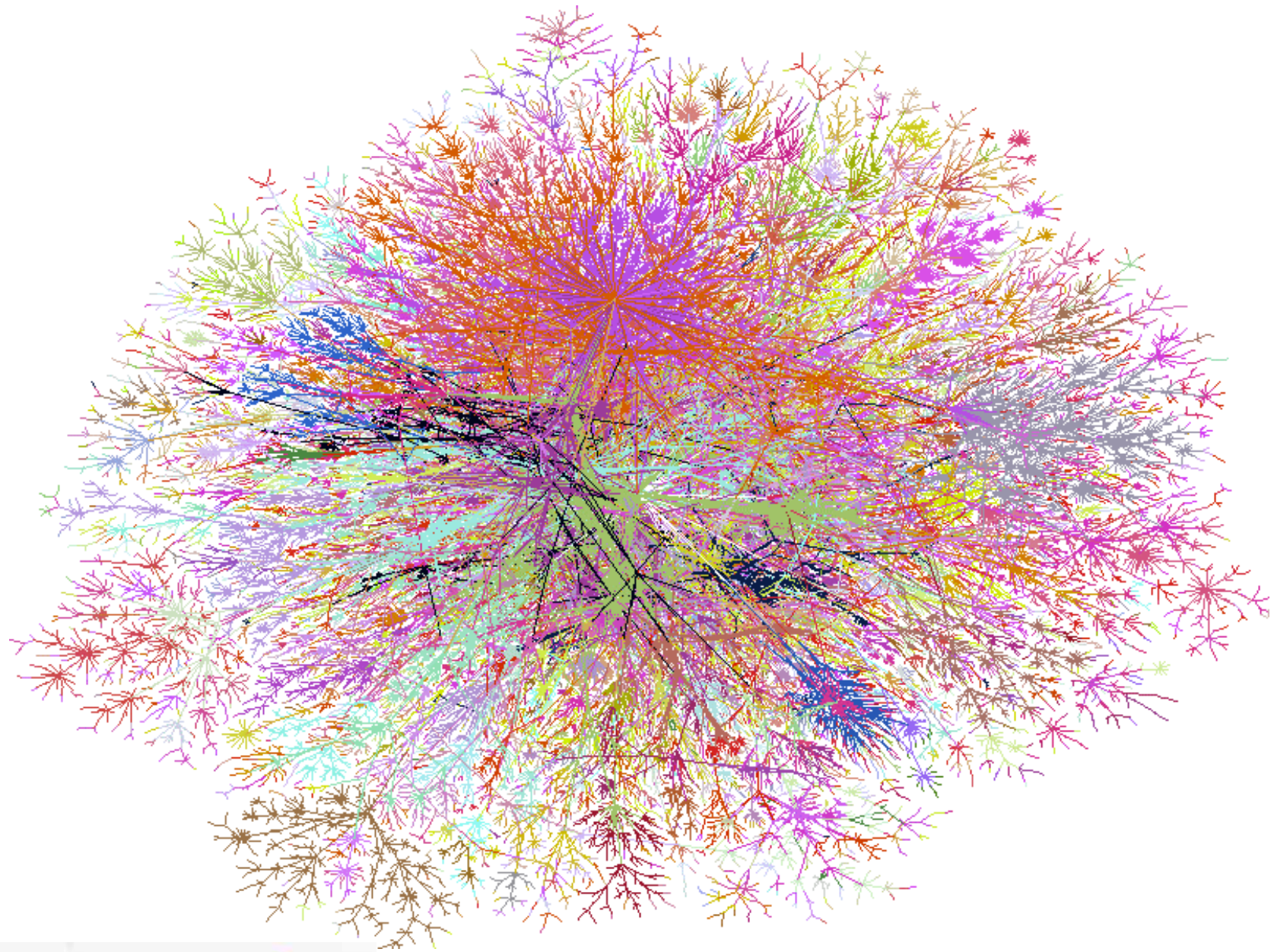
visualisation de l'information

- Visualisation de l'information
très grands graphes

D. Auber, M. Delest

Y. Chiacofa, G. Melançon, J.M. Fedou

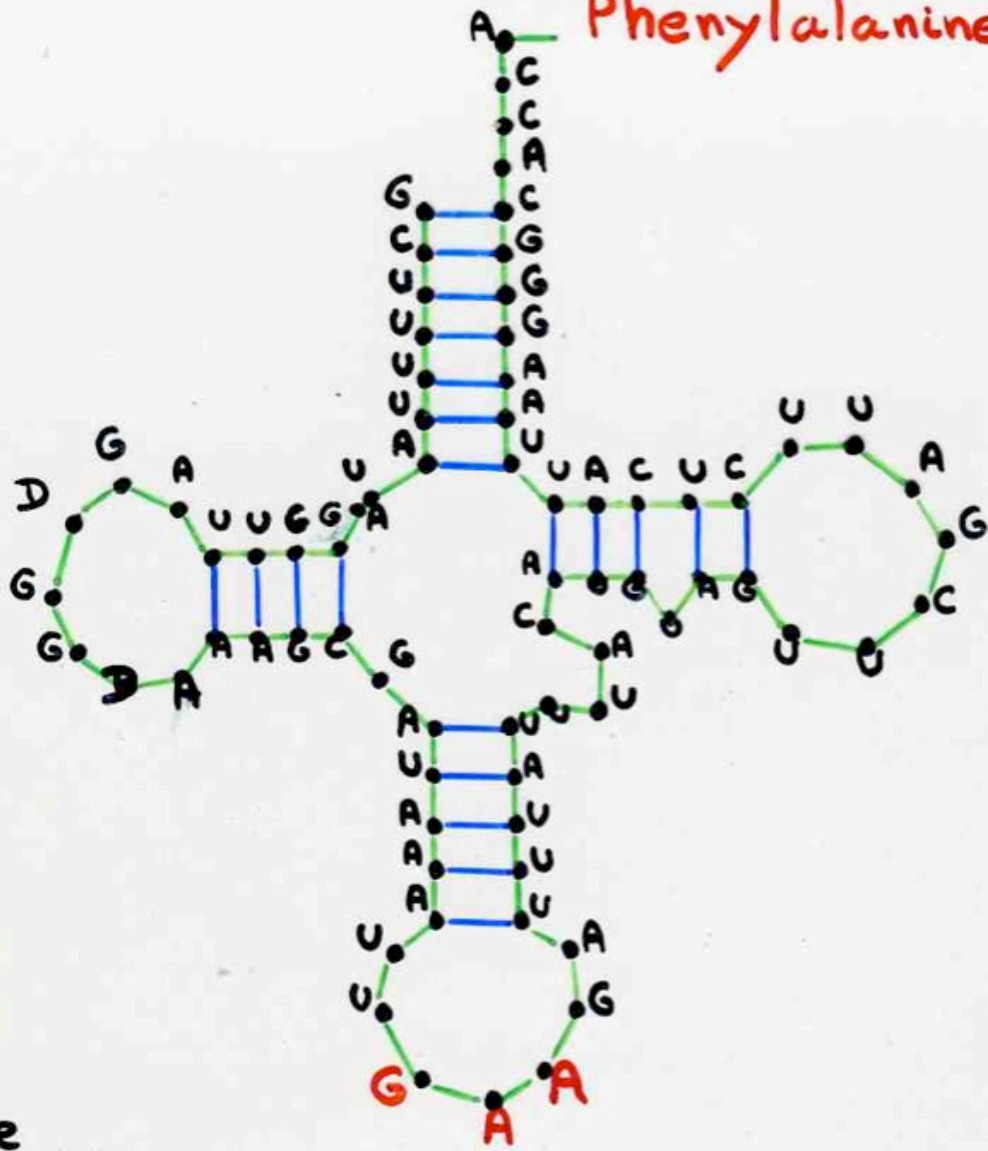
extension de l'analyse de Horton-Strahler
des arbres aux graphes



logiciel Tulip
D. Auber

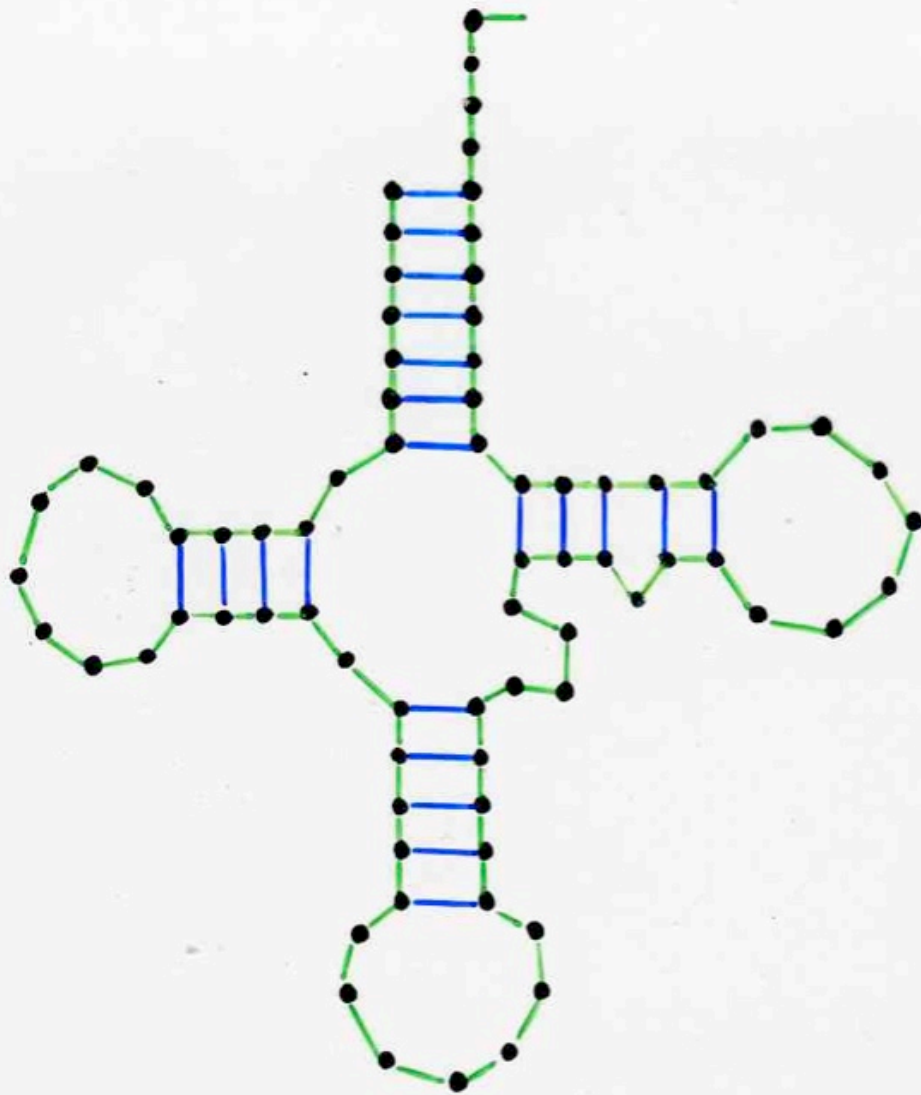
des arbres dans les molécules ...

Phenylalanine

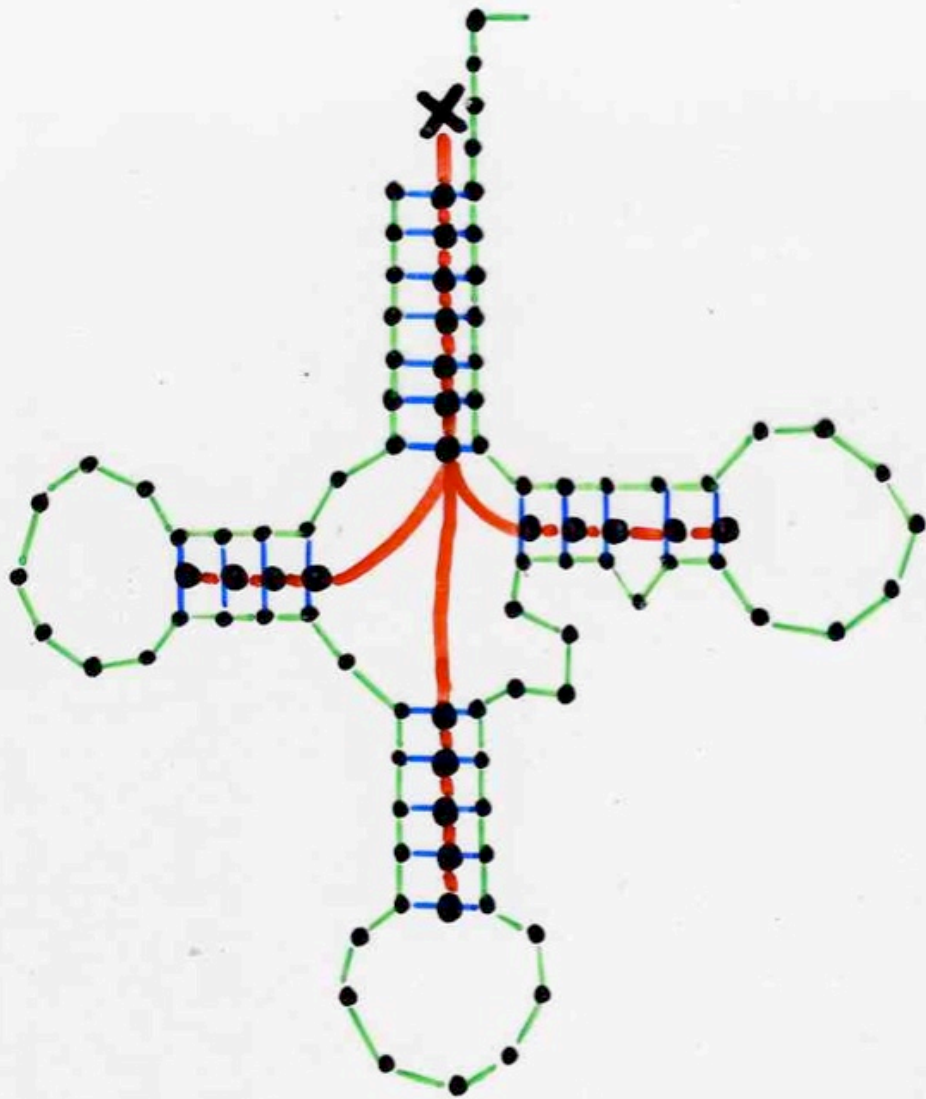


- A denine
- U racyle
- G uanine
- C ytosine

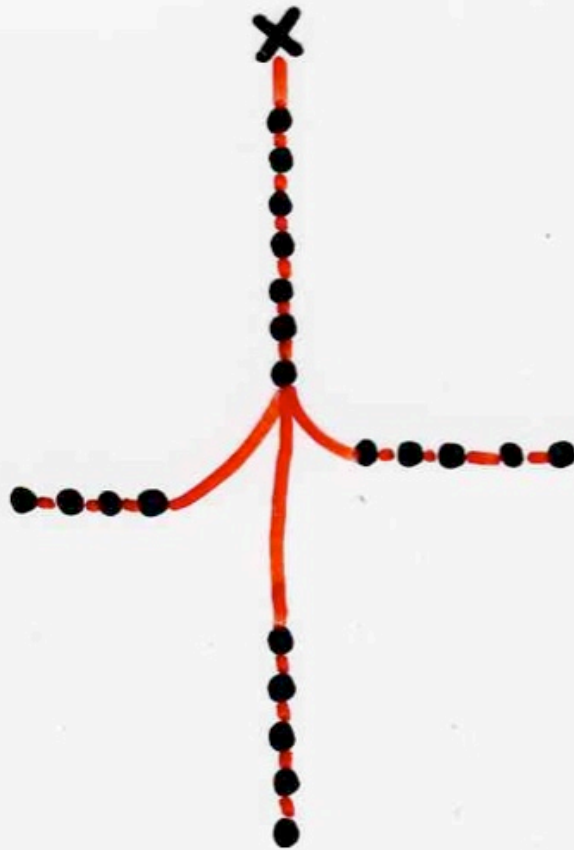
tARN^{Phe}



tARN^{Phe}



tARN^{Phe}



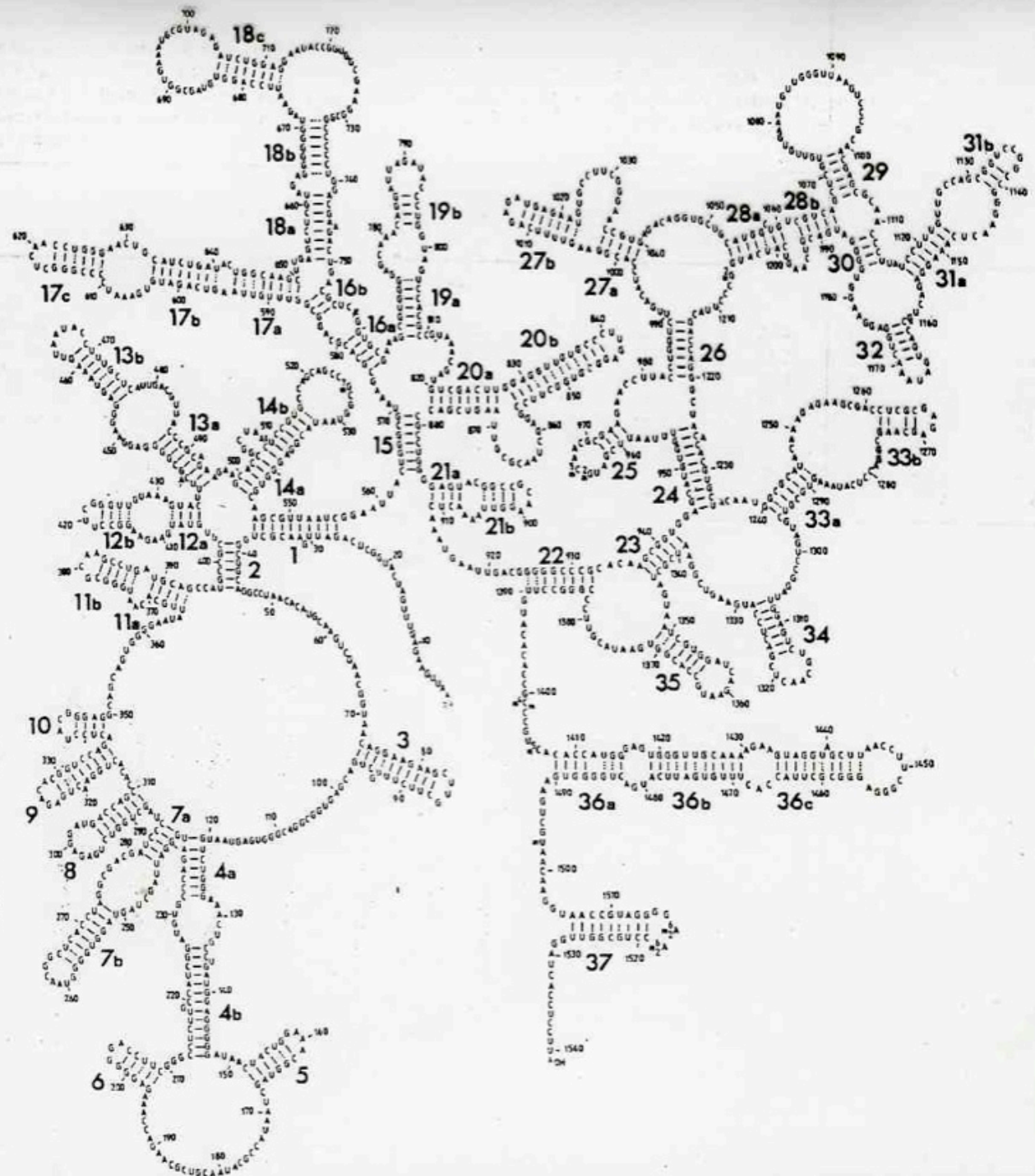


Fig. 1. Secondary structure model of the 16-S RNA from *E. coli*. This model has been fully described elsewhere [18]. The various secondary structure motifs are numbered for reference. Base-pairings 2 and 23 are included in this up-dated scheme and slight modifications have been introduced into helices 18b and 33b

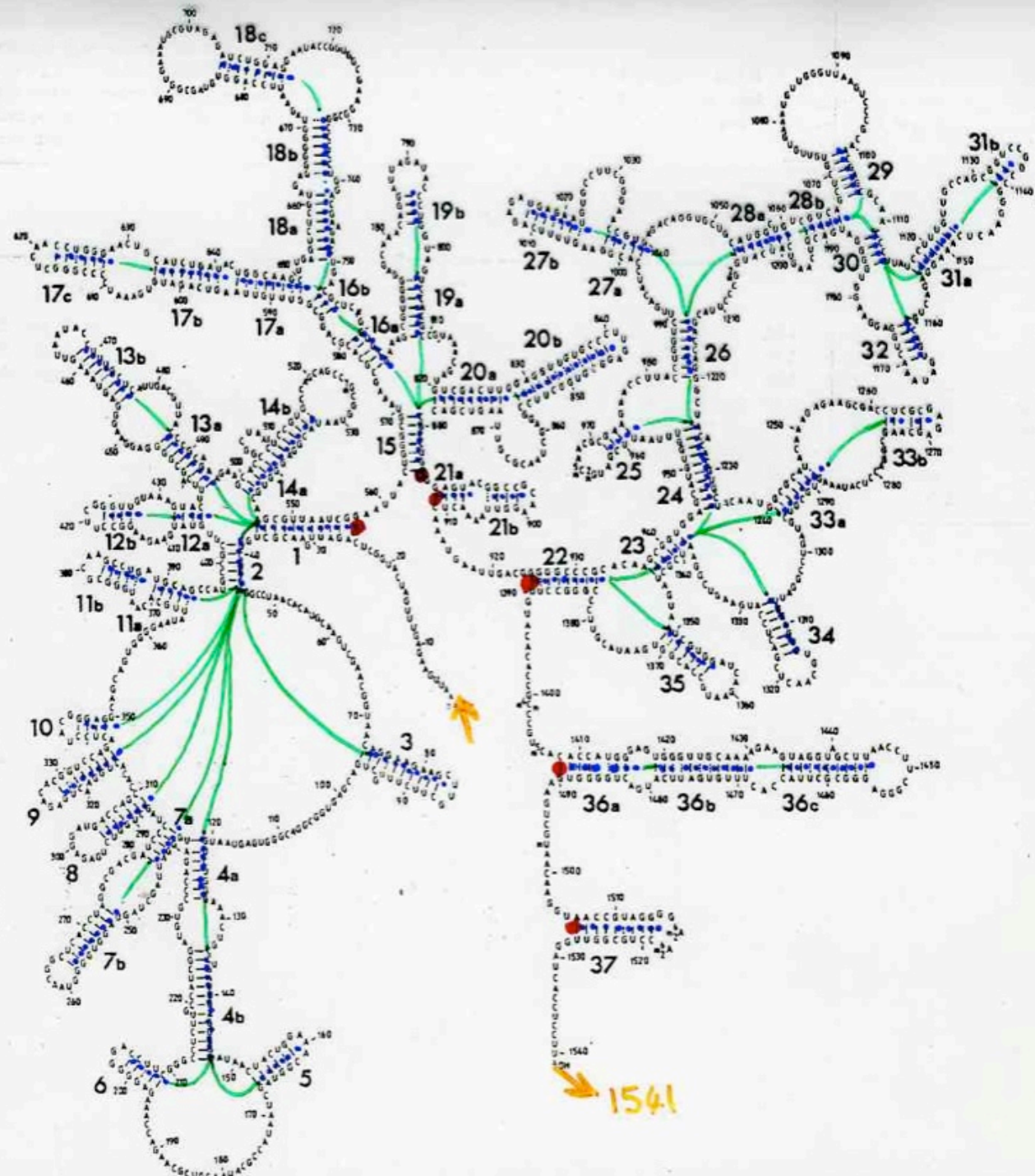
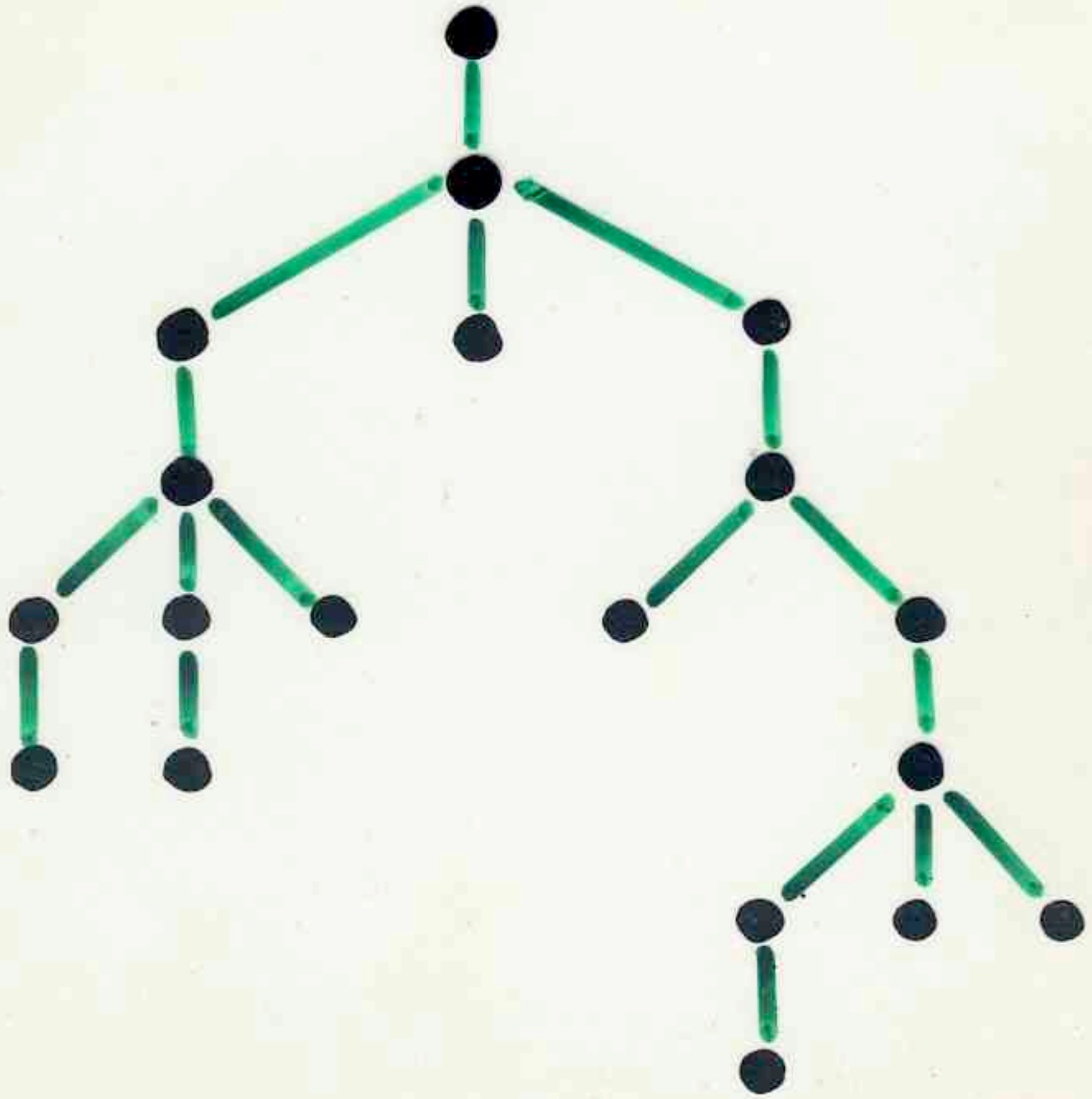
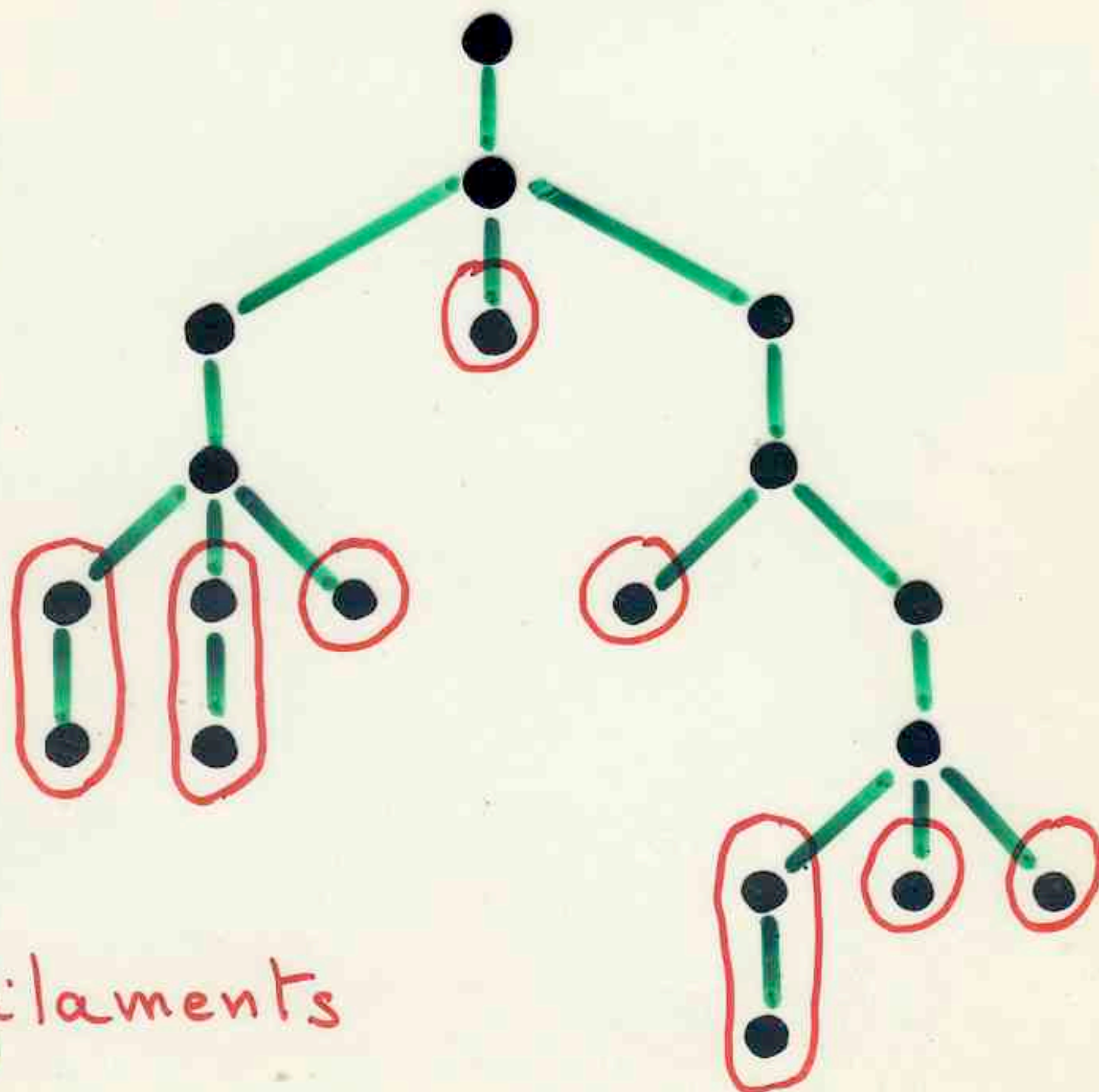


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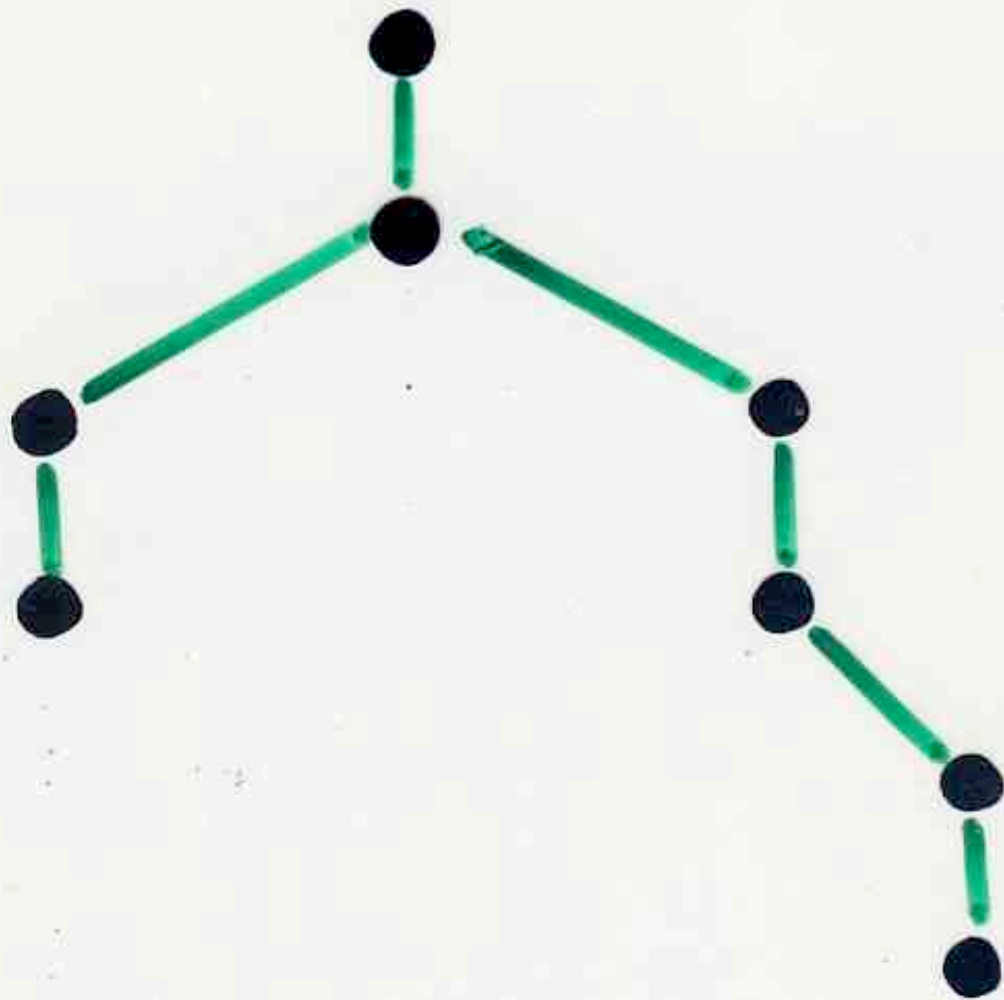
«complexité» ou «ordre»
d'une molécule

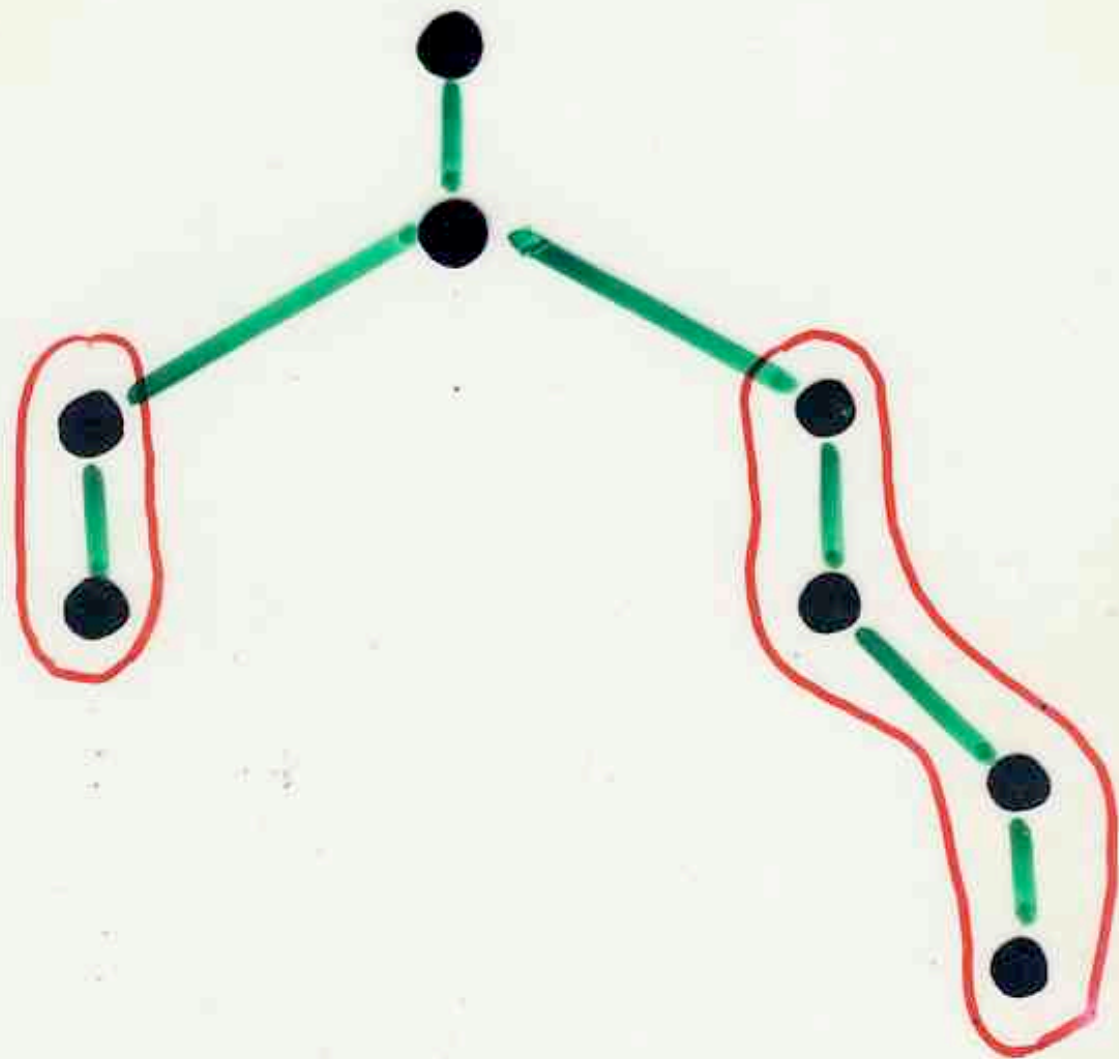
M. Waterman





filaments











$F_{n,k}$ = nombre de
forêts d'arbres
ayant n sommets
et d'ordre k

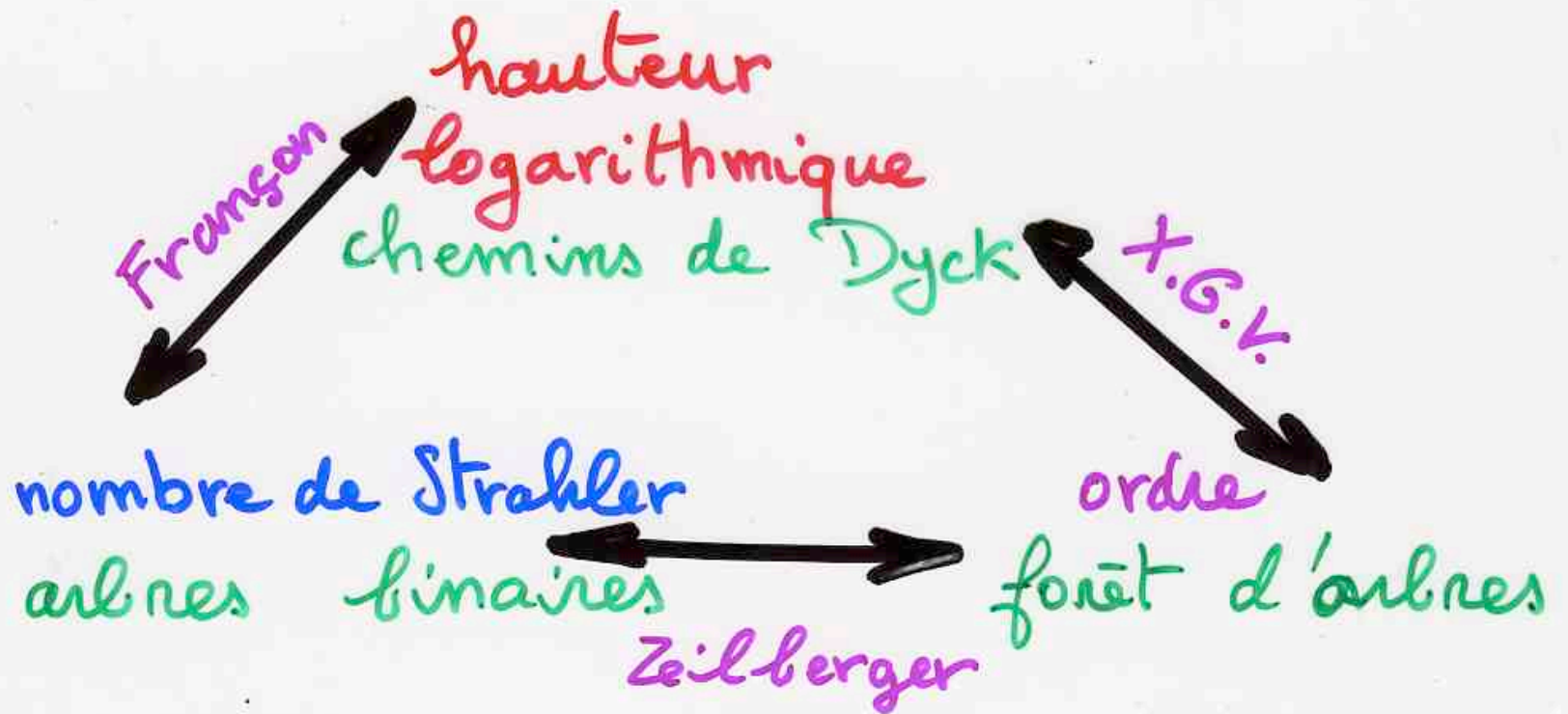
$F_{n,k}$ = nombre de
forêts d'arbres
ayant n sommets
et d'ordre k

= $S_{n,k}$

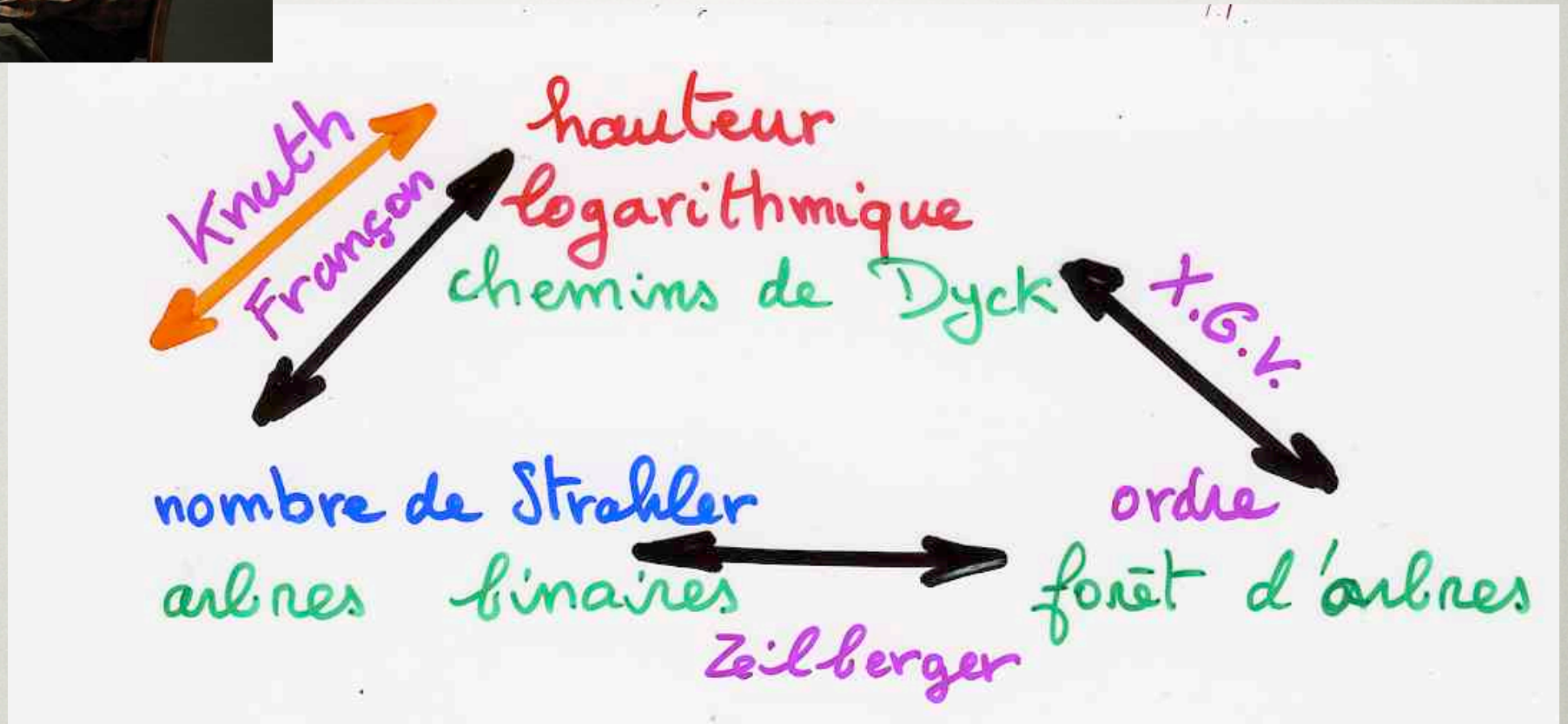
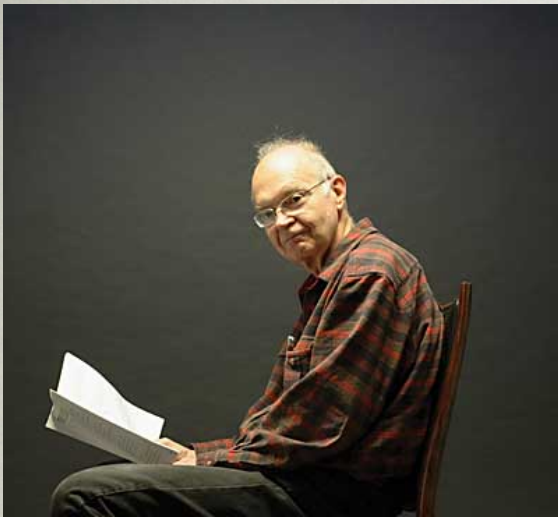
à nouveau
même
distribution !!

Vauchassade de Chaumont
X. V. (1985) (2001)

D. Zeilberger (1985)



Donald Knuth

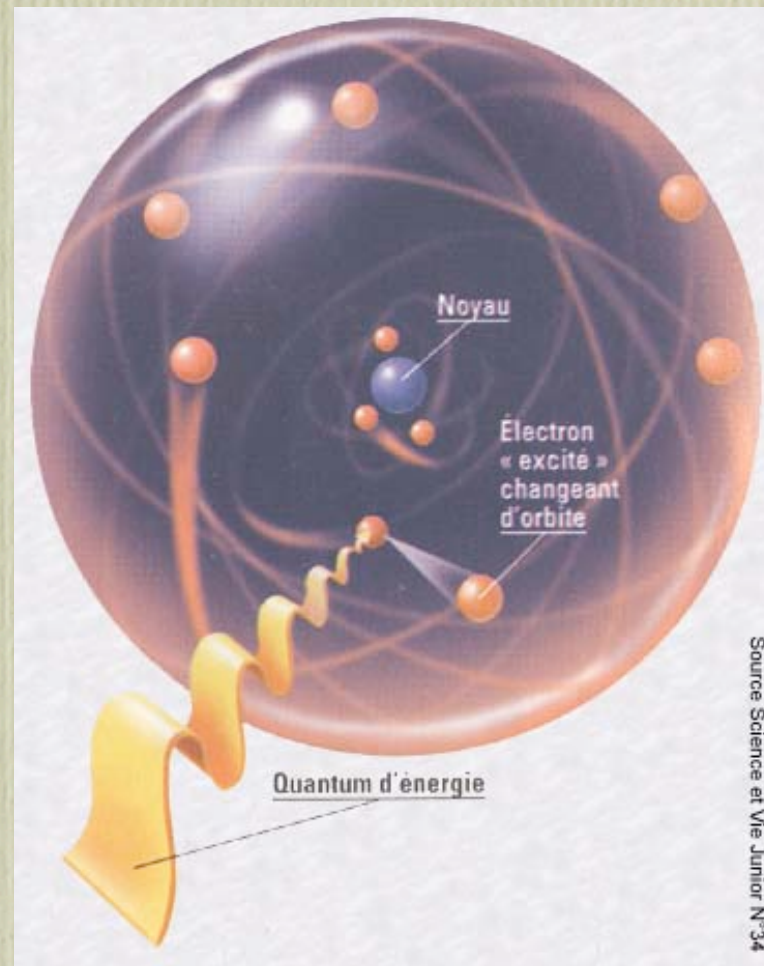


LE JARDIN ZEN

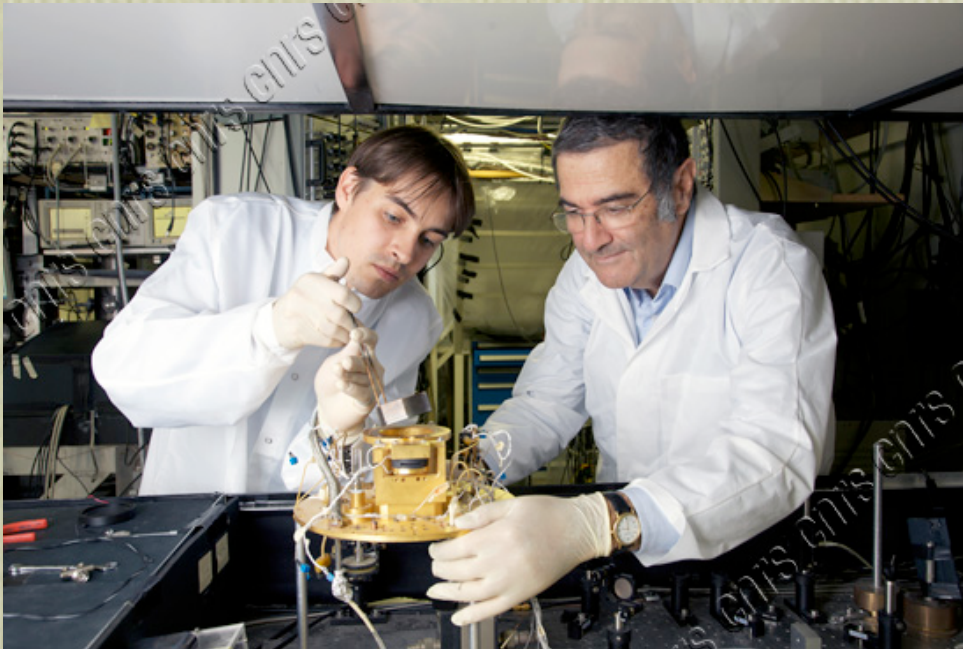
L'infinitement petit

Des arbres dans les grains de lumière ?

le monde quantique



le monde quantique



Serge Haroche
Nobel de physique 2012

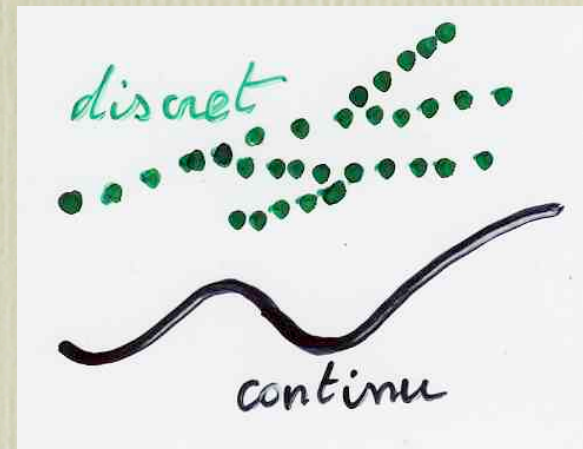
mécanique quantique
très loin du bon sens usuel

les particules ont une tendance à exister

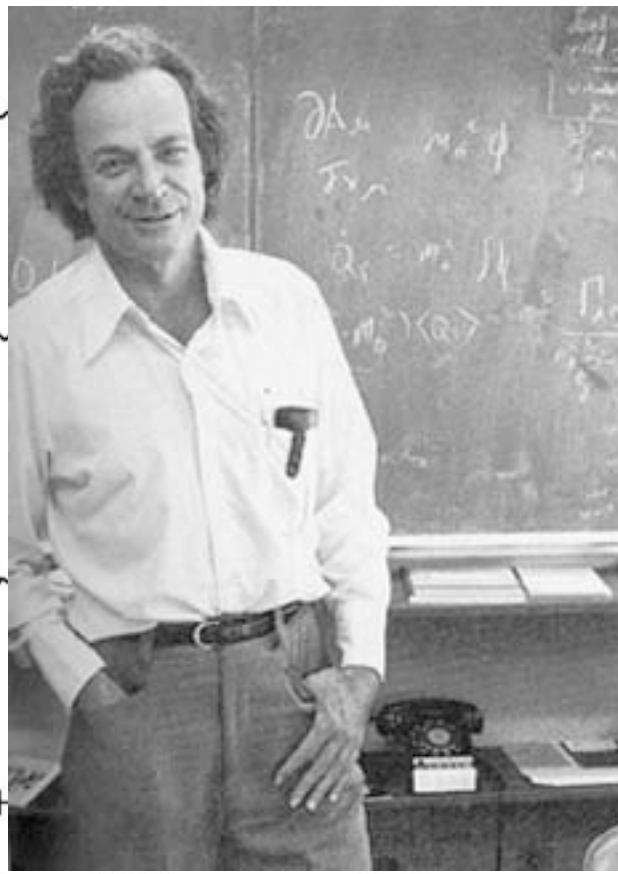
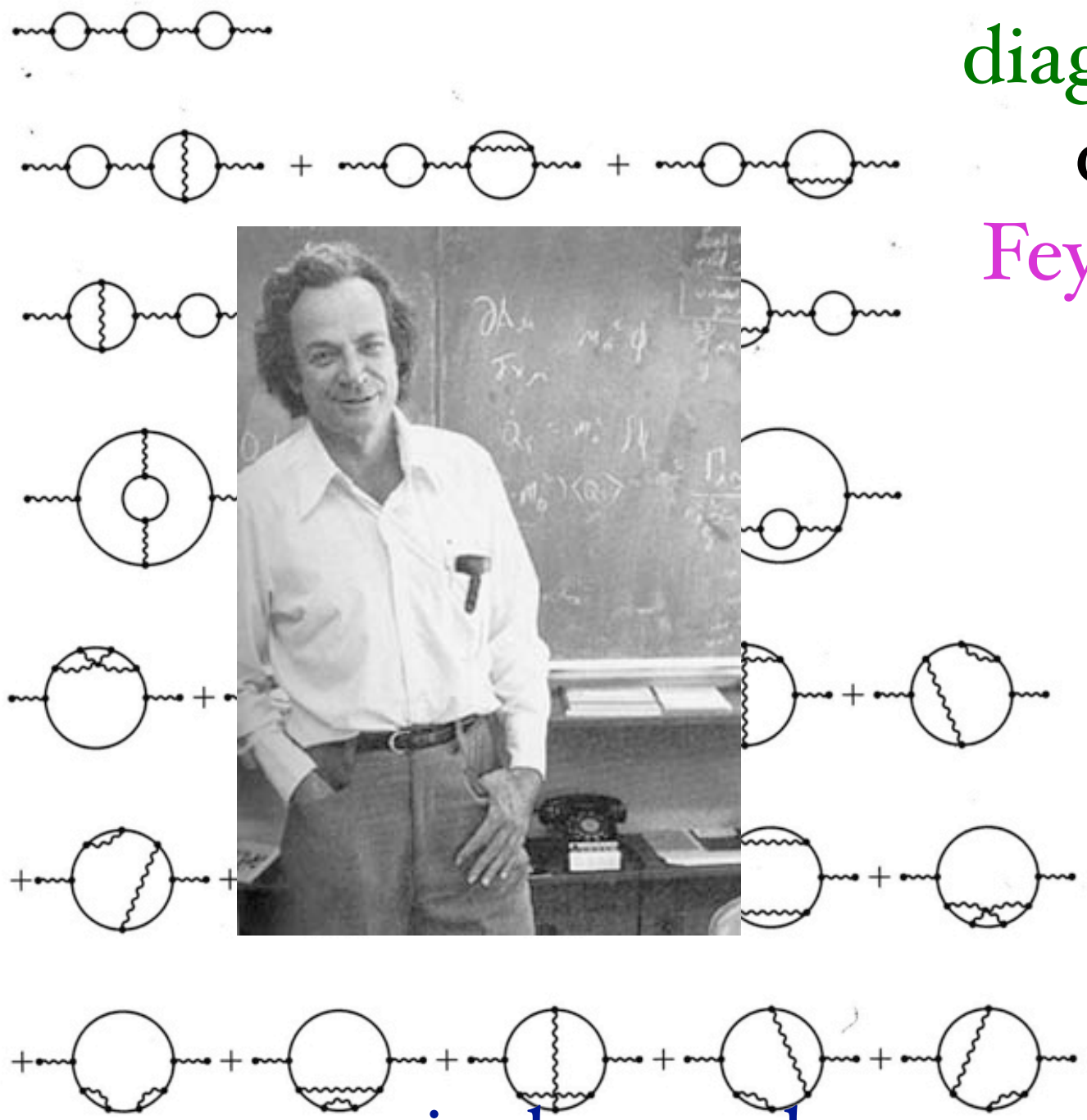
the fameux chat de Schrödinger, mort et vivant en même temps

espace, temps, matière, énergie:

continu ou discret ?



diagrams
de
Feynman



interactions entre particules, photons

somme infinie de quantités infinies ?!?

supprimer le double infini ...

renormalisation quantique

recette de cuisine

Diagrammes de Feynman

$$\sigma^\gamma(\Upsilon) = \text{diagram 1}$$

$$\sigma^\gamma(\Upsilon) = \text{diagram 2} + \text{diagram 3}$$

$$\sigma^\gamma(\Upsilon) = \text{diagram 4} + \text{diagram 5} + \text{diagram 6}$$

$$\sigma^\gamma(\Upsilon) = \text{diagram 7} + \text{diagram 8}$$

$$\sigma^\gamma(\Upsilon) = \text{diagram 9} + \text{diagram 10} + \text{diagram 11}$$

$$\text{diagram 12} + \text{diagram 13} + \text{diagram 14} + \text{diagram 15} + \text{diagram 16}$$

$$\text{diagram 17} + \text{diagram 18} + \text{diagram 19} + \text{diagram 20} + \text{diagram 21}$$

explication avec
les mathématiques
des arbres

A photograph of Alain Connes, a mathematician, in a lecture hall. He is standing, wearing a white shirt and dark trousers, gesturing with his hands as if speaking. The room features a large arched window with curtains, a green exit sign above a doorway, and a chalkboard with mathematical diagrams and text. A podium is visible in the foreground.

Alain Connes

les mathématiques d'Euclide , beaucoup de figures jusqu'à Newton
après, élimination des figures

Lagrange, traité de mécanique: pas une seule figure
équations, identités, abstraction pure



Joseph-Louis Lagrange
1736 - 1813

AVERTISSEMENT

DE LA DEUXIÈME ÉDITION.

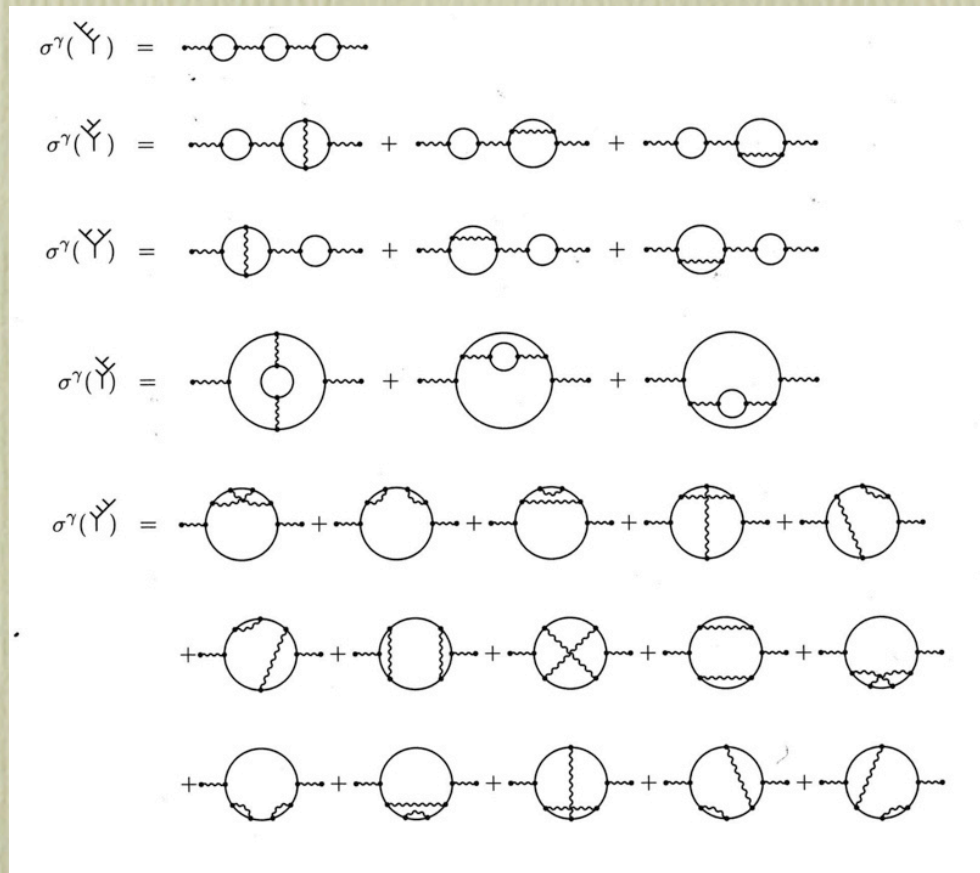
On a déjà plusieurs Traités de Mécanique, mais le plan de celui-ci est entièrement neuf. Je me suis proposé de réduire la théorie de cette Science, et l'art de résoudre les problèmes qui s'y rapportent, à des formules générales, dont le simple développement donne toutes les équations nécessaires pour la solution de chaque problème.

Cet Ouvrage aura d'ailleurs une autre utilité : il réunira et présentera sous un même point de vue les différents principes trouvés jusqu'ici pour faciliter la solution des questions de Mécanique, en montrera la liaison et la dépendance mutuelle, et mettra à portée de juger de leur justesse et de leur étendue.

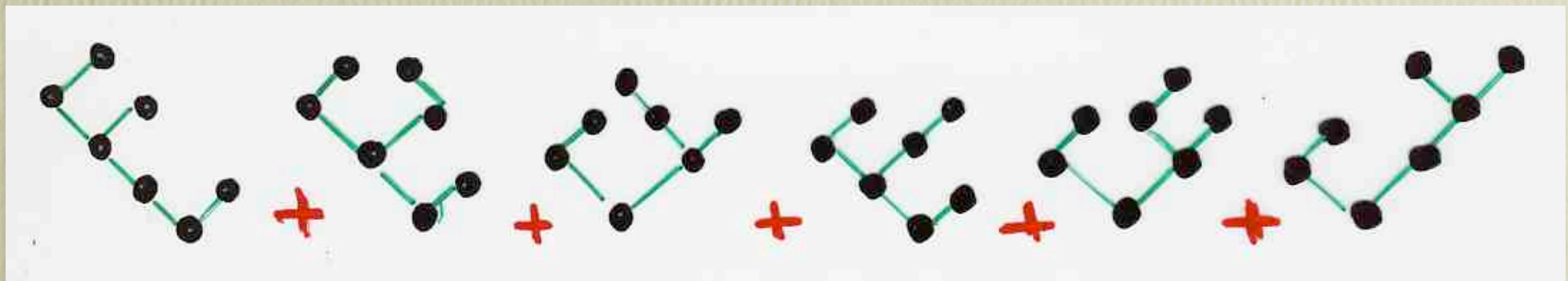
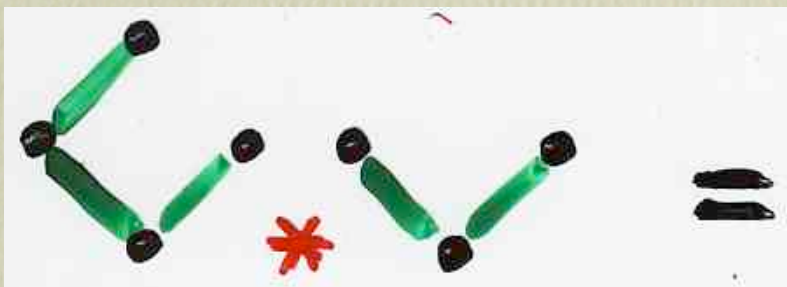
Je le divise en deux Parties : la Statique ou la Théorie de l'Équilibre, et la Dynamique ou la Théorie du Mouvement ; et, dans chacune de ces Parties, je traite séparément des corps solides et des fluides.

On ne trouvera point de Figures dans cet Ouvrage. Les méthodes que j'y expose ne demandent ni constructions, ni raisonnements géométriques ou mécaniques, mais seulement des opérations algébriques, assujetties à une marche régulière et uniforme. Ceux qui aiment l'Analyse verront avec plaisir la Mécanique en devenir une nouvelle branche, et me sauront gré d'en avoir étendu ainsi le domaine.

aujourd'hui, apparition des «figures», mais à un autre niveau



produit de deux arbres binaires

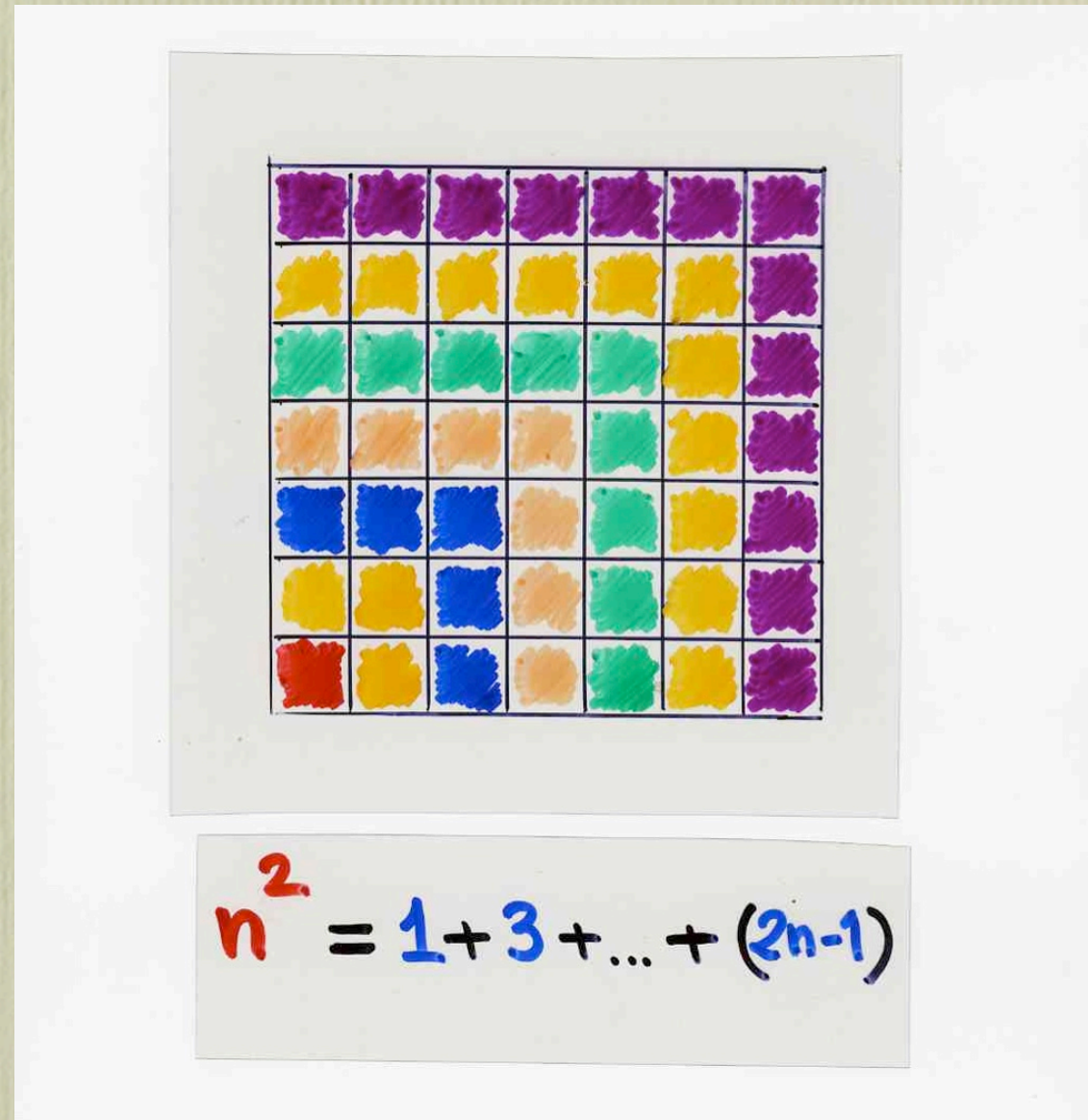


preuves avec des «figures»

preuves
combinatoires

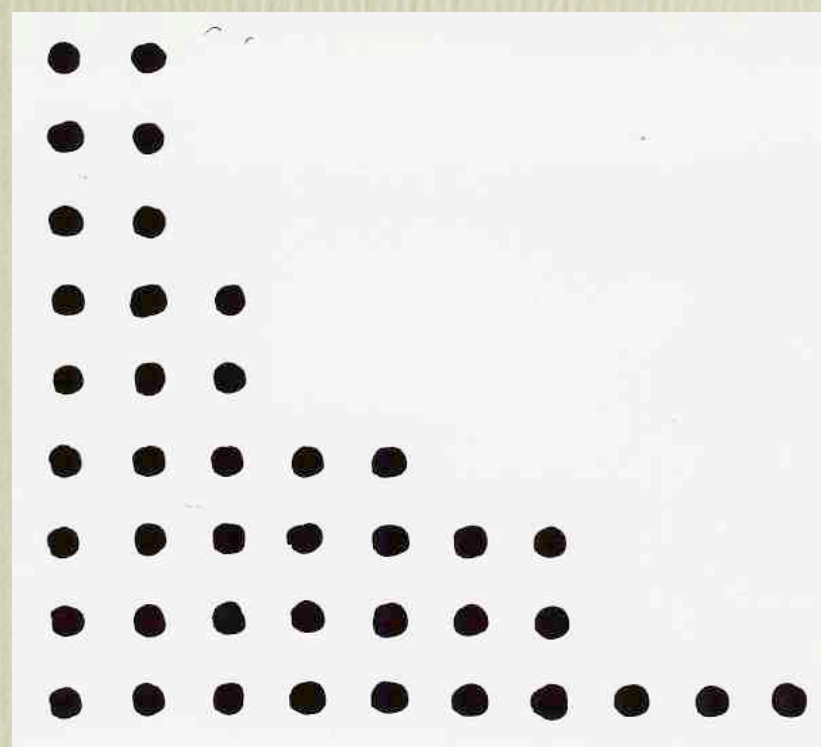
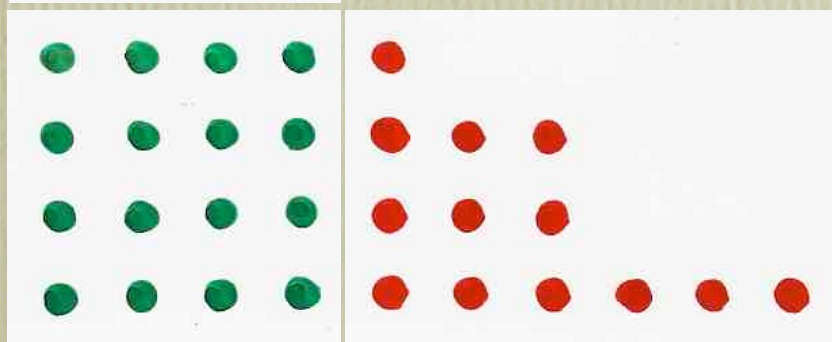
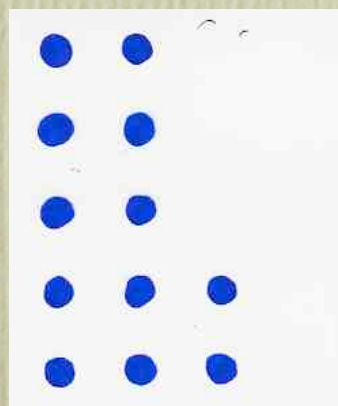


preuves combinatoires de certaines identités
avec des constructions de bijections, de correspondances
interprétations combinatoires



preuves combinatoires de certaines identités
 avec des constructions de bijections, de correspondances
 interprétations combinatoires

$$\sum_{m \geq 1} \frac{q^{m^2}}{[(1-q)(1-q^2)\dots(1-q^m)]^2} = \prod_{i \geq 1} \frac{1}{(1-q^i)}$$



Rogers - Ramanujan identities

$$R_I \quad \sum_{n \geq 0} \frac{q^{n^2}}{(1-q)(1-q^2)\dots(1-q^n)} = \prod_{\substack{i \equiv 1, 4 \\ \text{mod } 5}} \frac{1}{(1-q^i)}$$

$$R_{II} \quad \sum_{n \geq 0} \frac{q^{n^2+n}}{(1-q)(1-q^2)\dots(1-q^n)} = \prod_{\substack{i \equiv 2, 3 \\ \text{mod } 5}} \frac{1}{(1-q^i)}$$

Srinivasan
Ramanujan
(1887-1920)



"La fraction continue" de Ramanujan

$$\frac{1}{1 + \frac{1}{1 + \frac{q}{1 + \frac{q^2}{1 + \frac{q^3}{\dots}}}}} = \dots$$

$$\frac{\sum_{n \geq 0} \frac{q^{n^2+n}}{(1-q)(1-q^2)\dots(1-q^n)}}{\sum_{n \geq 0} \frac{q^{n^2}}{(1-q)(1-q^2)\dots(1-q^n)}}$$

$$R(q) = \prod_{n \geq 0} \frac{(1 - q^{5n+1})(1 - q^{5n+4})}{(1 - q^{5n+3})(1 - q^{5n+2})} = \frac{R_{II}}{R_I}$$

$$R(q) = \prod_{n \geq 0} \frac{(1 - q^{5n+1})(1 - q^{5n+4})}{(1 - q^{5n+3})(1 - q^{5n+2})} = \frac{R_{II}}{R_I}$$

$$t = -q [R(q)]^5$$

$$R(q) = \prod_{n \geq 0} \frac{(1-q^{5n+1})(1-q^{5n+4})}{(1-q^{5n+3})(1-q^{5n+2})} = \frac{R_{II}}{R_I}$$

$$t = -q [R(q)]^5$$

$$Y(q) = \prod_{n \geq 0} \frac{(1-q^{6n+2})(1-q^{6n+3})^2(1-q^{6n+4})(1-q^{5n+1})^2(1-q^{5n+4})^2(1-q^{5n})^2}{(1-q^{6n+1})(1-q^{6n+5})(1-q^{6n})^2(1-q^{5n+2})^3(1-q^{5n+3})^3}$$

$$R(q) = \prod_{n \geq 0} \frac{(1-q^{5n+1})(1-q^{5n+4})}{(1-q^{5n+3})(1-q^{5n+2})} = \frac{R_{II}}{R_I}$$

$$t = -q [R(q)]^5$$

$$\gamma(q) = \prod_{n \geq 0} \frac{(1-q^{6n+2})(1-q^{6n+3})^2(1-q^{6n+4})(1-q^{5n+1})^2(1-q^{5n+4})^2(1-q^{5n})^2}{(1-q^{6n+1})(1-q^{6n+5})(1-q^{6n})^2(1-q^{5n+2})^3(1-q^{5n+3})^3}$$

$$Z(t) = \gamma(q(t))$$

$$y \left(1 + 14t + 97t^2 + 415t^3 + 1180t^4 + 2321t^5 + 3247t^6 + 3300t^7 + 2475t^8 + 1375t^9 + 550t^{10} + 143t^{11} + 18t^{12} \right) +$$

$$y^2 \left(1 + 17t + 83t^2 + 601t^3 + 1647t^4 + 4606t^5 + 7809t^6 + 710t^7 + 124t^8 - 608t^9 - 440t^{10} - 92t^{11} - 36t^{12} \right) +$$

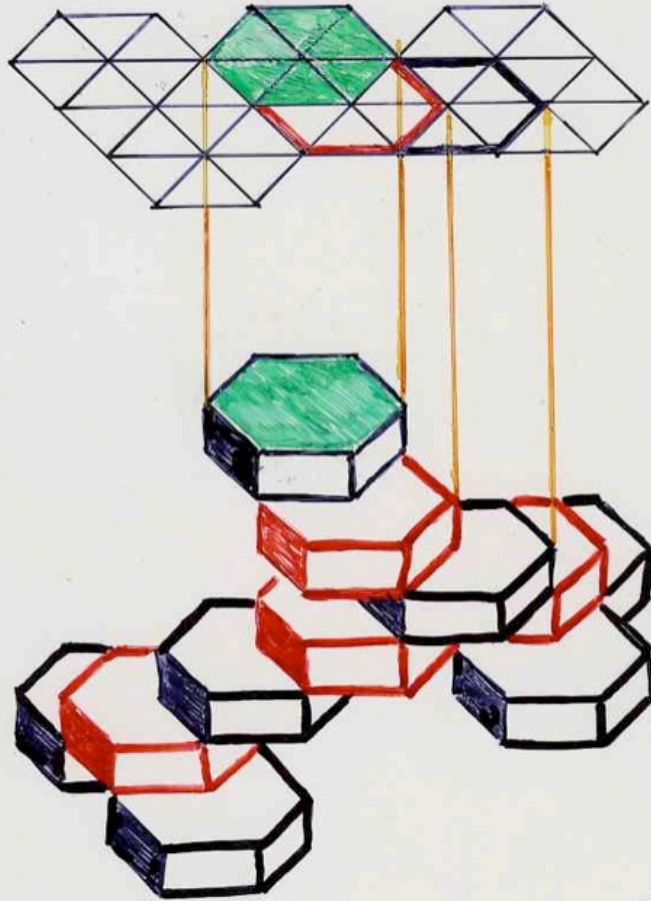
$$y^3 \left(3 + 50t + 381t^2 + 1715t^3 + 5040t^4 + 10130t^5 + 14062t^6 + 13002t^7 + 6930t^8 + 715t^9 - 1595t^{10} - 488t^{11} - 198t^{12} \right) +$$

$$y^4 \left(1 + 17t + 131t^2 + 595t^3 + 1765t^4 + 3574t^5 + 4939t^6 + 4356t^7 + 1815t^8 - 605t^9 - 1210t^{10} - 616t^{11} - 126t^{12} \right)$$

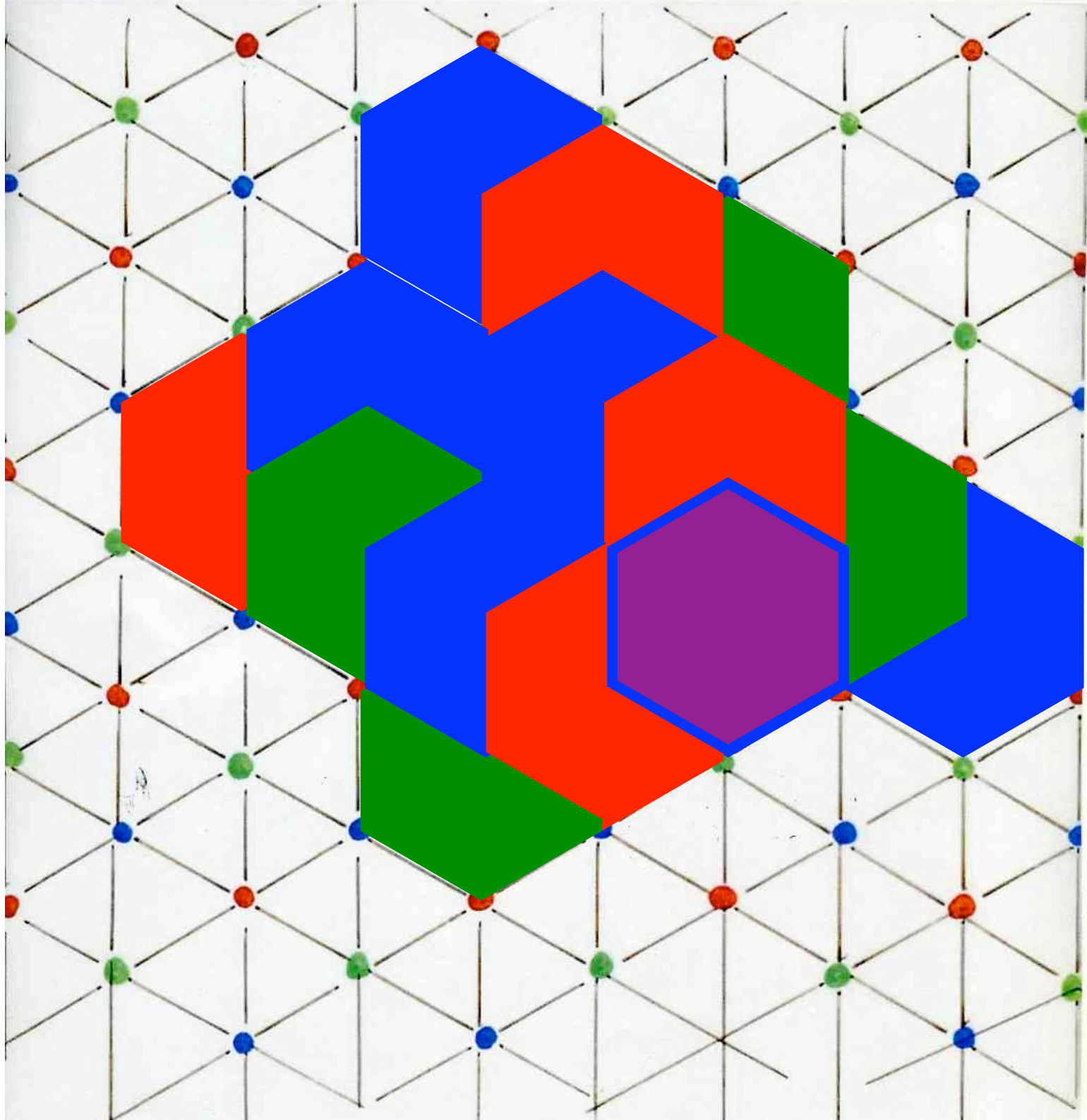
==

$$\left(t + 11t^2 + 55t^3 + 165t^4 + 330t^5 + 462t^6 + 462t^7 + 330t^8 + 165t^9 + 55t^{10} + 11t^{11} + t^{12} \right)$$

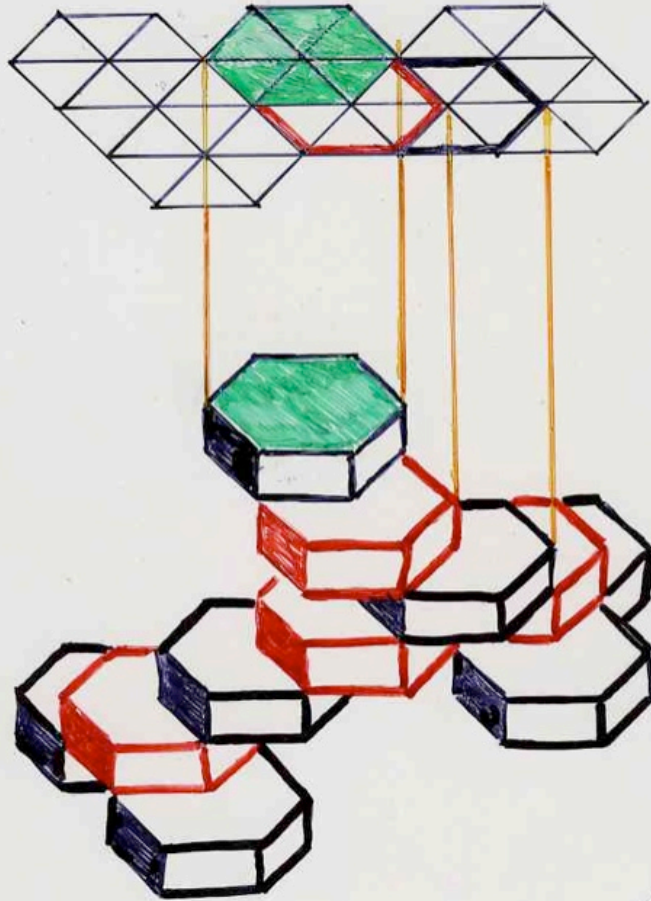
$$-p(-t) = y$$



Le concept d'empilements de pièces



$$-p(-t) = y$$



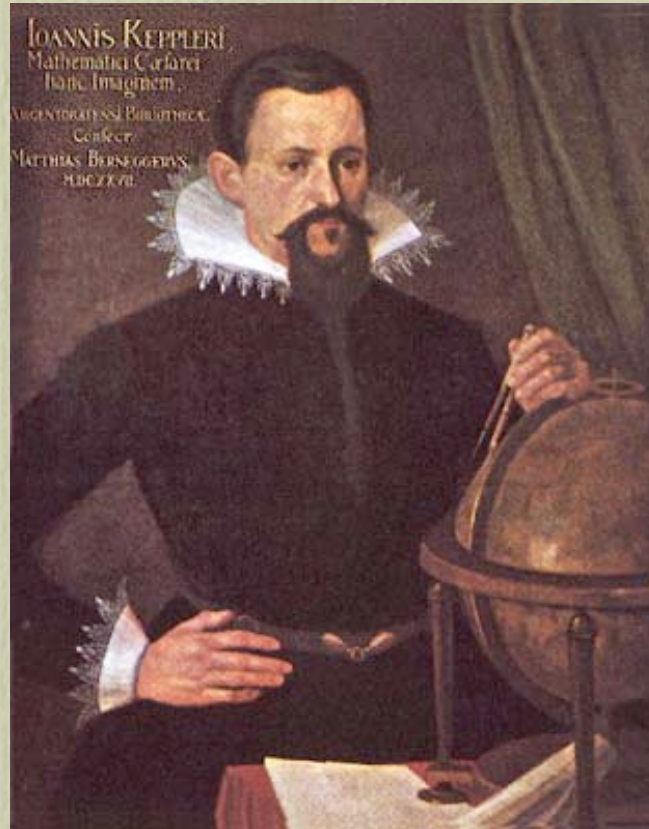
Physique combinatoire

SAINT-EXUPÉRY

L'infiniment grand

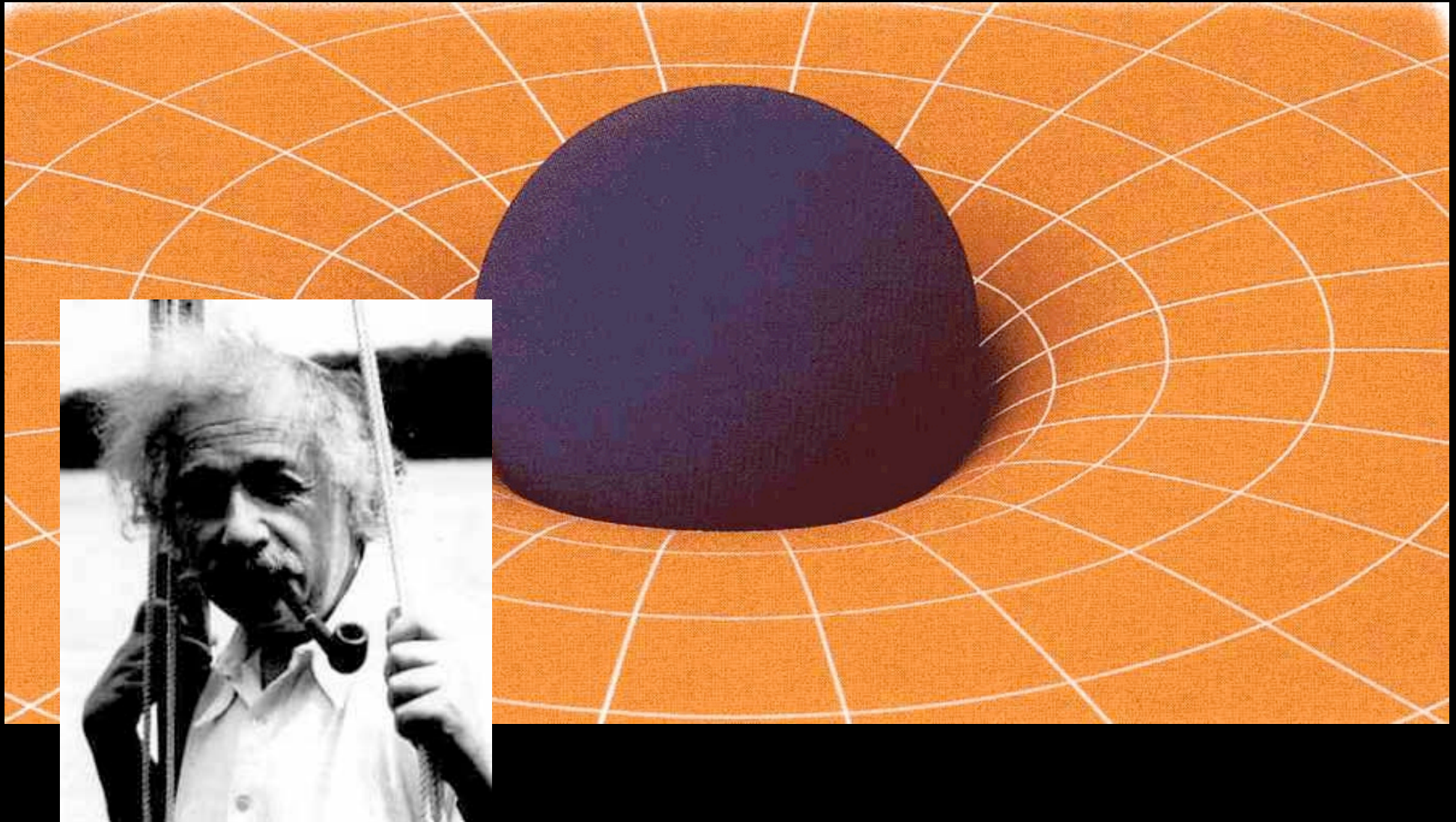
Des arbres dans les étoiles ?





géométrie et mécanique classique
Galileo, Kepler, Newton,...

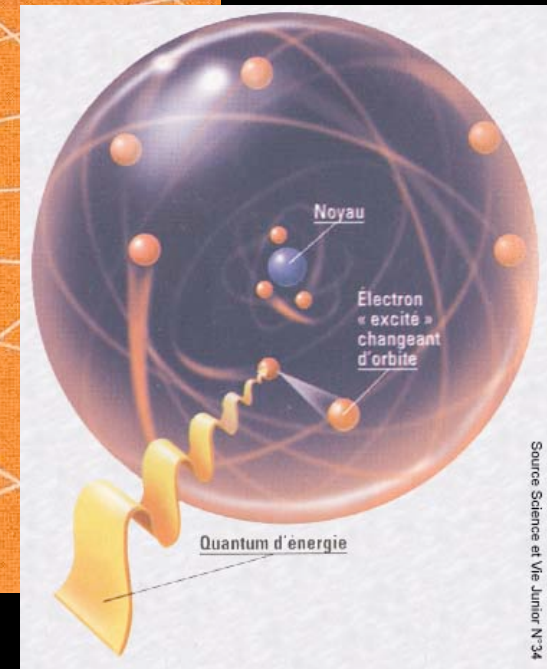
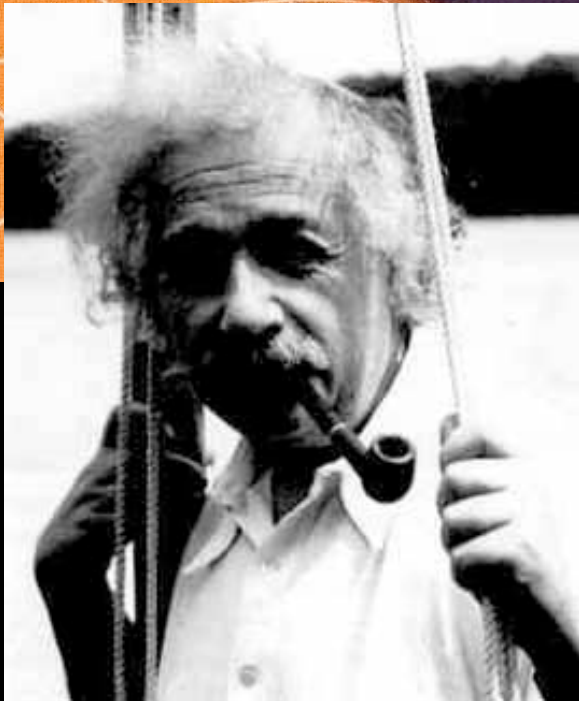
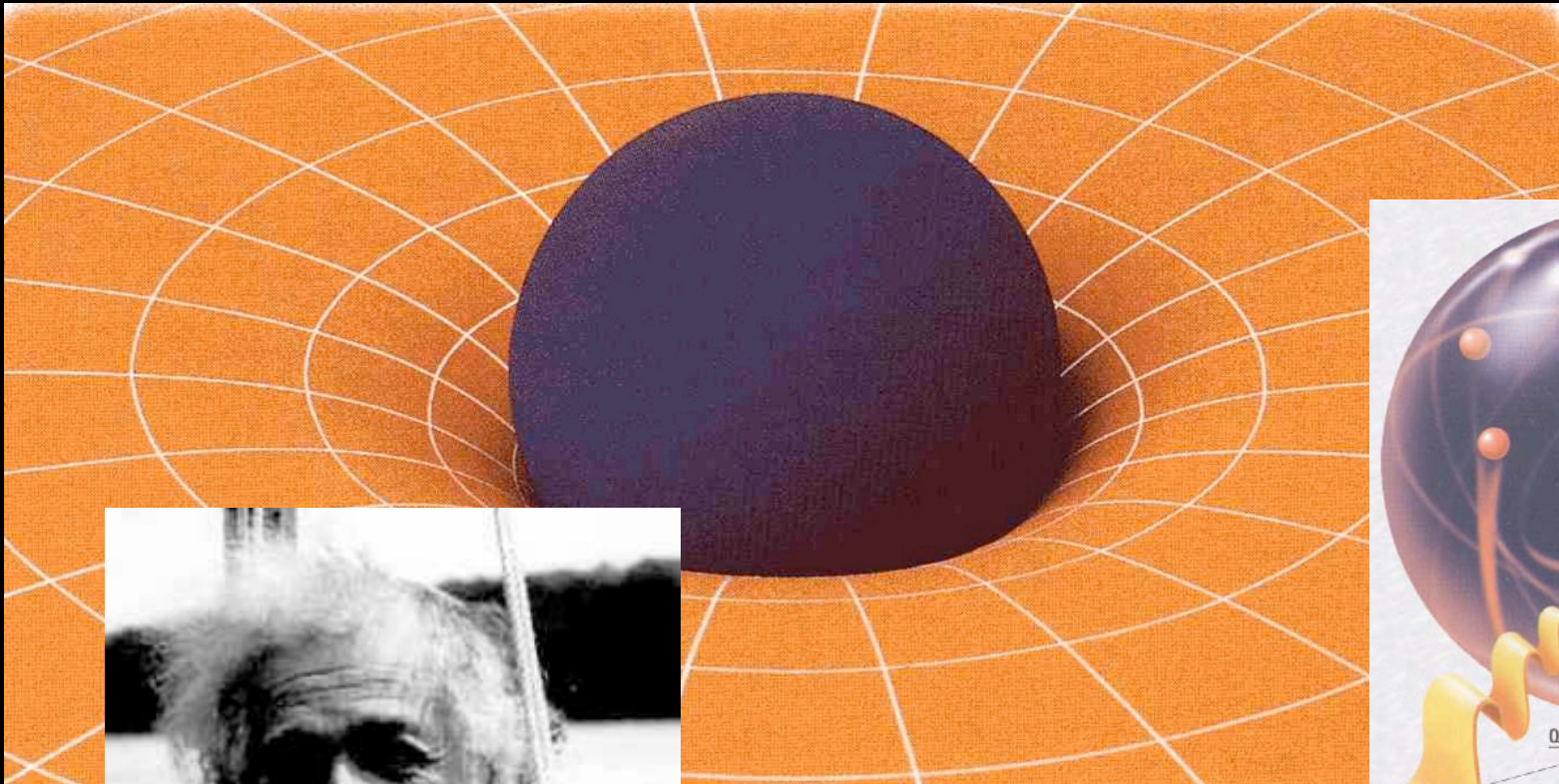
relativité générale





relativité générale

mécanique quantique



gravitation quantique

théorie des cordes

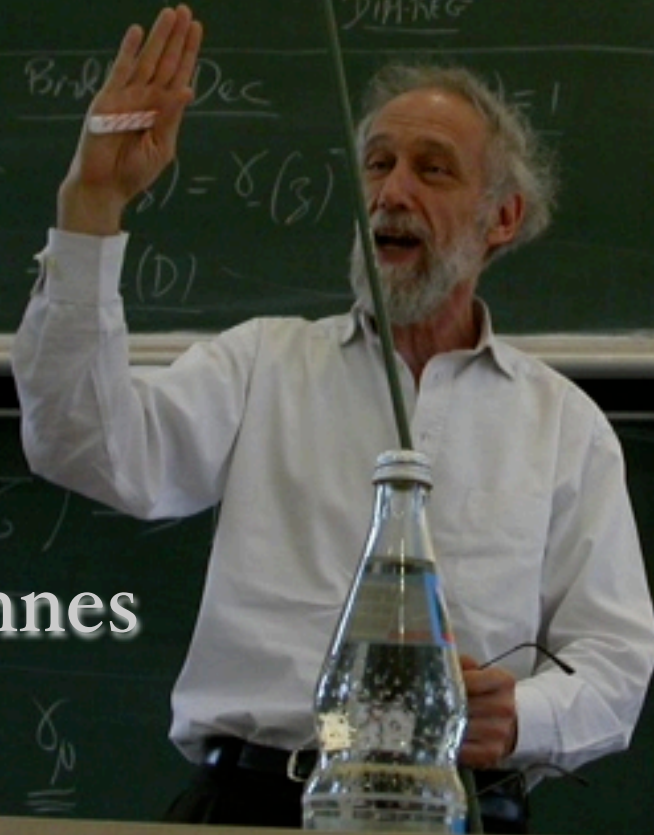
particule comme une corde de violon ... ?

chaque fréquence correspond à une particule.... ?

nombre de Catalan



géométrie non-commutative



Alain Connes

Universal Singular Fran

$$\gamma_W(z, v) = \text{Tr} e^{-\frac{1}{z} \int_0^v u^Y(e) \frac{du}{u}}$$

$$\gamma_W(-z, v) = \sum_{n \geq 0} \sum_{k_j > 0}$$

$$\frac{e(-k_1)e(-k_2) \cdots e(-k_n)}{k_1(k_1+k_2) \cdots (k_1+k_2+\cdots)}$$

Same coefficients as

Local Index Formula in NCC

$\int e^{-\frac{x^2}{\lambda}}$
 $\int e^{-\lambda k^2} d^D k = \left(\frac{\pi}{\lambda}\right)^{D/2}$
DIM-REG
D. KREIMER
Birk Dec
 $\chi(z) \in G$
 $\chi(z) = \chi(z)$
 (D)

$\text{Div}(\xi)$
M. MARCOLLI
 χ

gravitation quantique à boucles



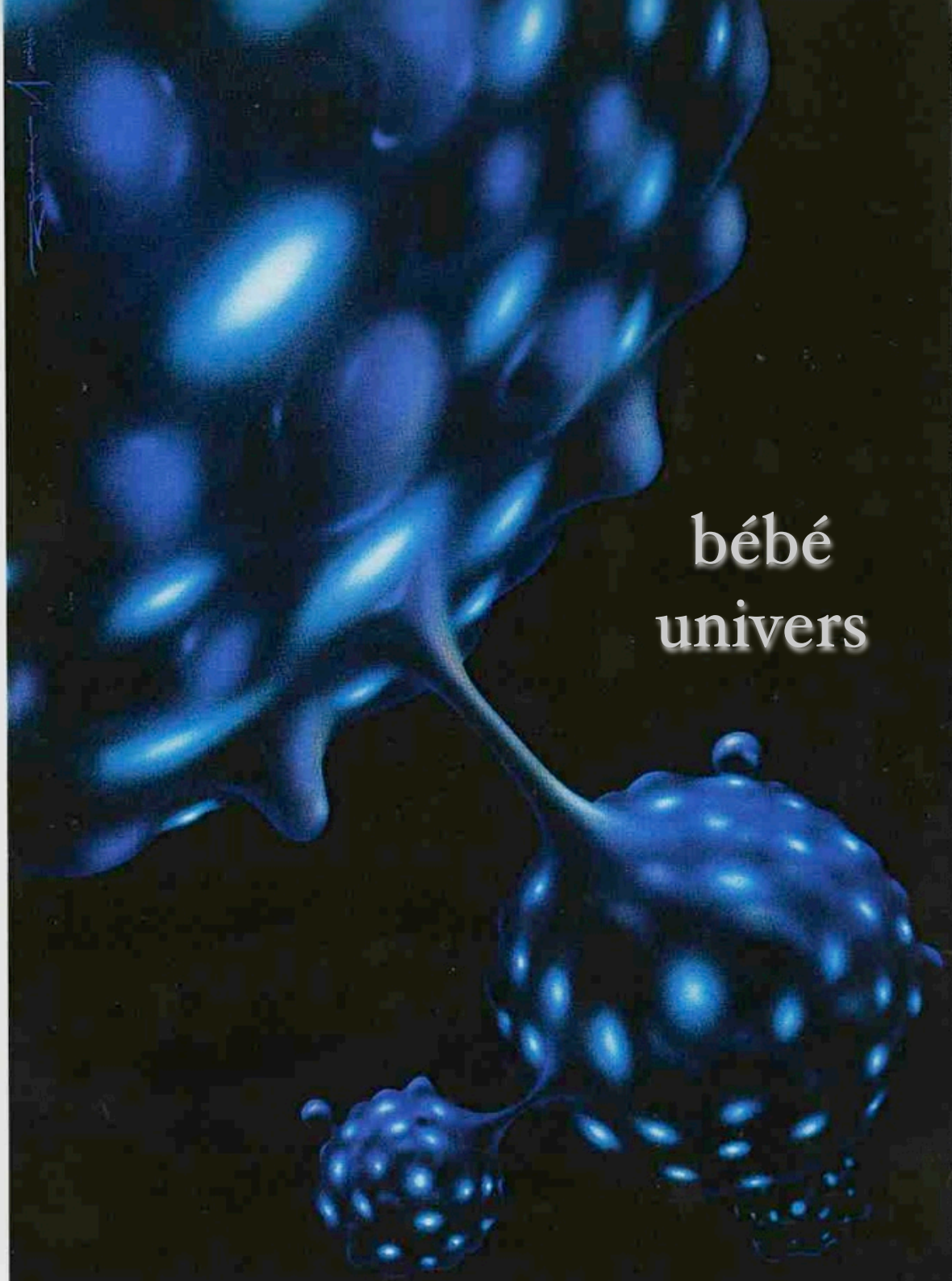
Carlo Rovelli

Et si le temps n'existait pas ?

écume de
l'espace-temps



Dessin
S. Numazawa
Ciel & Espace



bébé
univers

WILLIAM BLAKE

gravitation quantique

triangulations causales dynamiques





Deepak Dhar
TIFR Bombay

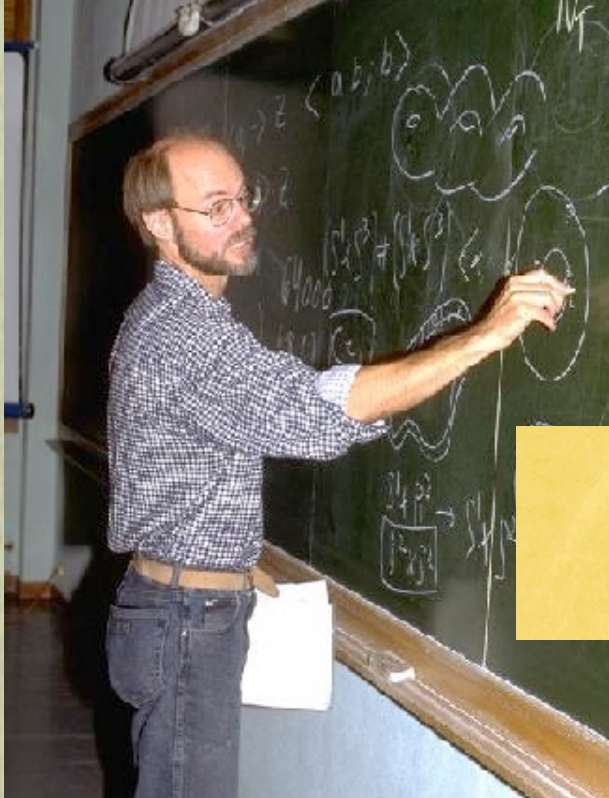
Xavier, you should have
a look at these papers:

— J. Ambjørn, R. Loll, "Non-perturbative Lorentzian quantum gravity and topology change", Nucl. Phys. B 536 (1998) 407-436
arXiv: hep-th/9805108

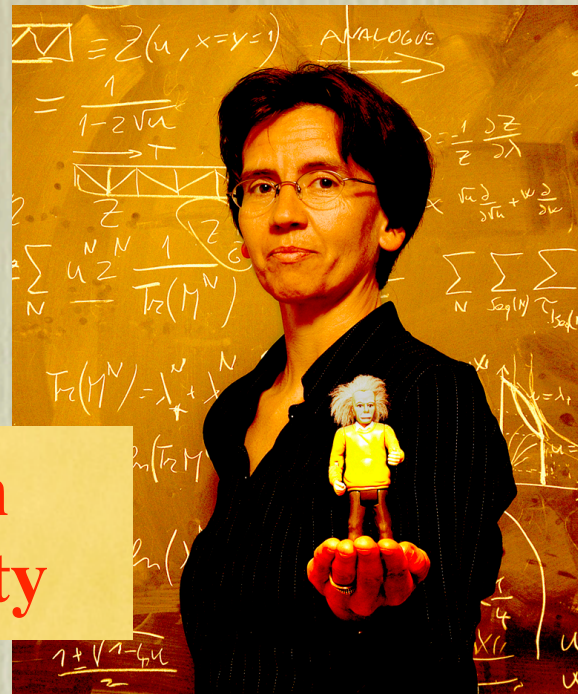
— P. Di Francesco, E. Guilteer, C. Kristjansen, "Integrale 2D Lorentzian gravity and random walks", Nucl. Phys. B 567 (2000) 515-553
arXiv: hep-th/9907084

gravitation

quantique



J. Ambjørn



R. Loll

**2D Lorentzian
quantum gravity**



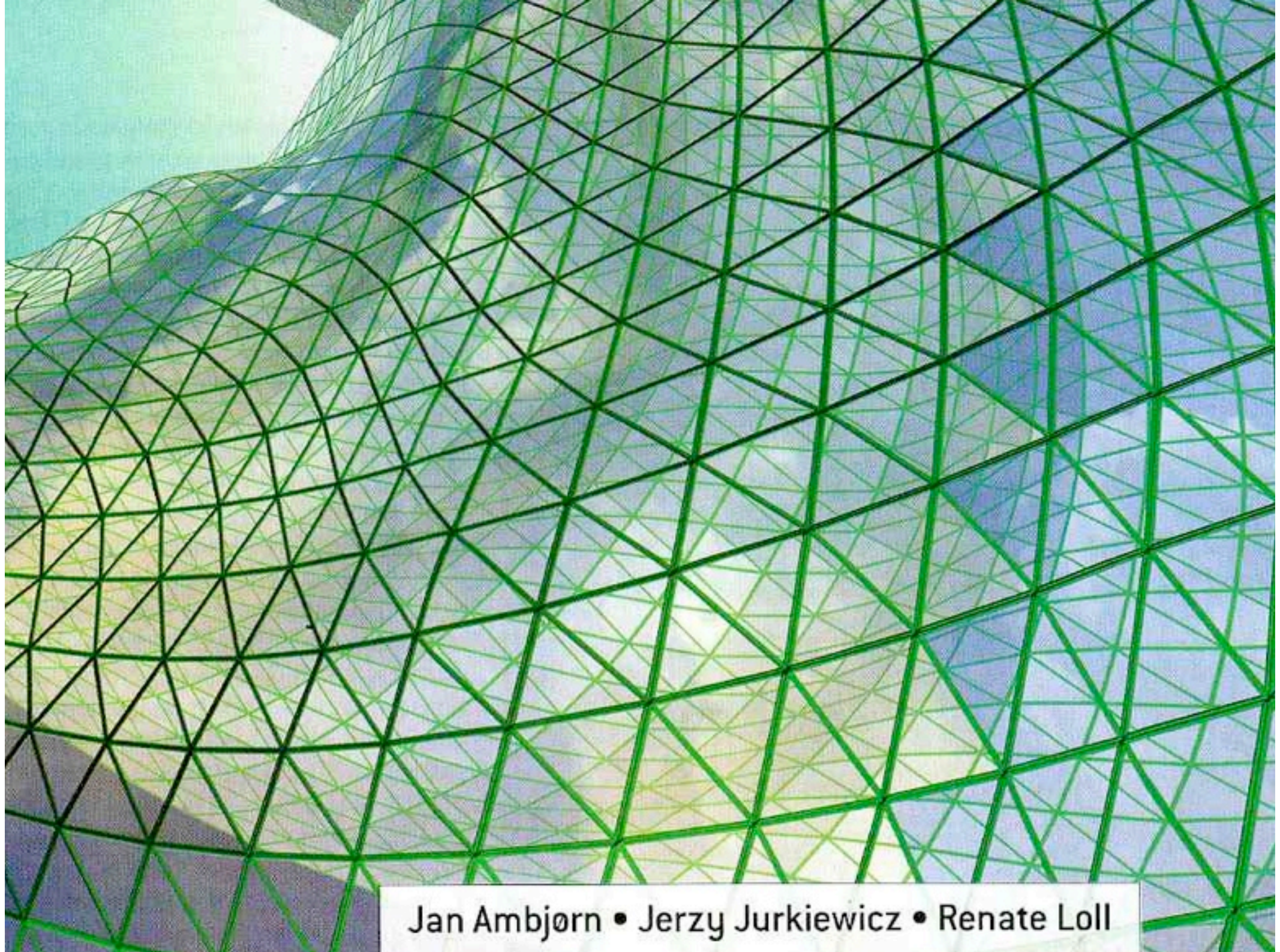
P. Di Francesco



E. Gitter

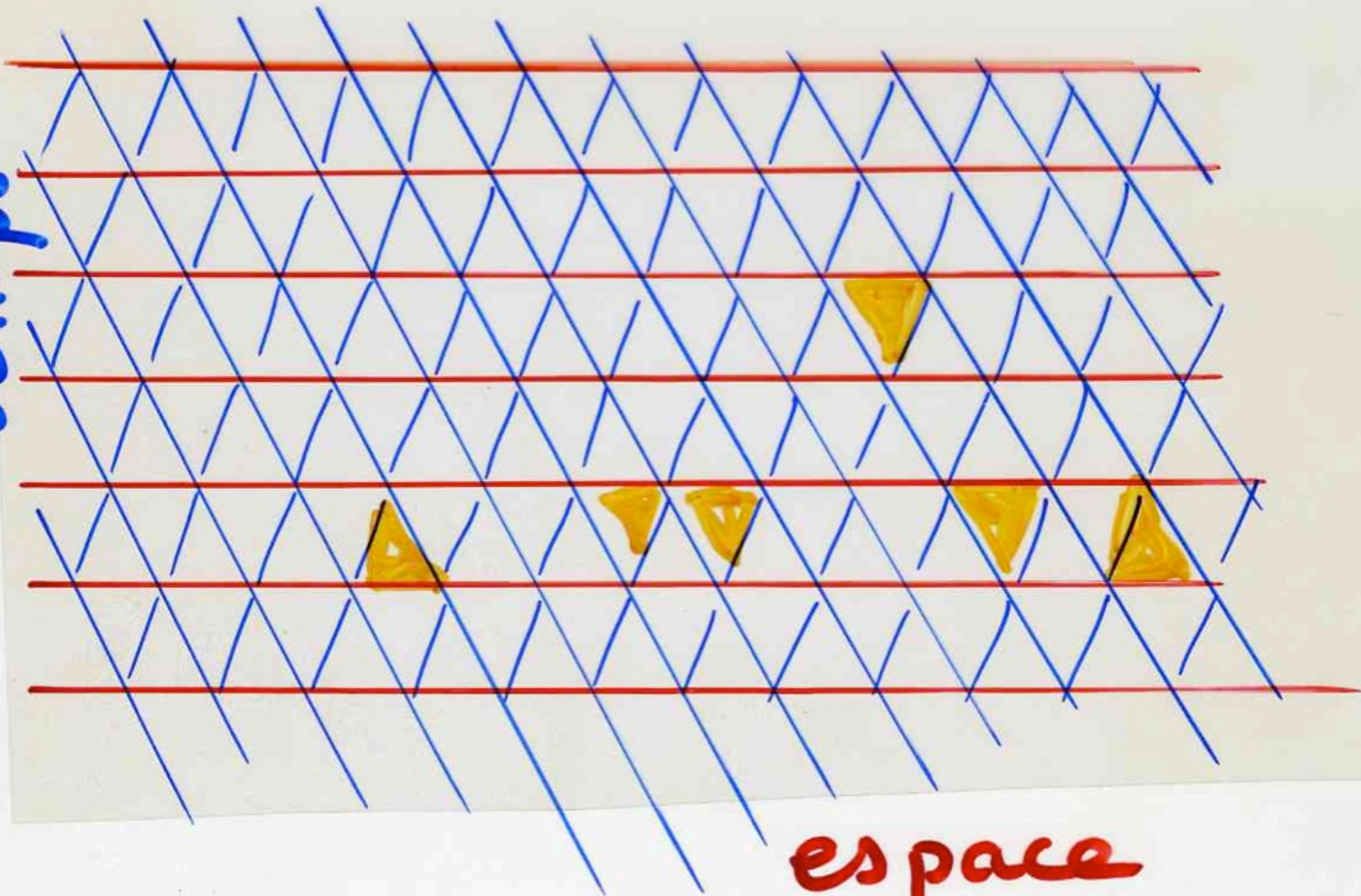


C. Kristjansen

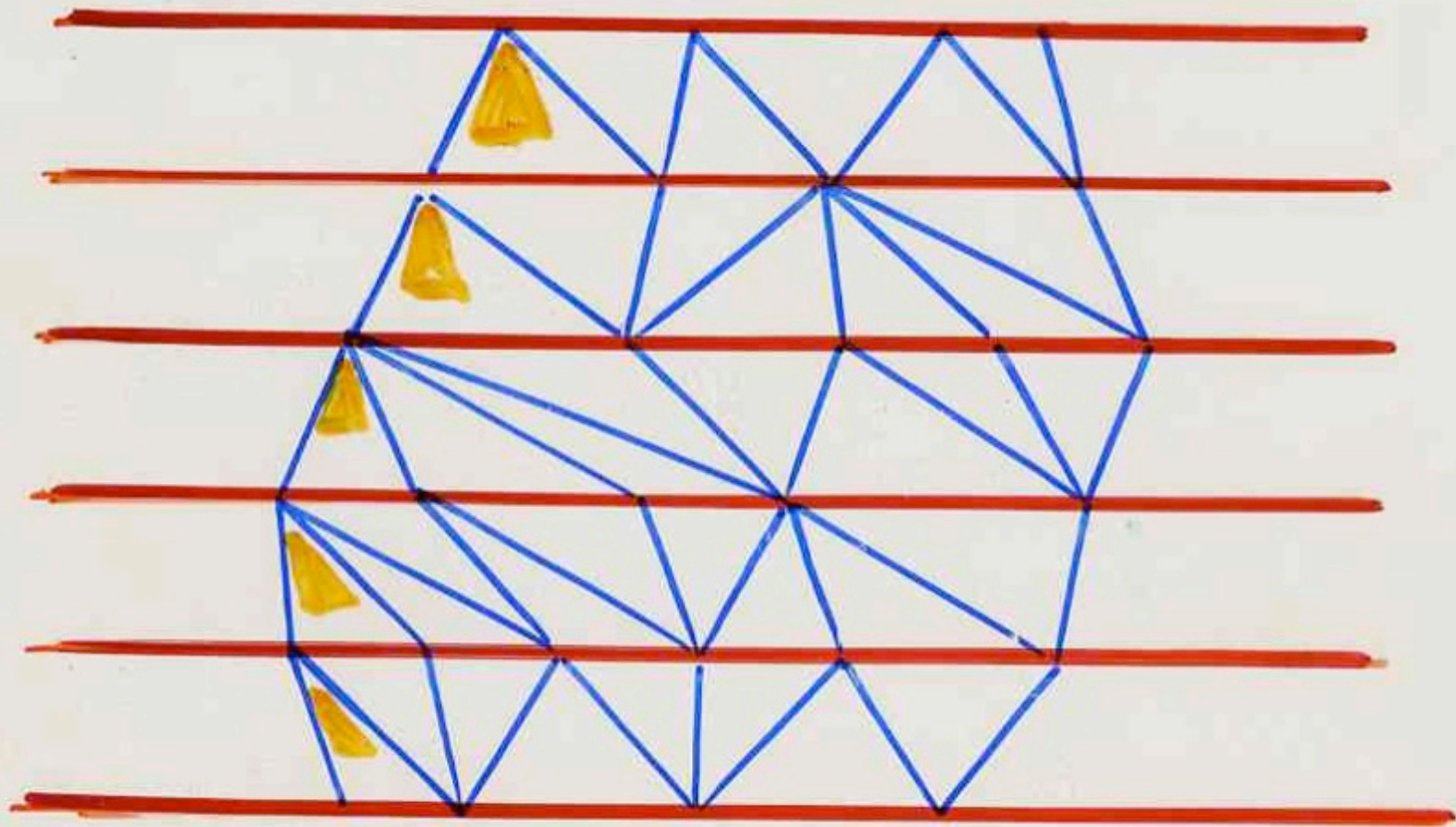


Jan Ambjørn • Jerzy Jurkiewicz • Renate Loll

temps



espace



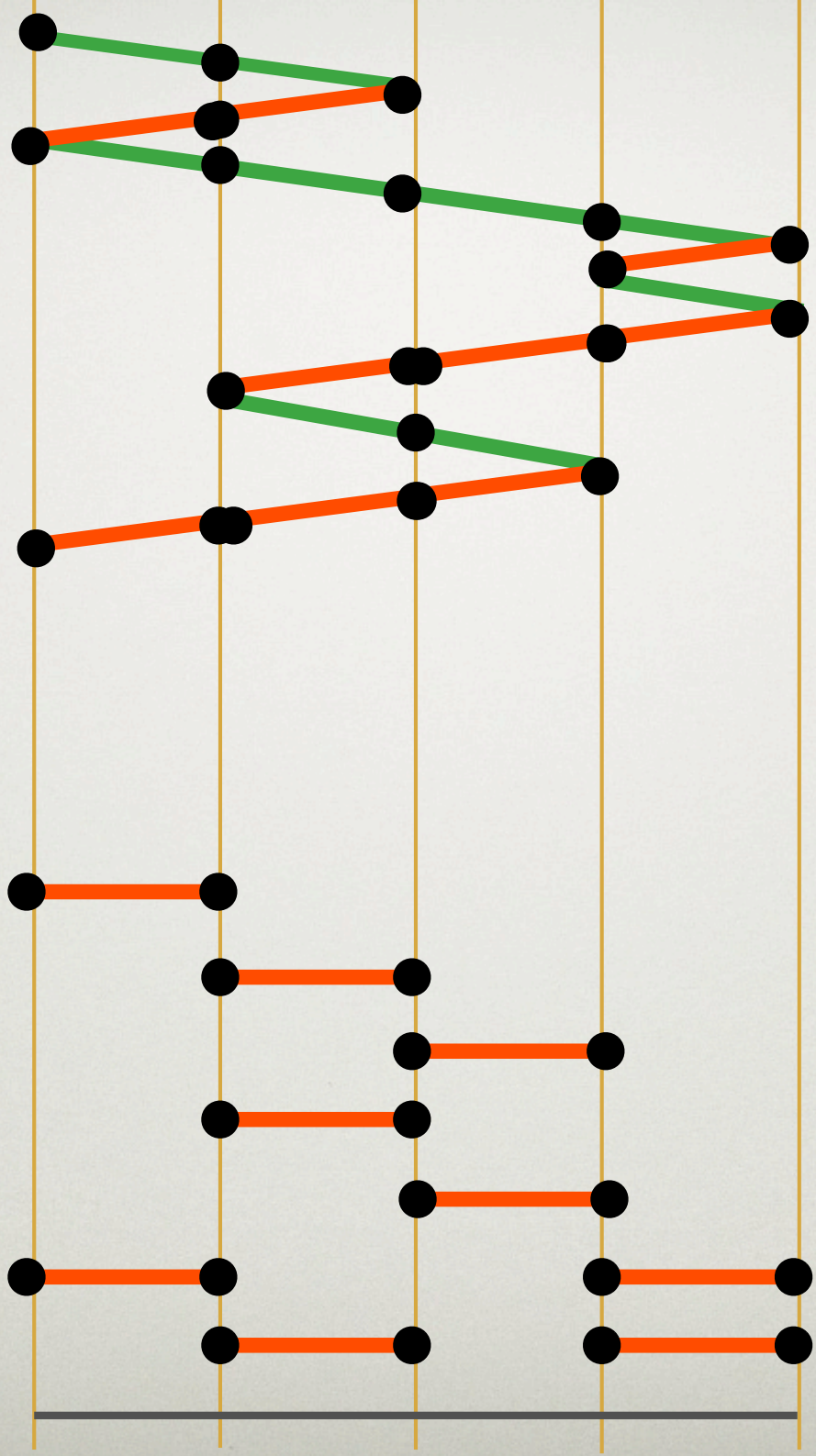
Catalan

number

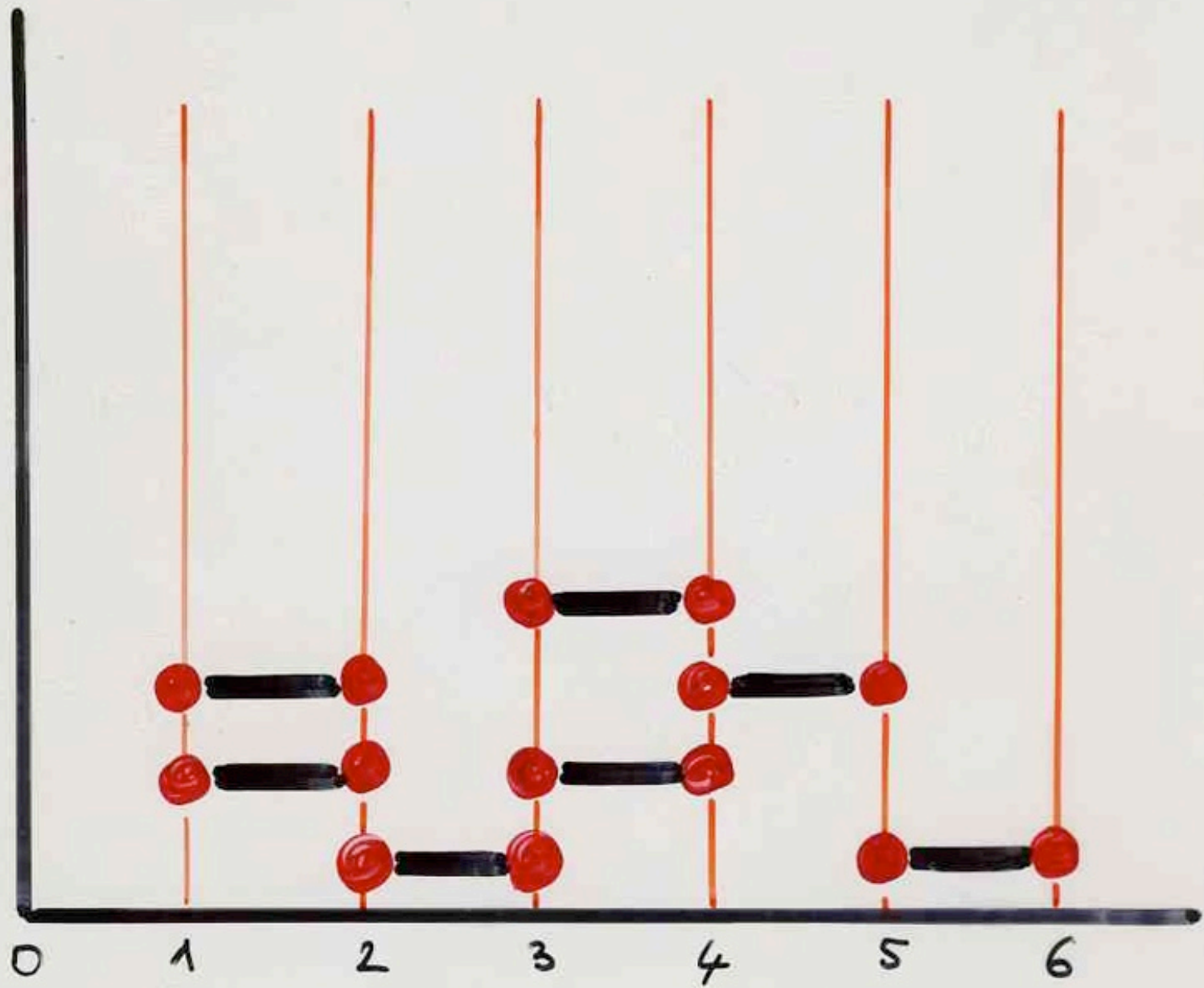


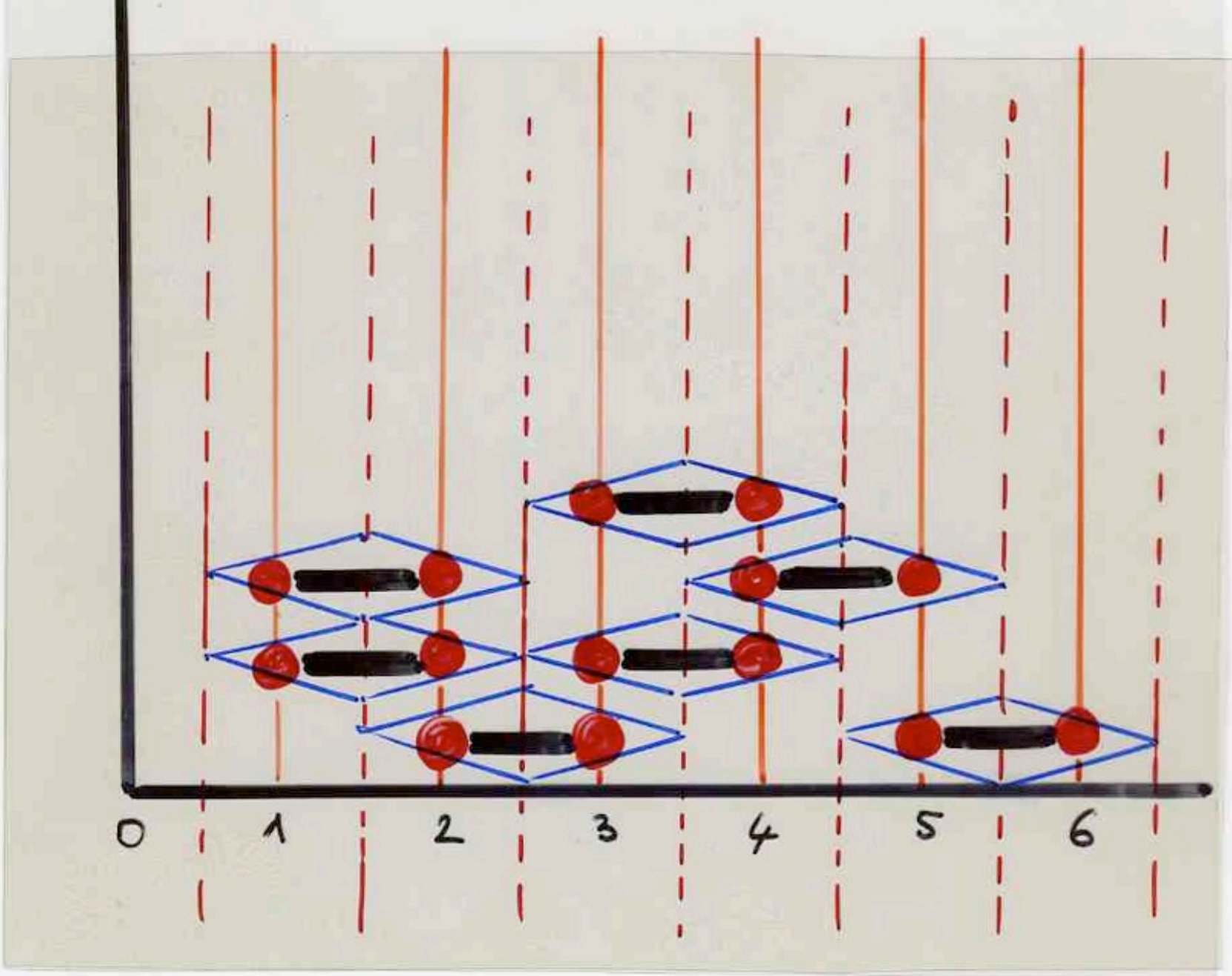
$$C_n = \frac{1}{(n+1)} \binom{2n}{n}$$

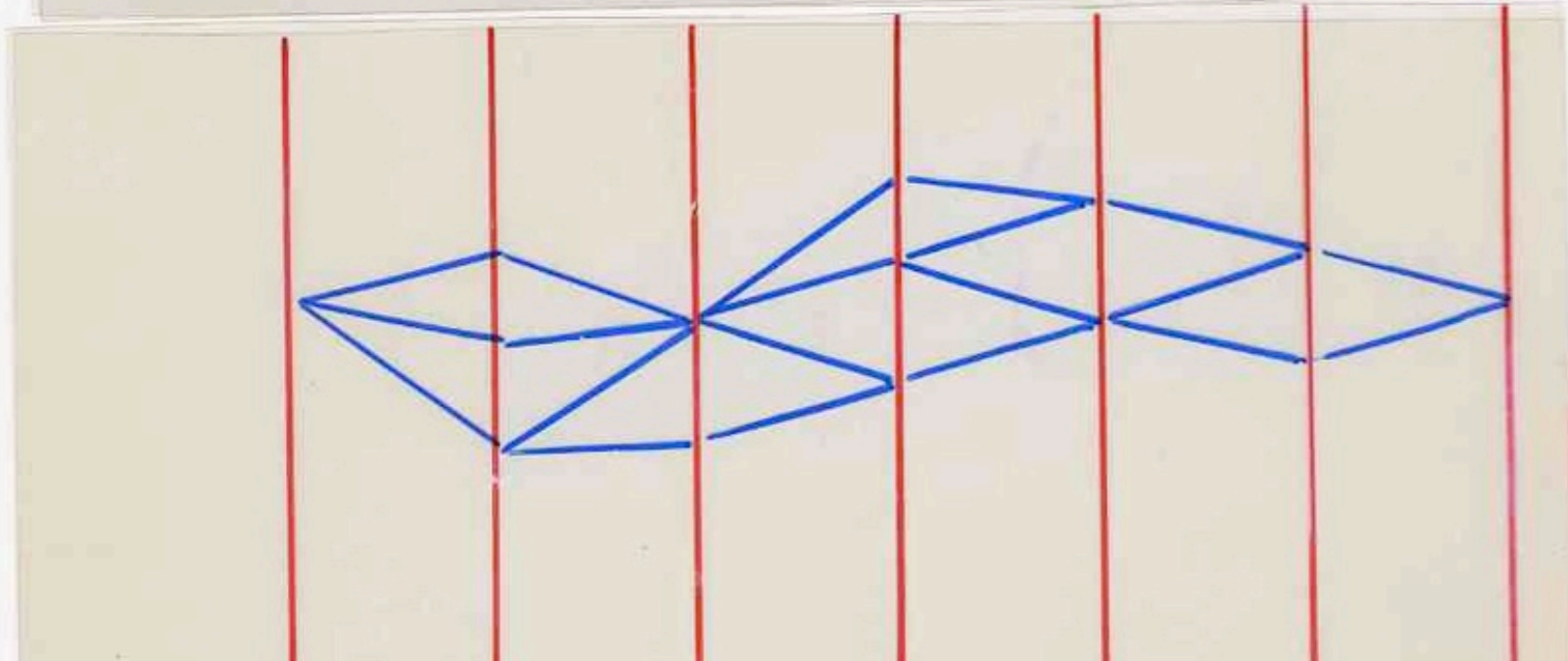
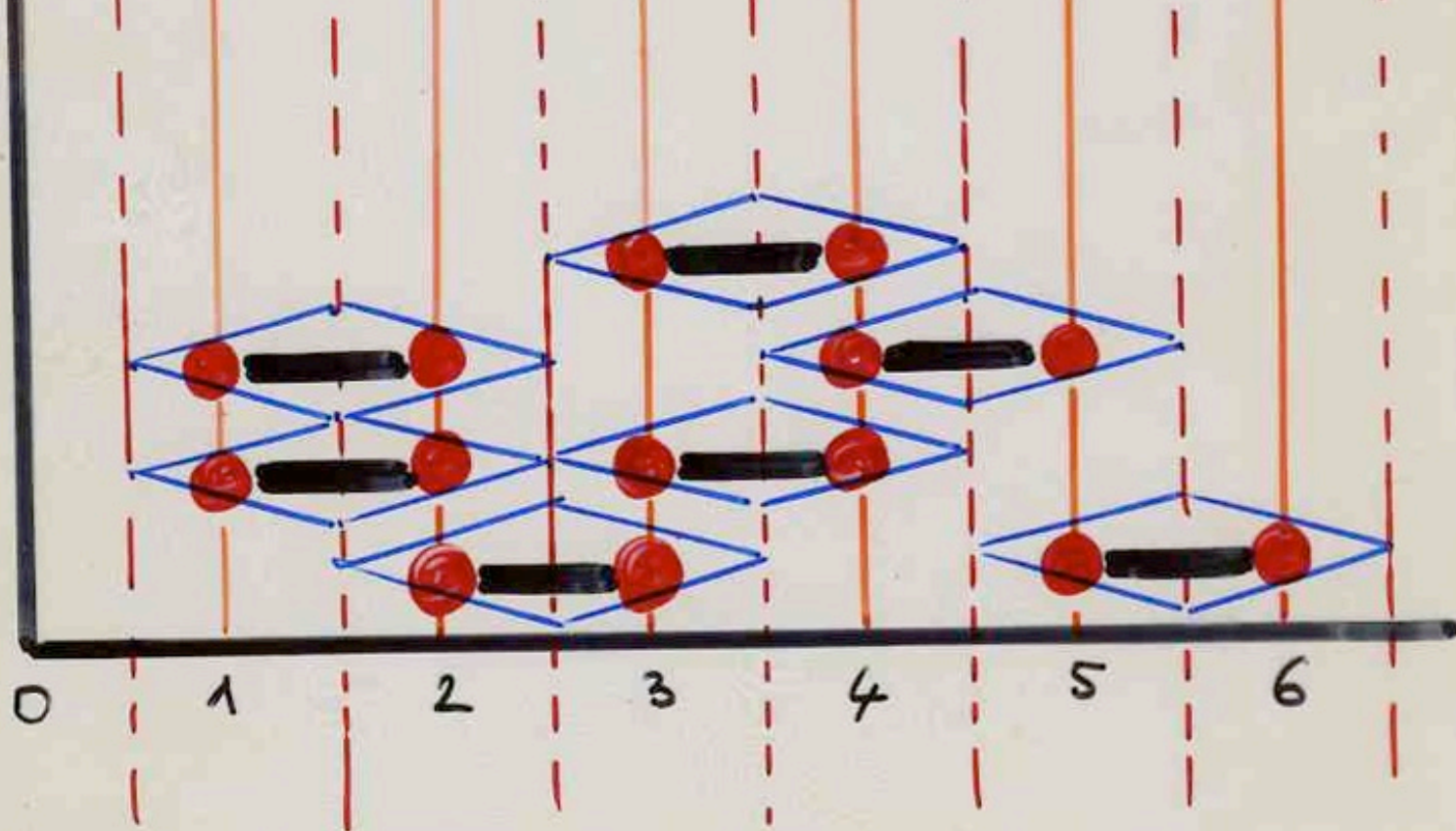
des chemins de Dyck
aux empilements de dominos

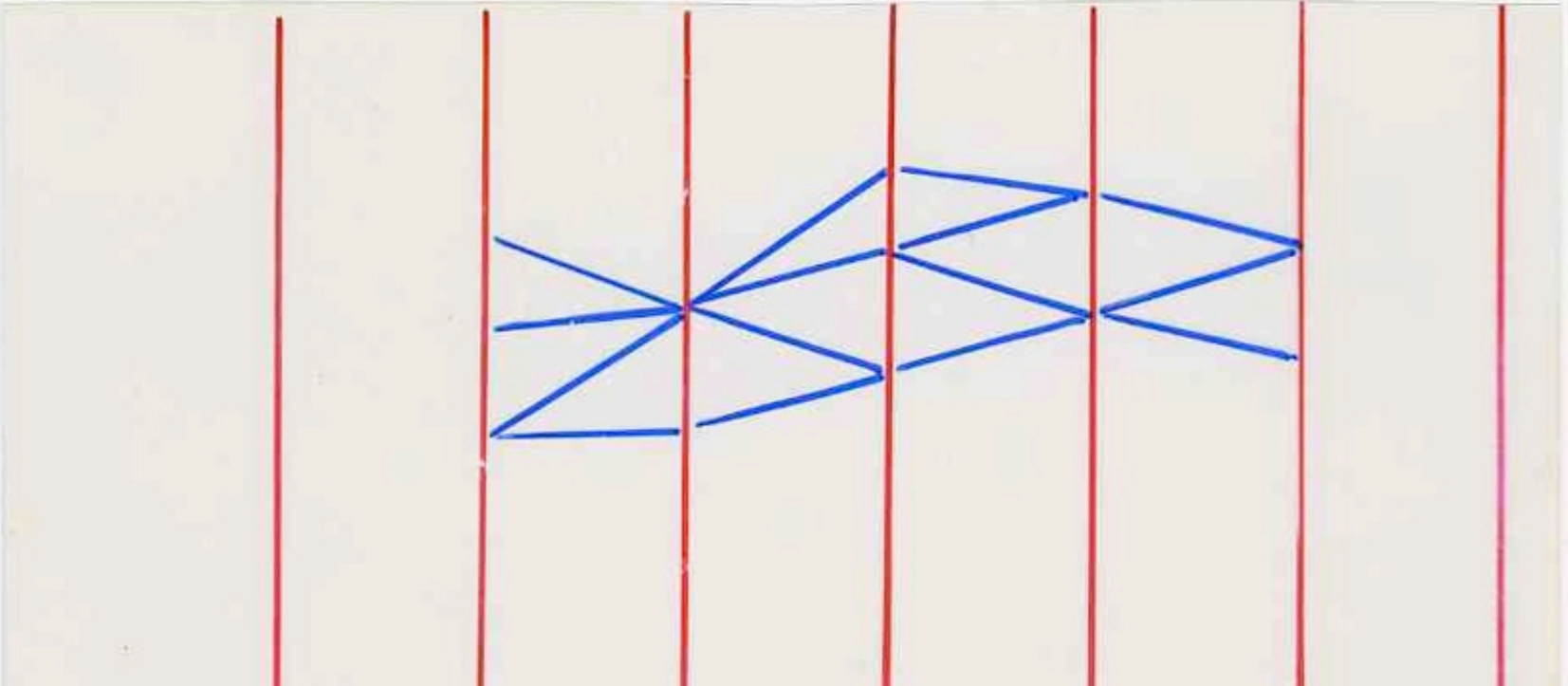
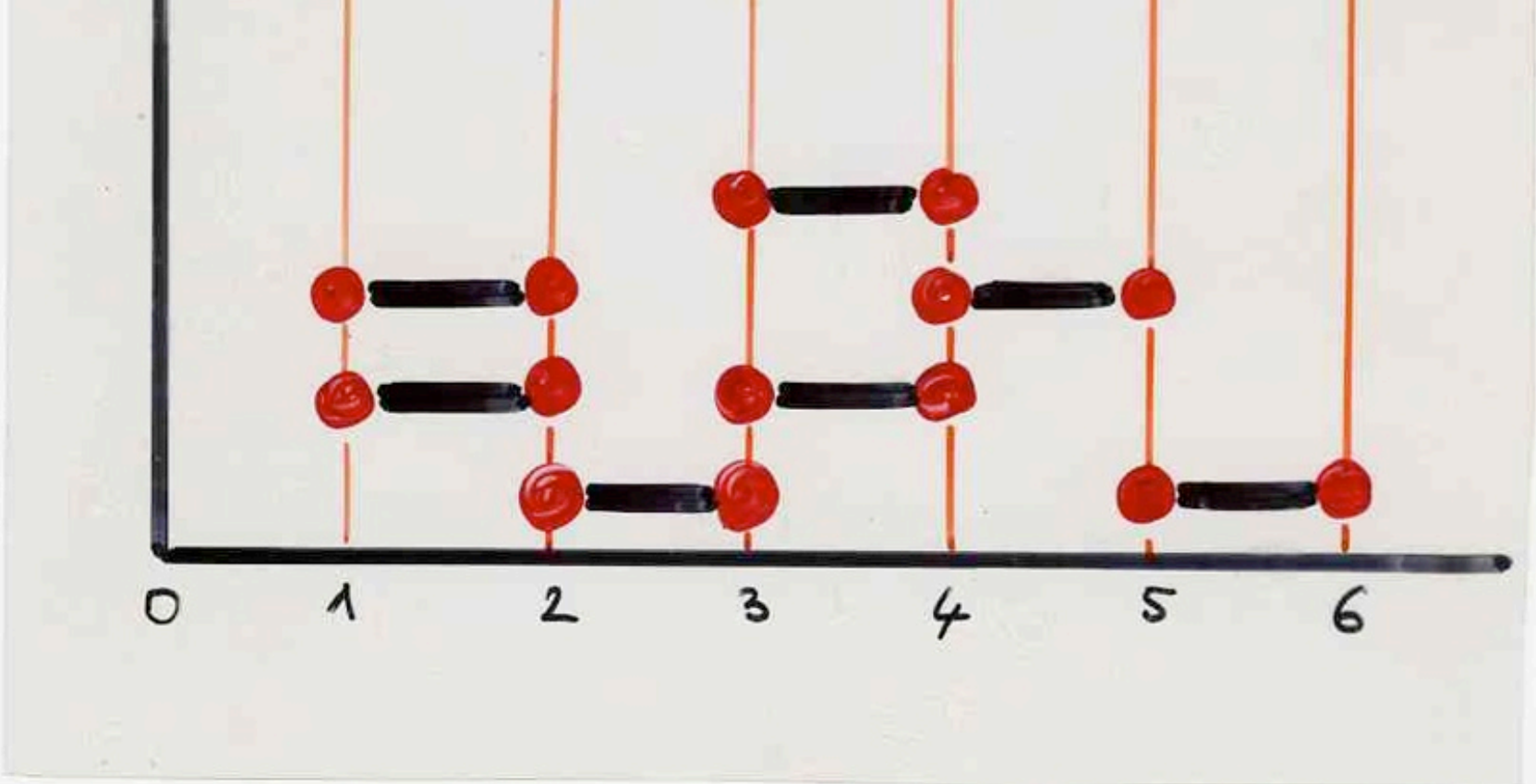


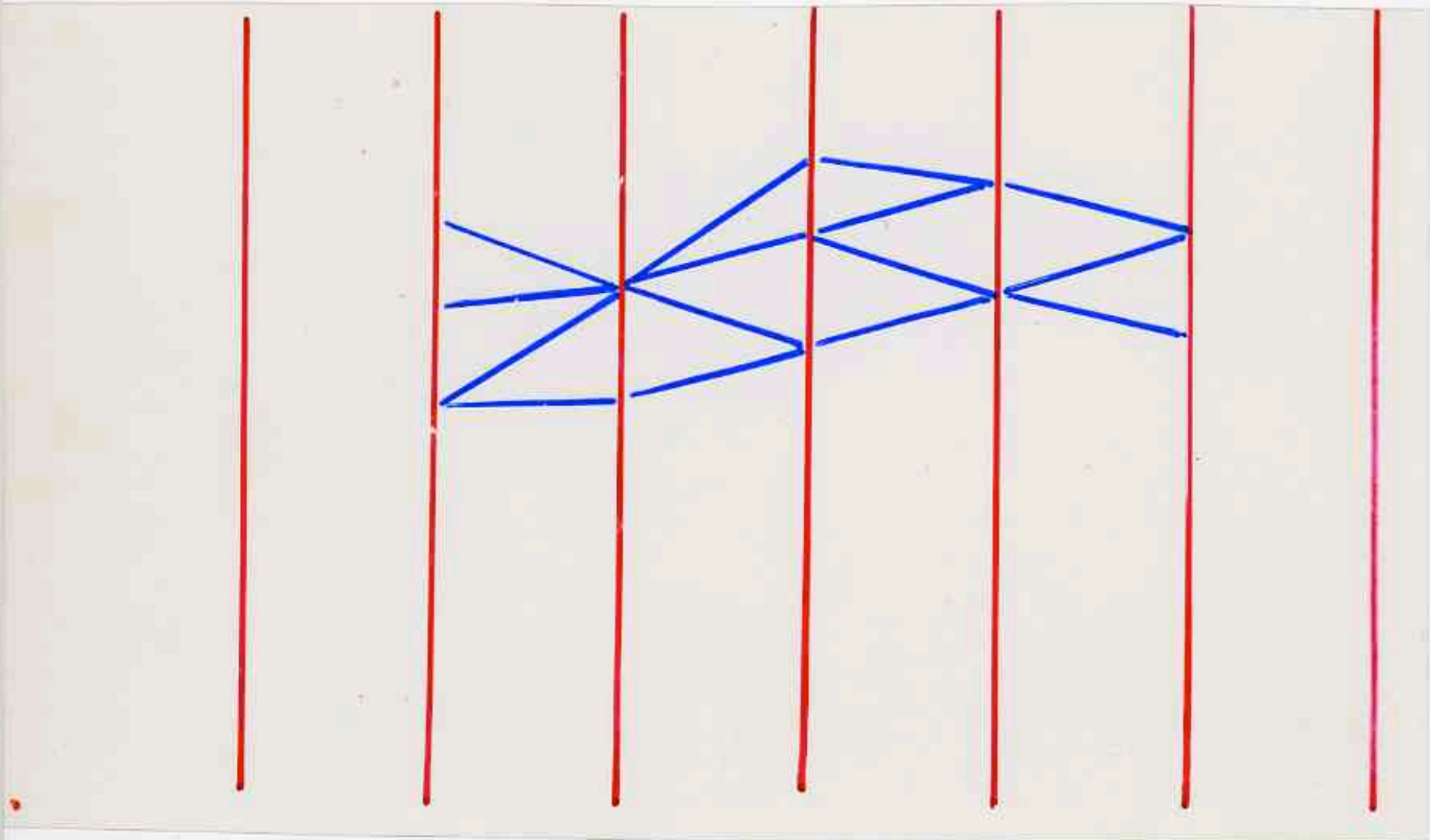
Des empilements de dominos
aux triangulations Lorentziennes

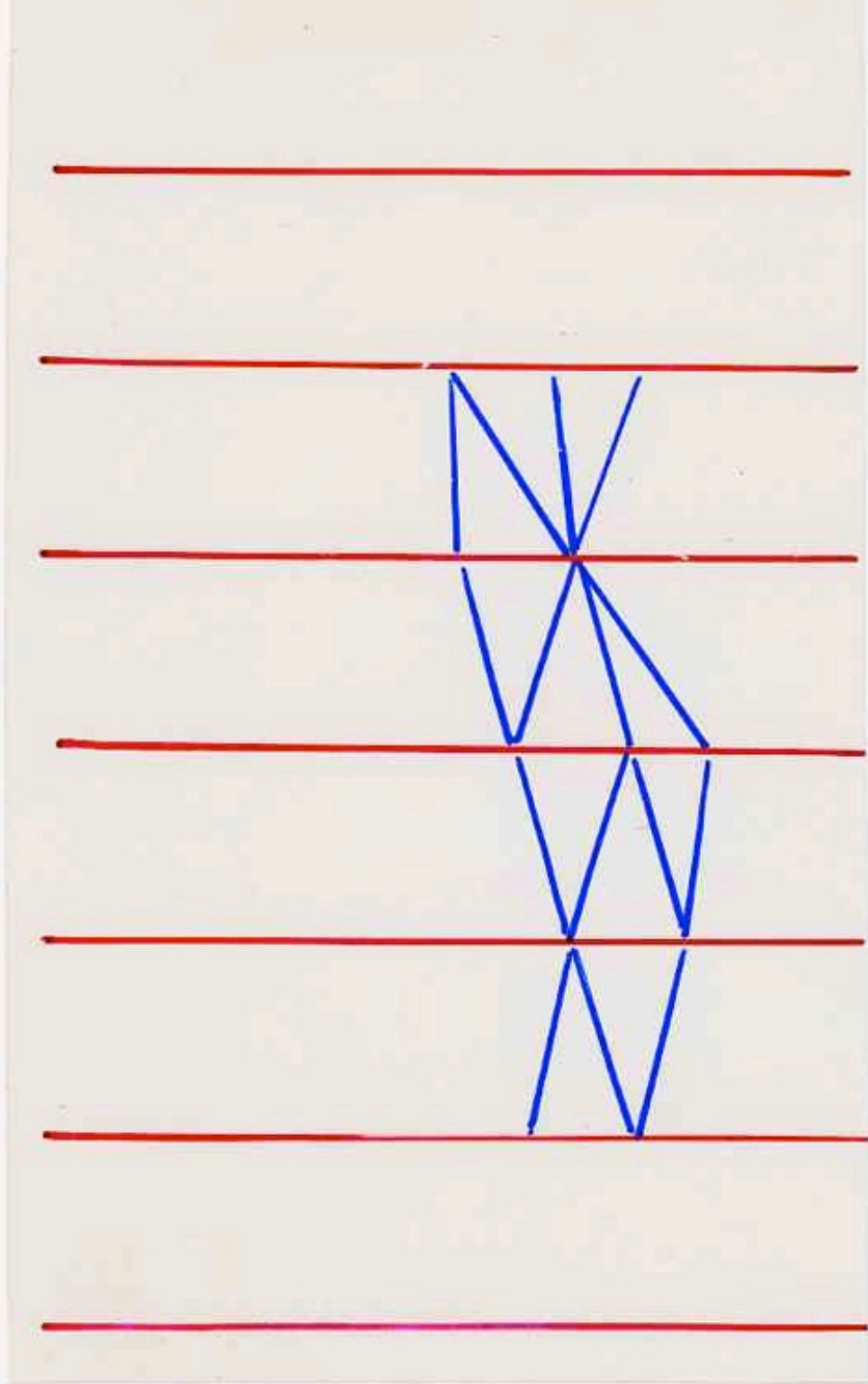




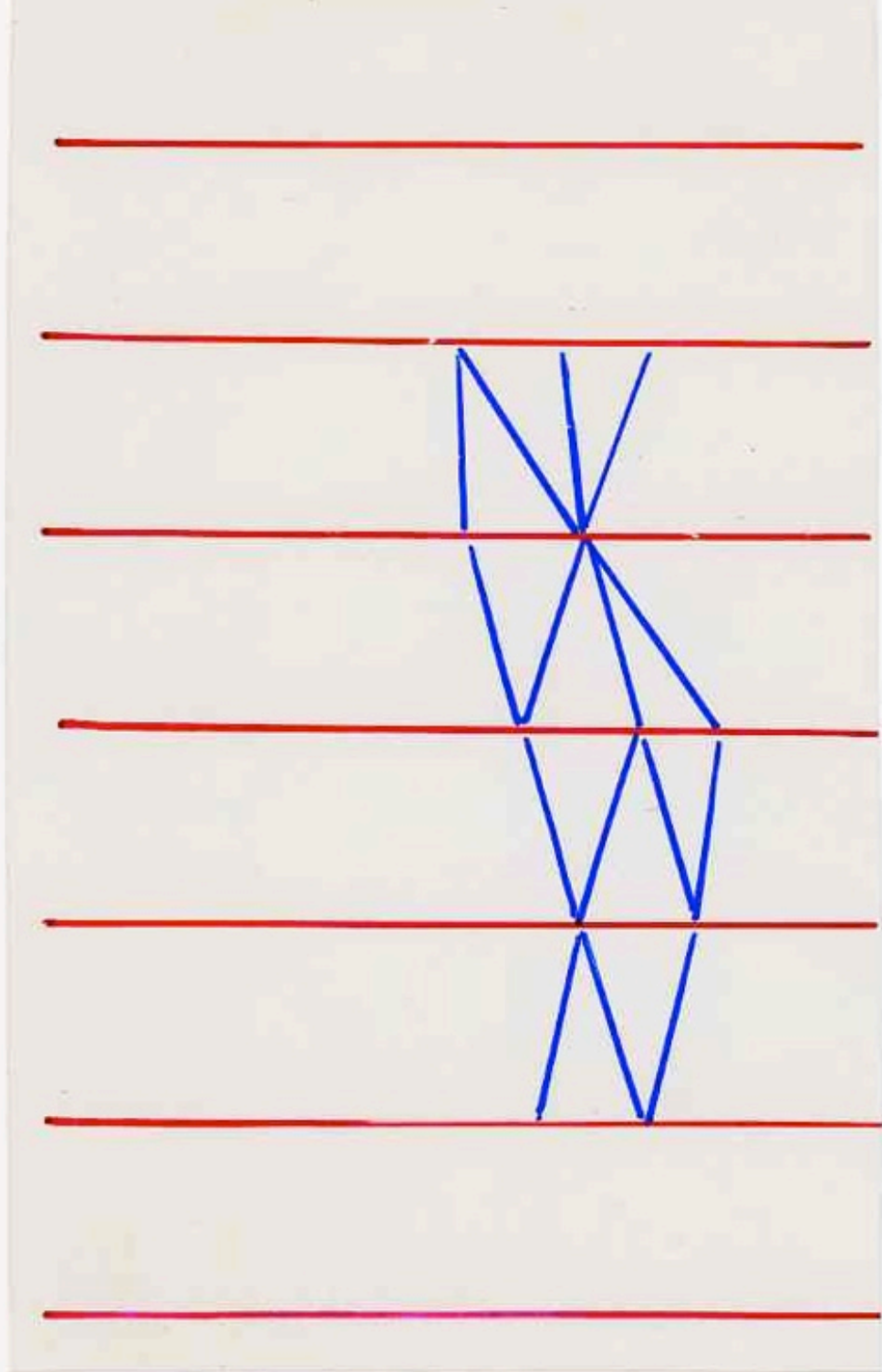


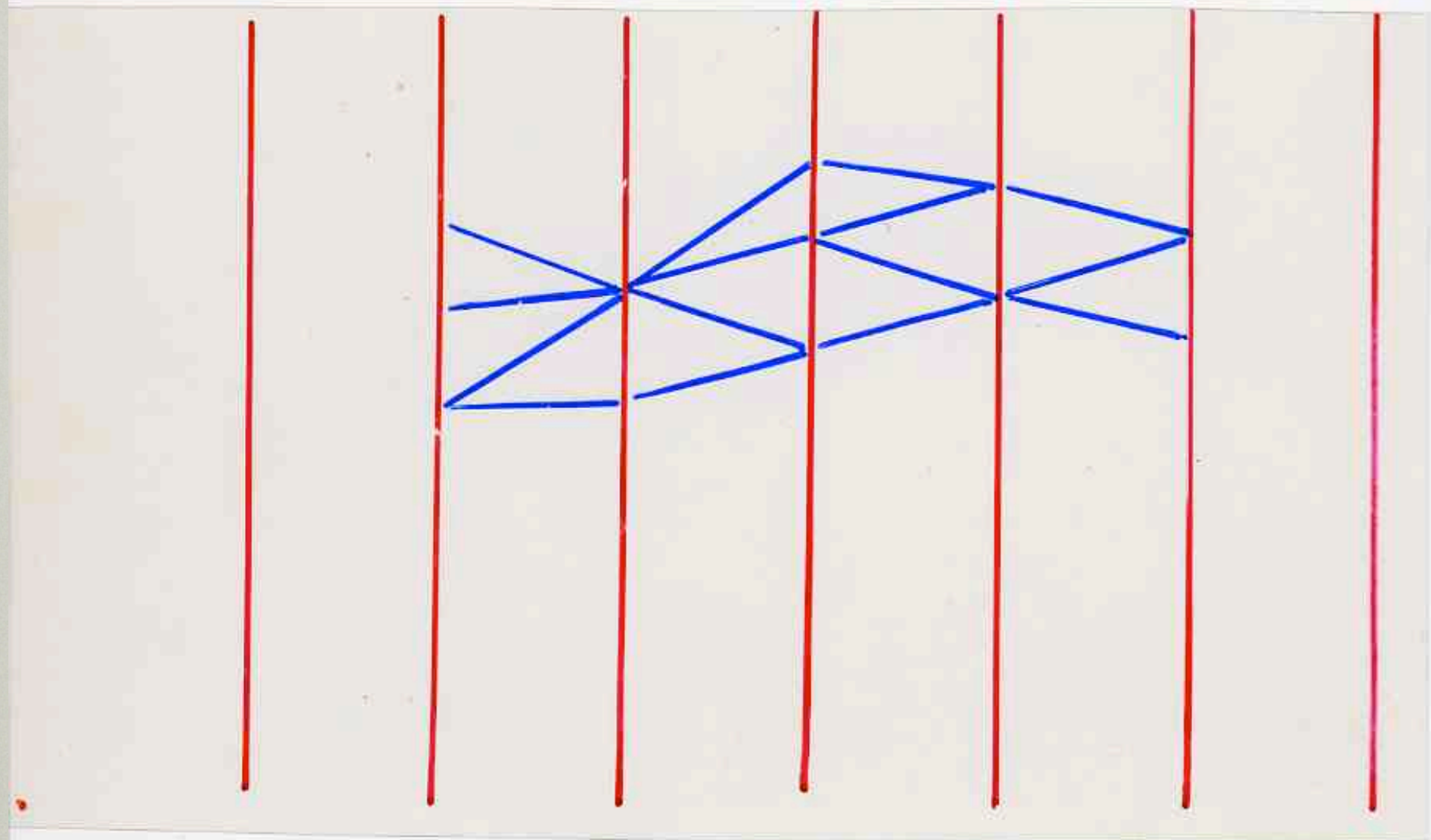


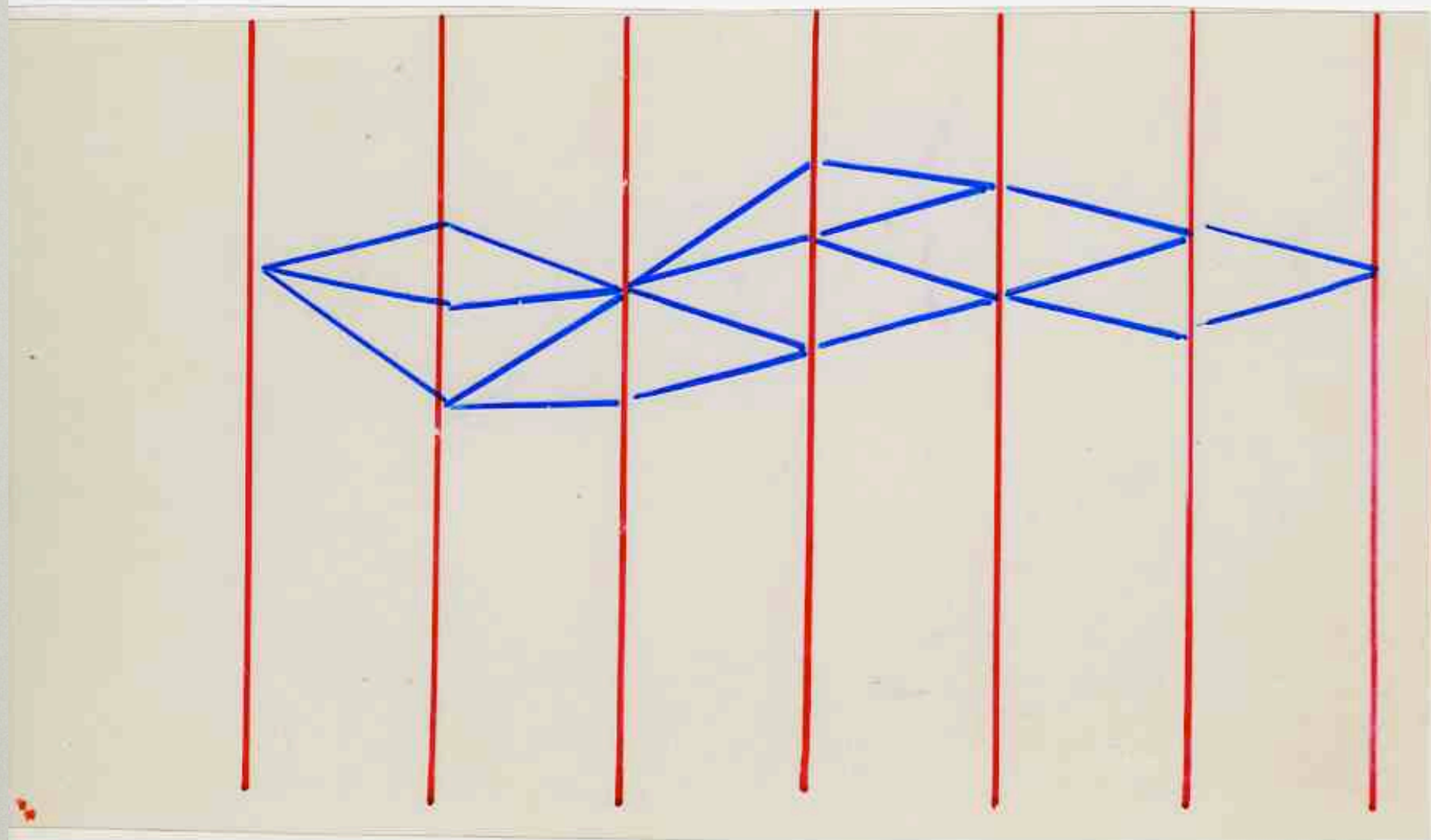


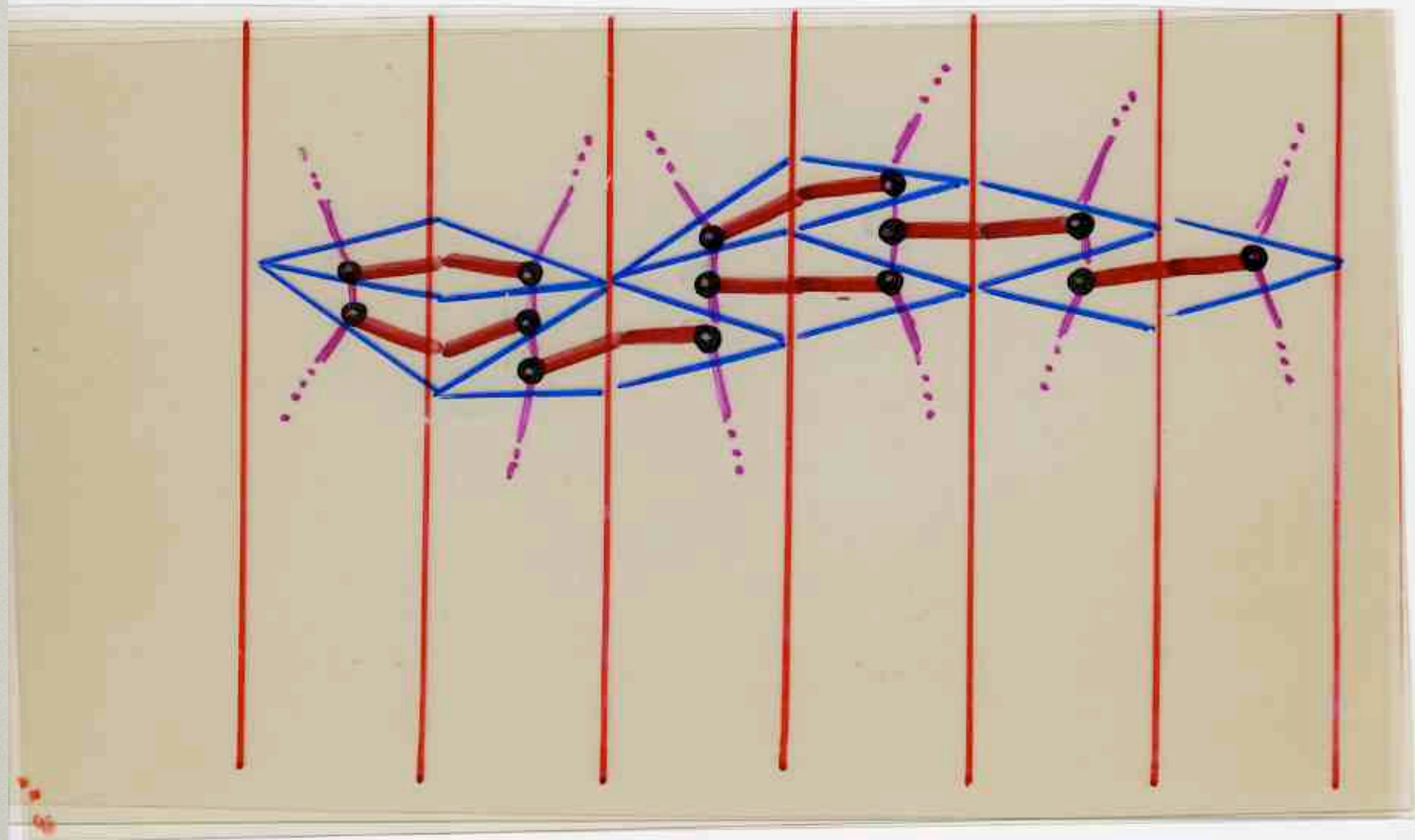


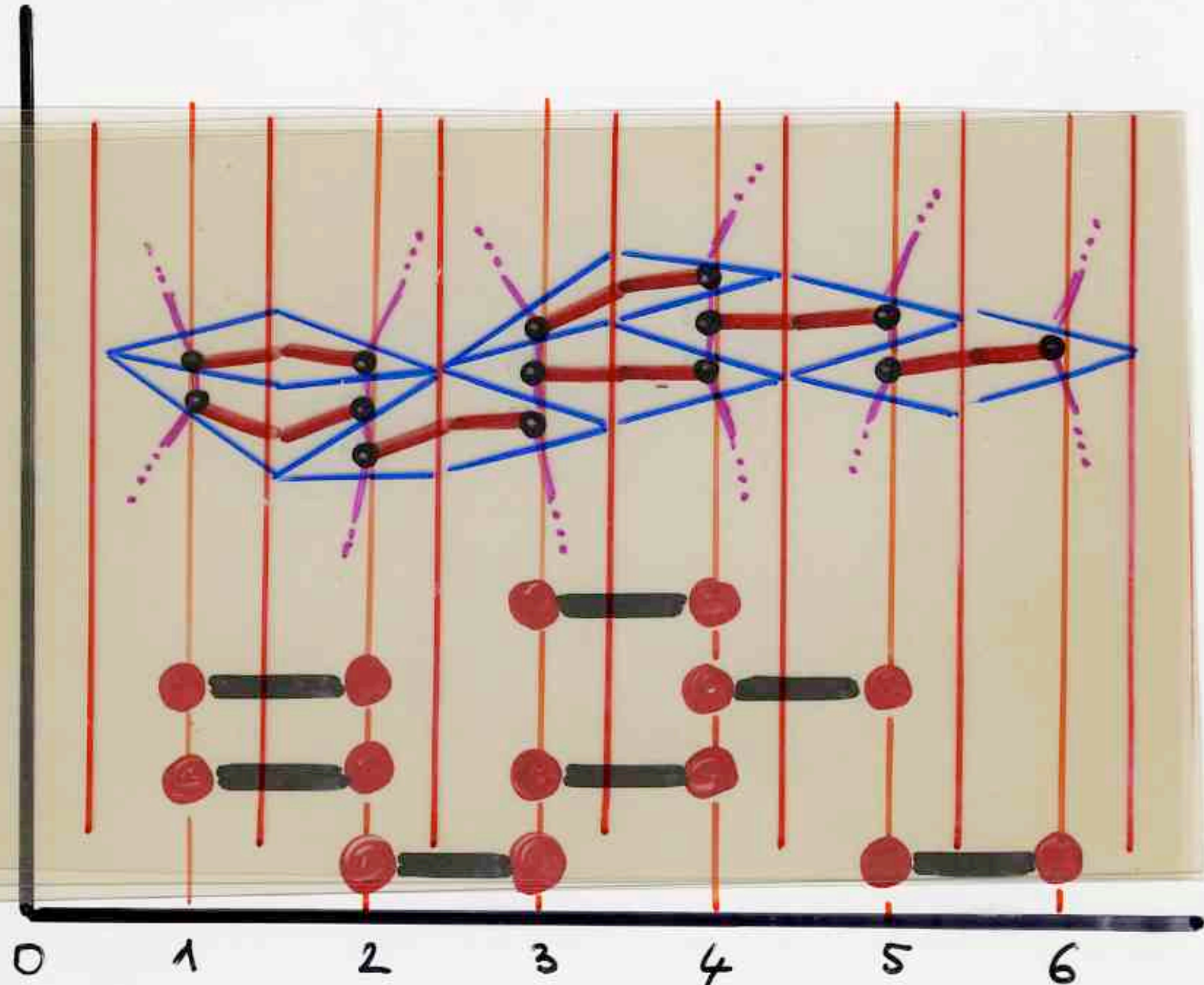
bijection réciproque

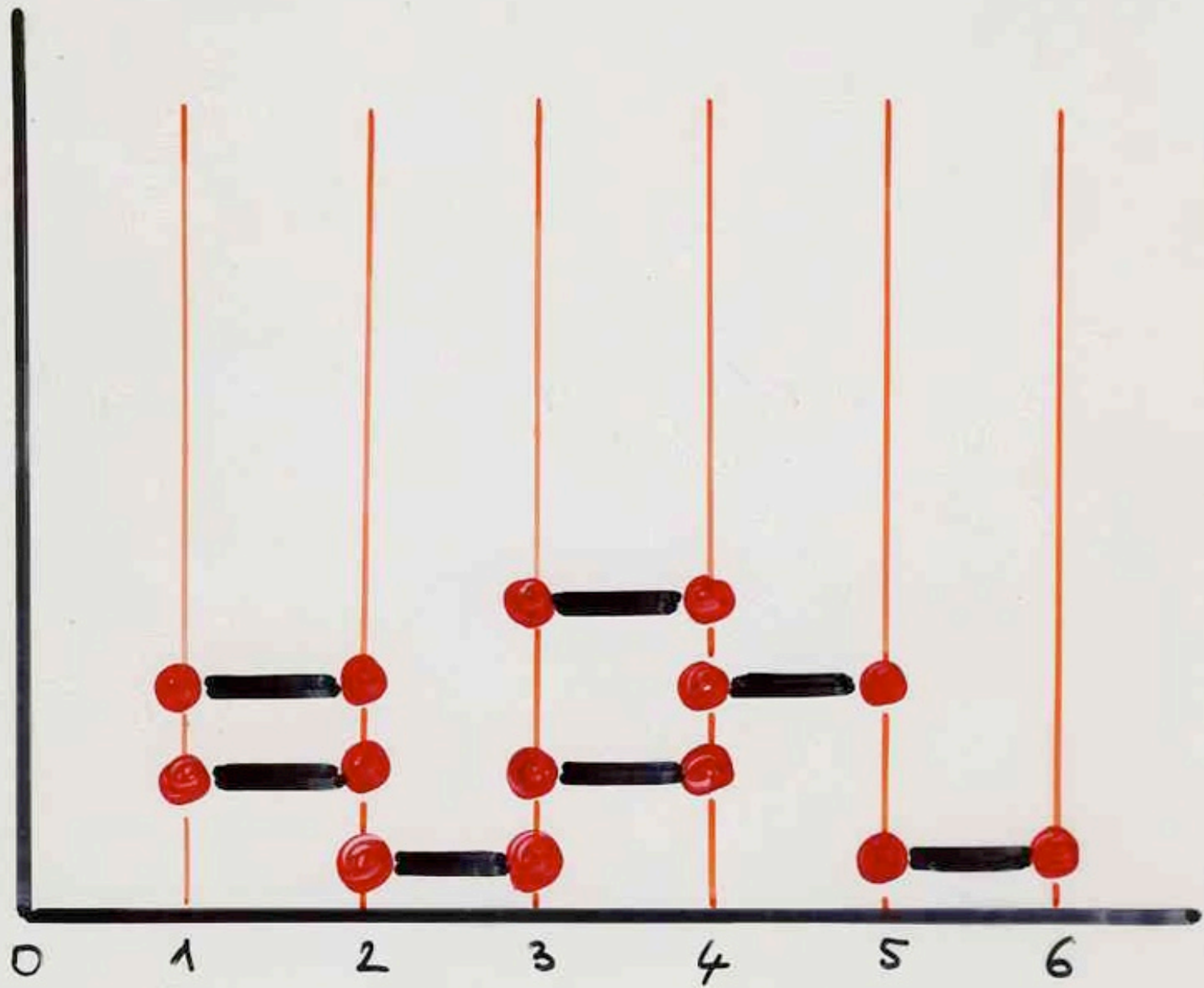












métamorphose:

triangulation

arbres binaire

chemin de Dyck

pyramide de dominos

triangulation Lorentzienne



L'ESPACE-TEMPS

MARCIA PIG LAGOS





Epilogue











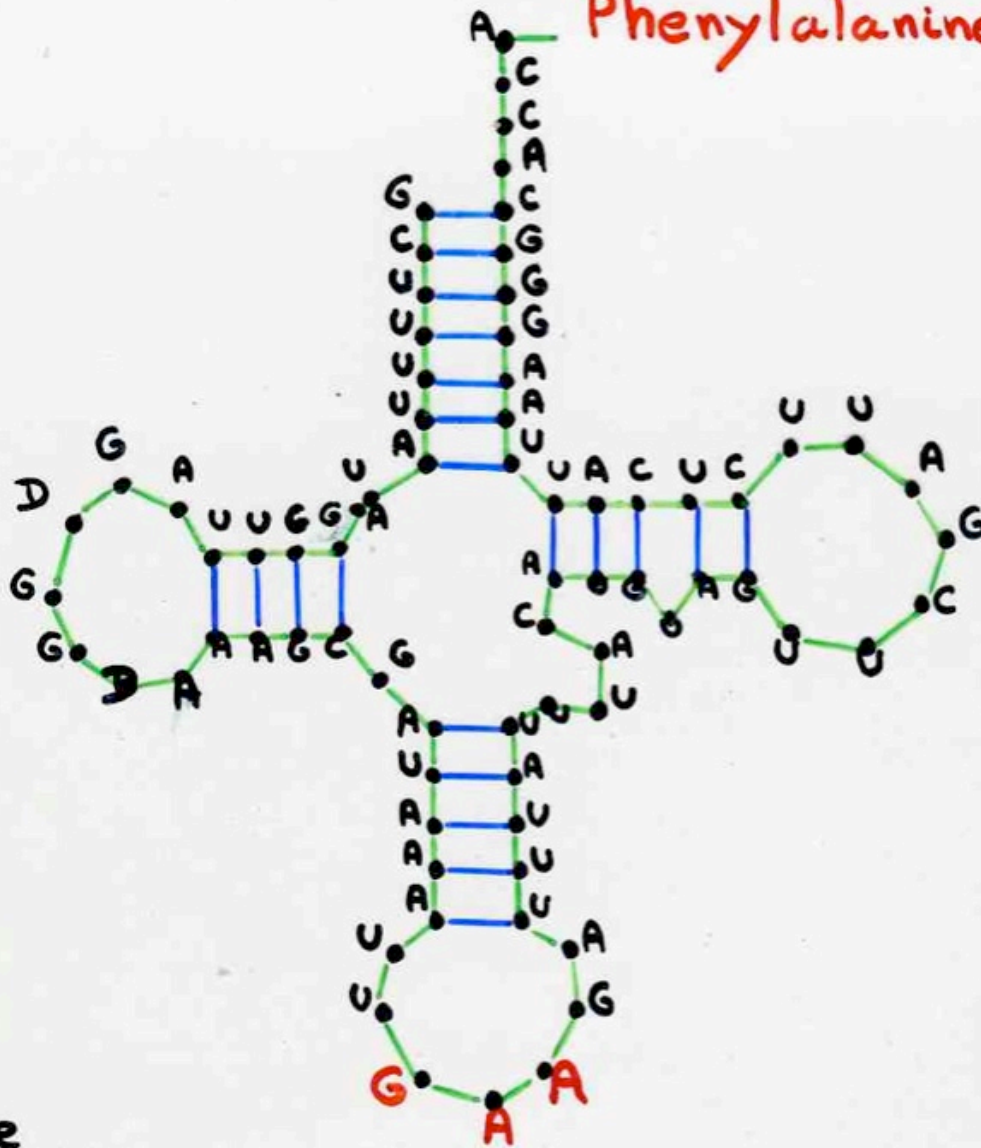
NATIONAL GEOGRAPHIC





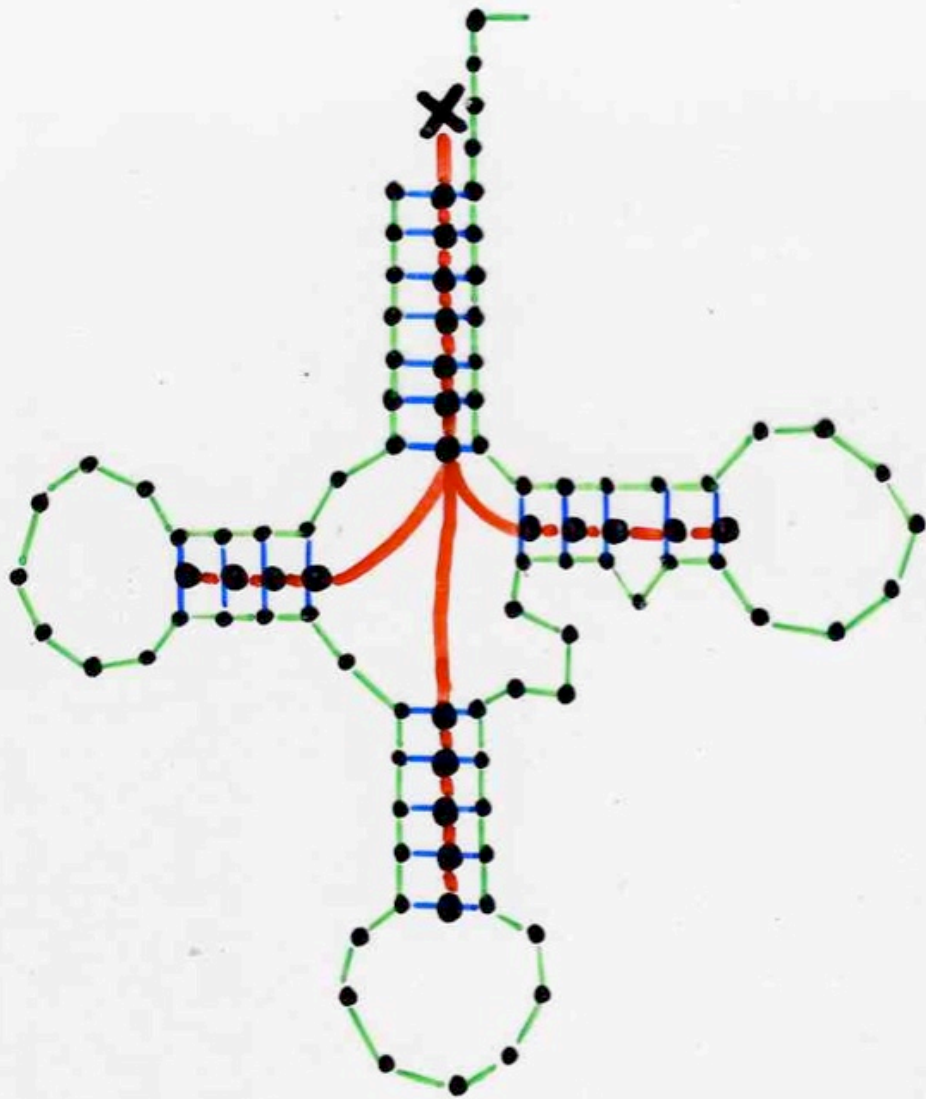
Trees everywhere

Phenylalanine

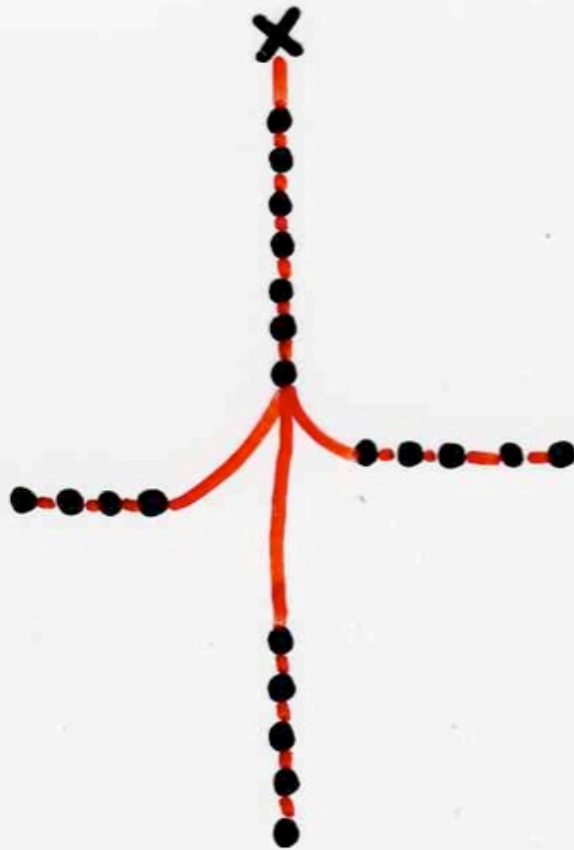


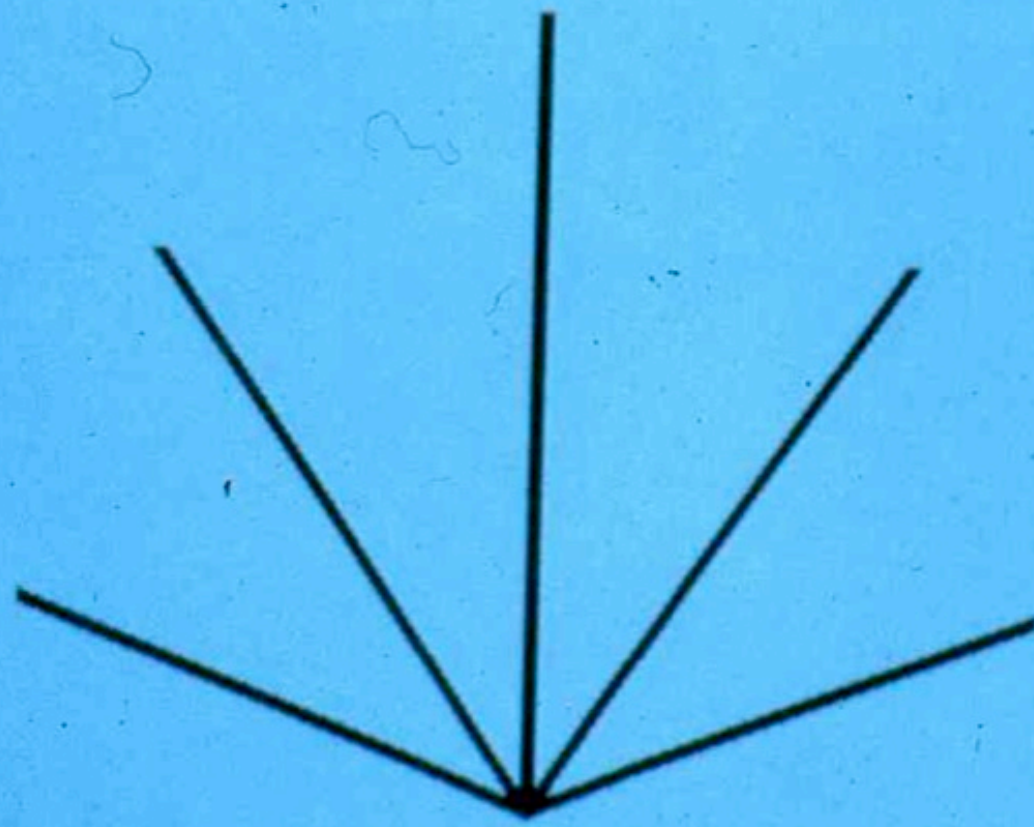
A denine
U racyle
G uanine
C ytosine

tARN^{Phe}



tARN^{Phe}















Il y a des arbres dans les étoiles,
des arbres dans les grains de lumière.

Les théories mathématiques s'interpellent,
s'entrecroisent, renaissent, se fondent entre elles.

Les grands Maîtres se parlent à travers les siècles
dans le jardin merveilleux des Mathématiques.

A night sky scene featuring a large, dark silhouette of a tree with intricate branches. The sky is filled with numerous bright, glowing stars and streaks of light, suggesting a meteor shower or a starry night. The foreground shows a rocky, uneven surface. The overall atmosphere is serene and magical.

C'est la fin
merci beaucoup !

violins:

G rard H.E. Duchamp

textes:

Le majestueux noyer

L'espace-temps

Marcia Pig Lagos

Association
Cont'Science

realisation:

Xavier Viennot

en vid o: G rard Duchamp
et Mariette Freudentheil

Remerciements

video:

atelier audiovisuel
Université Bordeaux I
Yves Descubes,
Franck Marmisse

aide technique video:

Christian Faurens,
SCRIME,
Université Bordeaux I
France

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